

DIRECT RECONSTRUCTION OF SPIRAL MRI USING LEAST SQUARES QUANTIZATION TABLE

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ABSTRACT

The least squares quantization table (LSQT) method is proposed to accelerate the direct Fourier transform for reconstructing images from nonuniformly sampled data, similar to the look-up table (LUT) and equal-phase-line (EPL) methods published recently. First, it classifies all the image pixels into several groups using the Lloyd-Max quantization scheme, and stores the representative value of each group in a small-size LSQT in advance. For each k -space data, the contribution is calculated only once for each group. Then, each image pixel is mapped into the nearest group and uses its representative value. The experiments show that the LSQT method requires far smaller memory size than the LUT method. Moreover, it is superior to the EPL and Kaiser-Bessel gridding methods in minimizing reconstruction error and requires fewer complex multiplications than the LUT and EPL methods. Additionally, the inherent parallel structure makes the LSQT method easily adaptable to a multiprocessor system.

Keywords: Image reconstruction, spiral MRI, Lloyd-Max quantization, least squares quantization table.

1. INTRODUCTION

Many methods have been proposed for the reconstruction of MRI data acquired using a non-Cartesian k -space trajectory, such as spiral [1, 2, 3, 4, 5]. As the most straightforward and accuracy solution, the direct Fourier transform method has some advantages over other methods such as non-uniform fast Fourier transform (NUFFT) and Kaiser-Bessel gridding [5, 6]. Firstly, it does not require convolution and post-compensation, and therefore minimizes the possibilities of introducing reconstruction errors. Secondly, the reconstructed image can be updated immediately following the most recent information after each individual data point is acquired. This would be important in applications such as MR fluoroscopy. Unfortunately, the high computational demand makes it impractical compared with methods that use the fast Fourier transform (FFT) [2, 3].

The Equal-Phase-Line (EPL) method and look-up table (LUT) method are two algorithms recently proposed to ac-

celerate the direct Fourier transform method [6, 7]. The EPL method distributes the contribution of a data point to the image pixels according to equal-phase lines defined by uniformly quantizing the phase interval of complex exponential function. This method however does not take into account the actual distribution of phases for each data acquired using non-Cartesian k -space trajectories, and therefore may cause a large quantization error and consequently a large reconstruction quality loss. Meanwhile, the LUT method is only efficient for small size images due to the huge memory required for storing a look-up table with size of $N^2 \times L$, where N^2 is the number of pixels in the image and L is the number of k -space data. In [7], where this method was proposed, the sizes of k -space data and reconstructed image in experiments are all 32×32 . The reconstruction time for larger size images were estimated by extrapolation. If the size of the look-up table can be reduced so that it is feasible for larger size images with only a little quality loss, the LUT method can still be appealing.

In this paper, we propose a least squares quantization table (LSQT) method to accelerate the direct Fourier transform to reconstruct MR images acquired using a spiral trajectory. Because the data in k -space contributes almost equally to various image pixels if they produce roughly the same phase, we can quantize the phase interval of the complex exponential function using the Lloyd-Max quantization technique and use the quantized values for reconstruction. In order to reduce the reconstruction loads, the representative values of each group are calculated and stored in a small size table in advance.

2. THE LEAST SQUARES QUANTIZATION TABLE METHOD

The direct Fourier transform method to reconstruct image I of size $N \times N$ from k -space data s with length L is [3, 6]:

$$I(x, y) = \sum_{p=1}^L s_p d_p \exp(j2\pi(xu_p + yv_p)), \quad (1)$$

where $x, y = [-N/2 : N/2 - 1]$ and $u, v = [-1/2 : 1/2]$, and d_p is the density compensation function. The contribution

of the p th data to the entire image space is therefore:

$$I_p(x, y) = s_p d_p \exp(j2\pi(xu_p + yv_p)). \quad (2)$$

It can be seen from (2) that for a given $\{u_p, v_p\}$, if two pixels (x_1, y_1) and (x_2, y_2) have the same phase such that $2\pi(x_1u_p + y_1v_p) = 2\pi(x_2u_p + y_2v_p)$, the data s_p has the same contribution to the two pixel locations. Also, if we define $C_p = xu_p + yv_p$, because of the periodic property of the complex exponential function, we can concentrate on the main phase band $[0, 2\pi)$, which corresponds to the decimal part of C_p in $[0, 1)$ denoted as $\langle C_p \rangle$. If C_p is negative, we add an integer to it to bring it to between 0 and 1, and $\langle C_p \rangle$ then is the resulting decimal part. Thus, each pixel (x, y) has a one-to-one correspondence to $\langle C_p \rangle$.

It is noted that if some pixels have the same $\langle C_p \rangle$, the acquired raw data in k -space has the same contribution to these pixels. Thus, all the pixels can be classified into groups, where each group is labeled by a different representative value and pixels in the same group receive the same contribution. If the contributions of the given data to all the groups are known, for a pixel, we only need to know to which group it belongs. Consequently, (2) is calculated only once for each group instead of for each pixel in the direct Fourier transform method, which results in the reduction of computer loads without loss for accuracy. However, in practice the reduction is not substantial because the number of groups is usually just a bit smaller than the number of pixels [6].

We observe that it is possible to decrease the number of groups if we can accept a small loss of image quality. The tradeoff lies in the number of groups. Instead of requiring an exact value of $\langle C_p \rangle$ before two pixels can be classified into the same group, we use the Lloyd-Max quantization algorithm to classify all the pixels into only M groups where $M \ll N^2$. The group boundaries are calculated by quantizing the interval $[0, 1)$ into M bins in the least squares sense of quantization error [8, 9]. The representative value of each group is the centroid of the corresponding group, which is the optimal point to give the lowest quality loss.

Therefore, for the k -space data s with length L , we can pre-compute a least squares quantization table (LSQT) \mathcal{Q} of size $M \times L$, which maps each of the k -space location to a particular group and representative value. The construction of table can be accomplished off-line and reused for the same k -space trajectory, being independent of the object being imaged. After loading the table, when a data arrives, the contributions of the data to all the groups can be computed as

$$b_p(i) = s_p d_p \exp(j2\pi\mathcal{Q}(i, p)), \quad i = 1, 2, \dots, M. \quad (3)$$

If we define a larger M , the quantization error and reconstruction error will be smaller, but the LSQT size and required memory will be larger. Conversely, a smaller M needs less memory but gives a larger reconstruction error.

To complete the algorithm, we need to define a way to map each pixel to the corresponding group. The way is to

Table 1. The representative values of 16 groups of $\langle C_{745} \rangle$ using uniform quantization and Lloyd-Max quantization.

i	Uniform	Least squares	i	Uniform	Least squares
1	0	0.00004663	9	0.5	0.99922
2	0.0625	0.0001399	10	0.5625	0.99937
3	0.125	0.0002333	11	0.625	0.99948
4	0.1875	0.000327	12	0.6875	0.99958
5	0.25	0.0004202	13	0.75	0.99967
6	0.3175	0.0005173	14	0.8125	0.99977
7	0.375	0.0006313	15	0.875	0.99986
8	0.4375	0.0007843	16	0.9375	0.99995

search for an entry in the table whose representative value is closest to the current $\langle C_p \rangle$ [9], such as by using a binary-searching algorithm. Suppose the pixel (x, y) is mapped to the k th group, the contribution of the p th data to the pixel (x, y) will be approximated by $b_p(k)$. Simultaneously, considering the symmetric and periodic properties of $\langle C_p \rangle$, i.e. $\langle C_p(-x, -y) \rangle = -\langle C_p(x, y) \rangle$ and also $\langle C_p(-x, y) \rangle = -\langle C_p(x, -y) \rangle$, pixel $(-x, -y)$ can be classified into the $(M-k)$ th group, and can set the corresponding value $b_p(M-k)$ directly. Therefore, only pixels where $x \geq 0$ need to perform mapping [6].

3. EXPERIMENTAL RESULTS AND DISCUSSION

In our experiments, the actual scanned spiral data are obtained from <http://www-mrsrl.stanford.edu/~brian/gridding/>, which contains 13,392 complex data. The size of the image is 256×256 , and M takes the values 16, 32, 64 and 128.

Firstly, we illustrate the difference between uniform quantization and least squares quantization algorithms using, say, the 745th data. The original $\langle C_{745} \rangle$'s do not locate evenly in the entire interval $[0, 1)$, but rather only in two regions $[0, 0.00094969]$ and $[0.99905, 0.99999]$. Table 1 gives the representative values of 16 groups of $\langle C_{745} \rangle$ obtained using two different quantization algorithms. This table can be regarded as a codebook. We see that the representative values of uniform quantization algorithm are evenly distributed in the entire interval $[0, 1)$ while those of least squares quantization algorithm are distributed in the two regions where the original $\langle C_{745} \rangle$'s locate. It means the least squares quantization algorithm takes into account the actual distribution of original $\langle C_{745} \rangle$. It should be noted that when performing quantization operation, the values that are smaller to the first representative value or larger than the last representative value will be quantized to the first bin due to the periodic property of the complex exponential function. For example, when the original value of one $\langle C_{745} \rangle$ is 0.00094969, it is quantized to the first bin for uniform quantization algorithm with quantization error 0.00094969. The corresponding pixel is classified into the first group with $b_{745}(1) = s_{745}d_{745}$ according to (3). While

for least squares quantization algorithm, this (C_{745}) is quantized to the eighth bin with quantization error 0.00016539. The corresponding pixel is classified into the eighth group with $b_{745}(8) = s_{745}d_{745} \exp(j2\pi \times 0.0007843)$. Comparing the two, the least squares quantization algorithm gives the more accurate quantization result and contribution. Furthermore, all the 256×256 pixels for the 745th data are classified into one group receiving the same contribution using uniform quantization algorithm, while 16 groups using least squares quantization algorithm. In our experiments, for $M = 16, 32, 64$ and 128 , the quantization error of $256^2 \times 13392$ decimal parts for least squares quantization algorithm is 31.98%, 30.7%, 29.11% and 27.46% of that for uniform quantization.

The ideal image obtained with the direct Fourier transform method (see Fig. 1(a)) is used as a standard to compare the LSQT method with the EPL and Kaiser-Bessel gridding method. All methods are implemented in MATLAB and C++ languages and performed on a personal computer (Intel Pentium 4 2.4GHz processor with 512MB of RAM). The absolute difference image, normalized Root Mean Square (nRMS) error and Maximum Absolute Difference (MAD) between the ideal and reconstructed images are used to evaluate the reconstruction performance [4]. The reconstructed images are scaled to a range of gray levels [0–255] and MAD is normalized by dividing by 255. Fig. 1(b) is the absolute difference image amplified 100 times of the Kaiser-Bessel gridding method with oversampling ratio 2, kernel width 3 and shape parameter 13.9086, which is the optimal combination recommended in [2].

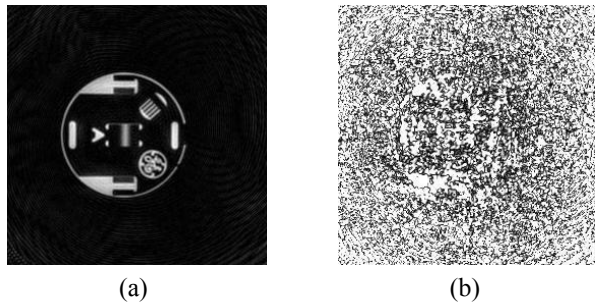


Fig. 1. (a) The “ideal” image obtained by using the direct Fourier transform method; (b) The absolute difference image amplified 100 times for the Kaiser-Bessel gridding method.

Fig. 2 shows the absolute difference images of the EPL and LSQT methods, where the first row is for the EPL method and the second row is for the LSQT method. All difference images are again amplified 100 times to improve contrast visibility. M takes the value of 16, 32, 64, and 128 for the first, second, third, and fourth columns respectively. For the EPL and LSQT with the same M , the latter distributes the error more evenly and shows a lower gray level for the peak difference region than the former. Also, with a larger M for each

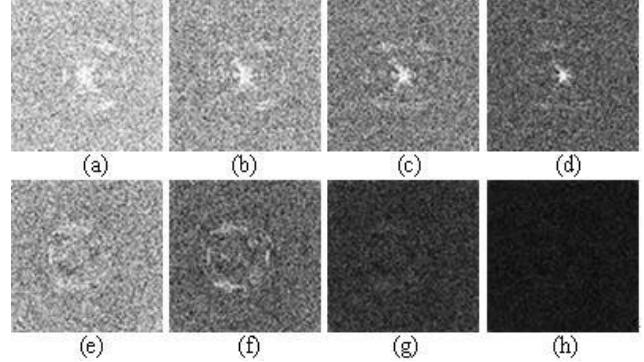


Fig. 2. The absolute difference images between reconstructed images and ideal image. The first row (a)–(d) (where M takes 16, 32, 64 and 128 respectively) is for the EPL method, and the second row (e)–(h) is for the LSQT method. All difference images are amplified 100 times.

method, the absolute difference is distributed more evenly over the entire image and its gray level becomes lower.

Fig. 3 (a) shows the nRMS against the required memory of the LSQT method for different M , and (b) and (c) show the nRMS and MAD of different methods against the number of groups. Firstly, for the LSQT method, the required memories for loading a table are about 3.27MB, 6.54MB, 13.08MB and 26.16MB for $M = 16, 32, 64,$ and 128 respectively. This is compared to 13,392MB for the LUT method. It is clear that reconstructing an image with such a size using the LUT method is impractical. Put another way, the required 16MB for constructing a 32×32 image from 1,024 data in [7] is enough to reconstruct a 256×256 image from a 13,392 data point spiral trajectory with $M = 64$ in our experiments. Secondly, the selection of M is flexible in trading off required memory against reconstruction error in our method. The LSQT method can be customized for the particular application by selecting a predetermined number of groups that corresponds to the desired required memory. If a high accuracy is desired, it is easy to implement by increasing the number of groups. Thirdly, the LSQT method is superior to the EPL and Kaiser-Bessel gridding method in reducing reconstruction error when M takes an appropriate value. Moreover, from fig. 1(b) to fig. 3, the LSQT method even when $M = 16$ performs better than the Kaiser-Bessel gridding.

When mapping the data s to image I , it requires about $(5/2)N \times N \times L$ complex multiplications for the direct Fourier transform method, with one real multiplication counted as $1/4$ complex multiplication [6]. For the LUT method, there are $N \times N \times L$ complex multiplications. The EPL method requires $L(1/2 + (3/2)M + (3/8)N \times N)$ and the LSQT method requires $L(1/2 + (3/2)M + (1/4)N \times N)$ complex multiplications. Because $M \ll N^2$, the required complex

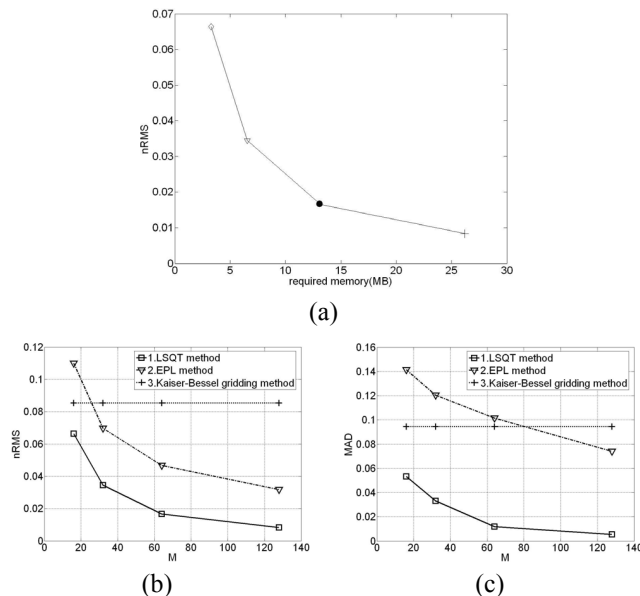


Fig. 3. (a) The nRMS against the required memory of the LSQT method for different M (“ \diamond ” — 16, “ ∇ ” — 32, “ \bullet ” — 64 and “ $+$ ” — 128); (b) and (c) The nRMS and MAD of different methods against the number of groups.

multiplications in the EPL and LSQT methods are approximately about 3/20 and 1/10 of that of direct Fourier transform.

When realizing these methods, the total reconstruction time includes three parts: calculating contribution for each data(3), computing $\langle C_p \rangle$ for each data and image pixel, and mapping pixels to the corresponding groups (no multiplication involved in this part). In our implementation, the direct Fourier transform time takes about 460s, and the EPL and LSQT are about 151s and 211s respectively. The extra time is in mapping operations in the EPL and LSQT methods. We do not compare the time for the LUT method because the large size table cannot be loaded into the memory. Though presently the LSQT method is more time-consuming than the Kaiser-Bessel gridding method, it could become faster than the latter theoretically with a sufficient number of processing units. The reason is that the table-based methods are highly amenable to parallelization, while in Kaiser-Bessel gridding method only the column FFTs could be parallelized for separate processors [7]. Some researchers have explored multiprocessor systems for reconstruction from nonuniformly sampled data [10]. Therefore, the inherent parallel structure can speed up the LSQT method by using a multiprocessor system.

4. CONCLUSION

In this paper, a LSQT based method is proposed to accelerate the direct reconstruction of spiral MRI with only a small loss of quality. It requires far smaller memory than the LUT

method when reconstructing images of the same size, which means it can be used to reconstruct large size images. Compared with the EPL and Kaiser-Bessel gridding method, the LSQT method provides reconstructions that are more accurate when an appropriate parameter is chosen. Additionally, it requires fewer operations of complex multiplications than the LUT and EPL methods. Thus, the LSQT method can be used to reconstruct large size high quality images and may be sped up by using a more efficient searching algorithm and a multiprocessor system.

5. REFERENCES

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