Restoration of Binary Images Using Positive Semidefinite Programming

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***Abstract*— We present a novel approach, using Positive Semidefinite (PSD) Programming, to restore blurred and noisy binary images when the point spread function (PSF) is known. The combinatorial nature of the problem is noted: binary image deconvolution requires the minimization of an energy function over binary variables, taking into account not only local similarity and spatial context, but also the relationship between individual pixel values and the PSF. Due to the high computational load the deconvolution process of a large image might face, we segment the binary image into smaller blocks before deconvolving each block. To suppress error propagation, we also process image blocks with different overlapping lines and columns. Superiority of the proposed PSD binary image restoration approach is confirmed by numerical experiments.**

I. Introduction

Binary image restoration is a problem arising from various applications such as stellar astronomy, fingerprint recognition, automated document handling etc. There exist numerous restoration techniques: Meloche and Zamar [1] developed the weighted mean square error (WMSE) method to restore noisy images. Hitchcock and Glasbey [2] identified a statistical model for digital image data, and proposed an inferential procedure to restore images of blob-like and filamentous objects. In other words, the method requires assumptions on the form of the image. Neifeld *et al.* [3] extended the Viterbi-based algorithm to restore binary images on a row-by-row basis. Gu *et al.* [4] used pulse coupled neural network to restore noisy binary images. Chan *et al.* [5] provided a convergent method to find a minimizer of the total-variation functional to restore noisy binary images. A major drawback of these algorithms is that they only deal with noisy binary images, whereas the more common case of image degradation, namely, blurring, is ignored. Generally, a blurred image with noise is modeled as

$$g(x,y) = f(x,y) * h(x,y) + n(x,y), \tag{1}$$

where $n(x,y)$ is an additive noise, and $f(x,y)$ and $g(x,y)$ represent the true and the final images, respectively. Here $h(x,y)$ is a linear shift-invariant blur, also known as the point spread function (PSF), and "$*$" denotes the 2-D linear convolution operator. The problem of recovering $f(x,y)$ from $g(x,y)$ is referred to as linear binary image restoration [6], [7]. Several deconvolution (deblurring) methods have been proposed such as Maximum Entropy Method [8], Weiner filter, One-Step Least Squares [9] etc. These methods suffer from problems including elimination of high-frequency components of the images as well as preprocessing requirements.

Recently, the development of Positive Semidefinite (PSD) programming has made possible the optimal solution to combinatorial problems such as binary partitioning, perceptual grouping and restoration. Keuchel *et al.* [10], [11] investigated the combinatorial problem of binary image denoising: for each pixel i in a noisy image, the pixel value g_i is known to originate from either of two prototypical values u_1 : 0 and u_2 : 255. In practice, g_i is real-valued due to noise corruption. To restore a discrete-valued image function represented by the vector $x \in \{-1,+1\}^n$ from the measurement g, the following functional is minimized:

$$\begin{aligned} z(x) &= \frac{1}{4}\sum_i ((u_2-u_1)x_i + u_2 + u_1 - 2g_i)^2 \\ &\quad + \frac{\lambda}{2}\sum_{<i,j>} (x_i - x_j)^2, \end{aligned} \tag{2}$$

in which λ is the smoothness term parameter, and the second term sums over all pairwise adjacent pixels on the regular image grid. In [12], we further explored the optimization process and developed binary image deconvolution techniques when the image is small and the size of PSFs is limited to below 3×3 because of the long time of restoring large images.

In this paper, using PSD Programming, we aim at restoring large binary images when the PSF is of arbitrary size. Moreover, *both image deblurring and denoising are simultaneously carried out*. The paper is organized as follows. Section II-A describes the binary combinatorial optimization of the problem and devises the energy function; Section II-B introduces the optimization approach to solve the combinatorial problem in Section II-A; the overlapping deconvolution method is proposed in Section II-C to deal with large binary image restoration. Experimental results are given in Section III. Finally, Section IV draws some concluding remarks and provides future insights.

II. Binary Image Restoration Using PSD Programming

In this section, we define the optimization method to solve the linear binary image restoration problem. First, we describe the binary combinatorial optimization of the problem and design the energy function. Next, we relax the combinatorial problem for computing suboptimal solutions. The overlapping deconvolution method is then introduced for handling large-size images.

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A. Combinatorial Optimization

When a true image f is blurred by a linear PSF h, the blurred image g is the convolution of f and h as in (1). Suppose the true image is of size u by v and h is a matrix of size m by n, in the computation of the convolution, h is flipped to form h_r, namely,

$$h_r(i,j) = h(m+1-i, n+1-j), \tag{3}$$

in which $h_r(i,j)$ and $h(i,j)$ denote the entries of h_r and h, respectively. For a specific pixel x_i in the discrete-valued image function represented by $x \in \{-1,+1\}^n$, the blurred pixel value would be the linear combination of x_i and its neighboring pixel values and entry values of the PSF:

$$x_i^b = \sum_{j=1}^{m}\sum_{k=1}^{n} x_{i+(j-\tilde{m})+(k-\tilde{n})u} h_r(j,k), \tag{4}$$

with $\tilde{m}$ and $\tilde{n}$ being the "center" entry of h_r, viz. $\tilde{m} = \left\lfloor \frac{m+1}{2} \right\rfloor$ and $\tilde{n} = \left\lfloor \frac{n+1}{2} \right\rfloor$. To restore the vector $x \in \{-1,+1\}^n$, the functional is changed by replacing x_i in (2) with x_i^b in the data-fitting term, and we have

$$\begin{aligned} z(x) &= \frac{1}{4}\sum_i((u_2-u_1)x_i^b + u_2 + u_1 - 2g_i)^2 \\ &\quad + \frac{\lambda}{2}\sum_{<i,j>}(x_i - x_j)^2. \end{aligned} \tag{5}$$

Up to constant terms, (5) leads to the following optimization problem:

$$\inf_x \frac{1}{4}x^T Q x + \frac{1}{2}b^T x, x \in \{-1,+1\}^n. \tag{6}$$

The constructions of b and Q are detailed below.

We use $g_{(hs,vs)}$ to denote the image shifting of g by hs lines and vs columns, in which $hs > 0$ means shifting hs lines down, otherwise $|hs|$ up; and $vs > 0$ means shifting vs columns right, otherwise $|vs|$ left. To represent $\sum_i (u_2-u_1)(u_2+u_1-2g_i)\sum_{j=1}^{m}\sum_{k=1}^{n} x_{i+(j-\tilde{m})+(k-\tilde{n})u} h_r(j,k)$ in the form of $b^T x$, b_i is defined as:

$$b_i = \sum_{j=1}^{m}\sum_{k=1}^{n}(u_2-u_1)(u_2+u_1-2g_{(j-\tilde{m},k-\tilde{n})i})h_r(j,k), \tag{7}$$

where g is the blurred image. To represent the summation

$$\sum_i \frac{1}{4}(u_2-u_1)^2\left(\sum_{j=1}^{m}\sum_{k=1}^{n} x_{i+(j-\tilde{m})+(k-\tilde{n})u} h_r(j,k)\right)^2, \tag{8}$$

it is noted that for any i, x_i^2 is either 1 or 0. The summation $\sum_i x_{i+(j-\tilde{m})+(k-\tilde{n})u} x_{i+(\hat{j}-\tilde{m})+(\hat{k}-\tilde{n})u}$, wherein $j = \hat{j}$ and $k = \hat{k}$ do not happen concurrently, differs from $\sum_i x_i x_{i+(j-\hat{j})+(k-\hat{k})u}$ in that for an arbitrary $\sum_i x_i x_{i+p+qu}$, if $p > 0$ ($p \le 0$) then the summation does not include the last p (first p) lines in the image; if $q > 0$ ($q \le 0$), the summation does not include the last q (first q) columns in the image. Q of size uv by uv is constructed in the way that when the summation region of $\sum_i x_{i+(j-\tilde{m})+(k-\tilde{n})u} x_{i+(\hat{j}-\tilde{m})+(\hat{k}-\tilde{n})u}$ is identified and we define $T = (u_2-u_1)^2/2$, for line i, $Q(i, i+(j-\hat{j})+(k-\hat{k})u)$ will be added with value $2 \times T \times h_r(j,k)h_r(\hat{j},\hat{k})$. There are altogether $\frac{mn\times(mn-1)}{2}$ summation terms in (8). The last term of (5) can also be represented in Q, namely, for every line i of Q, $Q(i,i-1)$ and $Q(i,i+1)$ is added with value -2λ.

B. Positive Semidefinite Relaxation

The objective function of (6) can be further homogenized in the following way:

$$x^T Q x + 2b^T x = \begin{pmatrix} x \\ 1 \end{pmatrix}^T L \begin{pmatrix} x \\ 1 \end{pmatrix}, L = \begin{pmatrix} Q & b \\ b^T & 0 \end{pmatrix}. \tag{9}$$

Denoting the Lagrangian multiplier variables with $y_i, i = 1,...,n$, (9) can be relaxed to

$$z_d := \sup_y e^T y, L - D(y) \in S_+^n, \tag{10}$$

where $D(y)$ denotes the diagonal matrix with y_i as the diagonal values, and e is a vector of all entries being 1 and S_+^n is the positive semidefinite cone, and therefore (10) is a convex optimization problem. The dual problem to (10) is obtained as

$$z_d := \inf_{x \in S_+^n} L \bullet X, D(X) = I, \tag{11}$$

which again is convex.

The optimization solver SeDuMi [13] is used to compute optimal solutions X^* to (11). Then the combinatorial solution x to the original problem (9) can be found by using the randomized-hyperplane technique proposed by Goemans and Williamson [14].

C. Overlapping Deconvolution Method over Image Blocks

The computation time quickly grows with the number of variables in the problem such that the restoration of binary images with size around 100×100 is impractical in reasonable time. To overcome this, we segment the image to smaller (e.g., 10×10) blocks before applying the approach to each block. An obvious problem is that without taking into account the pixels on the boundary lines or columns of the block, error inevitably increases because the computation of x_i^b requires its adjacent pixels within the range of the PSF matrix as illustrated in (4).

Consequently, a tradeoff has to be made between the computation time and quality of the restored images. To this end, we adopt a hybrid scheme with overlapping lines and columns among image blocks. For example, an image block of size 16×16 is restored but only the pixels in the center 10×10 block is retained.

III. Experimental Results

The original binary image is shown in Figure 1(a). Figures 1(b) and 1(c) are the images blurred by h_3 without and with noise, respectively, in which h_3 is $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Figures 1(d) and 1(e) are the images blurred by h_5 without and with noise, respectively, in which

TABLE I

RESTORATION ACCURACY: FIGURE 2

Figure 2	a	b	c	d
Accuracy	0.9402	0.9644	0.9857	0.9888
Figure 2	e	f	g	h
Accuracy	0.9153	0.9291	0.9358	0.9362

h_5 is a randomly generated and normalized 5×5 matrix $\begin{bmatrix} 0.0686 & 0.0550 & 0.0444 & 0.0293 & 0.0042 \\ 0.0167 & 0.0330 & 0.0572 & 0.0676 & 0.0255 \\ 0.0438 & 0.0013 & 0.0666 & 0.0662 & 0.0587 \\ 0.0351 & 0.0593 & 0.0533 & 0.0296 & 0.0007 \\ 0.0644 & 0.0321 & 0.0127 & 0.0645 & 0.0100 \end{bmatrix}$. In both cases, the added noise is white Gaussian with mean 0 and variance 0.01.

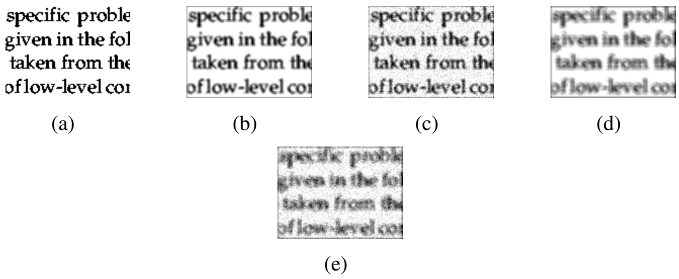

Fig. 1. (a) The true image. (b) The image blurred by h_3 and without noise. (c) The image blurred by h_3 and with noise. (d) The image blurred by h_5 and without noise. (e) The image blurred by h_5 and with noise.

Figures 2(a)-(d) show the results of restoration of Figure 1(b) without overlapping, with overlapping of 1 to 3 lines and 1 to 3 columns, respectively. Figures 2(e)-(h) show the results of restoration of Figure 1(c) without overlapping, with overlapping of 1 to 3 lines and 1 to 3 columns, respectively. Figures 3(a)-(f) show the results of restoration of Figure 1(d) without overlapping, with overlapping of 1 to 5 lines and 1 to 5 columns, respectively. Figures 3(g)-(l) show the results of restoration of Figure 1(e) without overlapping, with overlapping of 1 to 5 lines and 1 to 5 columns, respectively.

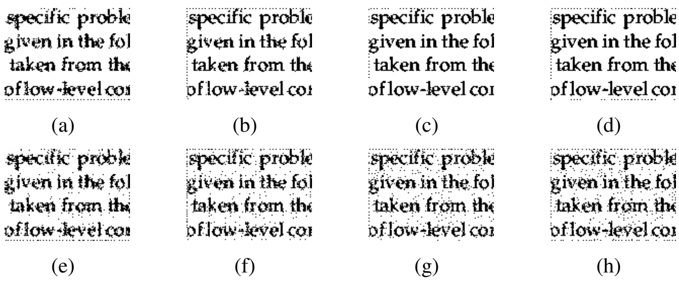

Fig. 2. Restoration results of Figure 1(b) and Figure 1(c).

From Tables I and II, where accuracy is obtained by comparing the restored image and the original image pixel by pixel, it can be seen that when the overlapping of the image block increases, the deconvolution (deblurring) has a better improvement than the denoising. This happens because in the restoration process, some pixels with small values in the blurred image will be deconvolved by the PSF to have much bigger values, and thus are recognized as 255 instead of the true value 0. The results also show that although the restorations have high accuracy, the quality of the restored images are not satisfying if the accuracy is below 90%.

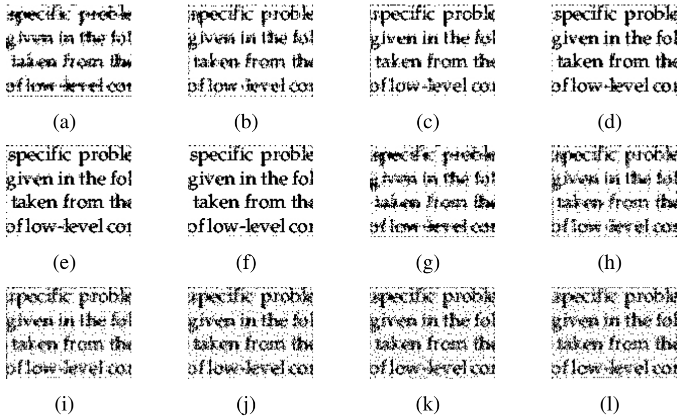

Fig. 3. Restoration results of Figure 1(d) and Figure 1(e).

TABLE II

RESTORATION ACCURACY: FIGURE 3

Figure 3	a	b	c	d	e	f
Accuracy	0.8815	0.9083	0.9287	0.9602	0.9695	0.9796
Figure 3	g	h	i	j	k	l
Accuracy	0.8606	0.8689	0.8734	0.8806	0.8764	0.8786

In summary, although the proposed optimization approach has some difficulty in restoring images with heavy noise, it restores blurred images when the noise is small with very high accuracy.

IV. CONCLUSIONS AND FUTURE WORK

This paper has presented a novel binary image restoration technique using Positive Semidefinite (PSD) Programming. Deblurring and denoising are performed simultaneously. For restoration of large images, we have adopted an overlapping scheme that segments the image into smaller-size blocks before applying the technique to each block. Advantages of the proposed optimization method are threefold: it is accurate, noise resistive, and simple to code without introducing extra parameters. Numerical experiments have demonstrated the excellent performance of the proposed technique in binary image restoration. Future work might lead to blind restoration wherein the point spread function (PSF) is unknown.

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