

# Spiral MRI Reconstruction using Least Squares Quantization Table

Dong Liang, Edmund Y. Lam, George S. K. Fung, and Xin Zhang

Department of Electrical and Electronic Engineering  
The University of Hong Kong, Pokfulam Road, Hong Kong  
{dliang, elam, skfung, xinzhang}@eee.hku.hk

**Abstract.** Recently, the authors introduced least squares quantization table (LSQT) method to accelerate the direct Fourier transform to reconstruct magnetic resonance images acquired using a spiral trajectory. In this paper, we will discuss the LSQT further in its adaptability, reusability and choice of the number of groups. The experimental results show that the LSQT method has better adaptability for the different reconstruction cases than the equal phase line (EPL) and Kaiser-Bessel gridding methods. Additionally, it can be reused for reconstructing different images of varied sizes.

**Key words:** Image reconstruction; spiral trajectory; least squares quantization table; adaptability; reusability.

## 1 Introduction

Recently, non-Cartesian scanning in  $k$ -space, such as spiral, has received increased attention in Magnetic resonance imaging (MRI). However, when the data is acquired using a non-Cartesian  $k$ -space trajectory, image reconstruction can no longer be accomplished with a simple FFT. Therefore, the problem of reconstructing image from a set of nonuniformly sampled data has drawn more attention in past few years. The Kaiser-Bessel gridding is the most widely used method to solve this problem, where nonuniformly sampled data are first re-sampled onto a Cartesian grid and then a FFT is applied to reconstruct the image. But too many steps in Kaiser-Bessel gridding increase the possibilities of introducing reconstruction errors. As the most straightforward and accuracy solution, the direct Fourier transform [1] has some advantages over other methods. Unfortunately, the high computational demand makes it impractical compared with methods that use the fast Fourier transform (FFT) [2]. Two algorithms, Equal-Phase-Line (EPL) and look-up table (LUT), were proposed to solve this problem [3], [4]. The EPL method does not take into account the actual distribution of phases for each data, and therefore may cause a large quantization error and consequently a large reconstruction quality loss. Meanwhile, the LUT method is only efficient for small size images due to the huge memory required for storing a large size look-up table.

More recently, the authors introduced a new algorithm for accelerating the direct Fourier transform to reconstruct MR images acquired using a spiral trajectory. The algorithm, called least squares quantization table (LSQT) [5], reduces the computation loads of direct Fourier transform with only a little quality loss. The results were shown to be more accurate than EPL and require less memory than LUT. In this paper, we will report further results in the LSQT method in terms of its adaptability, reusability and choice of the number of groups.

## 2 The Least Squares Quantization Table Method

We summarize the LSQT method in this section. Further details can be found in [5].

When direct Fourier transform method is applied to reconstruct image  $I$  of size  $N \times N$  from  $k$ -space data  $s$  with length  $L$ . The contribution of the  $p$ th data to the entire image space is [3]:

$$I_p(x, y) = s_p d_p \exp(j2\pi(xu_p + yv_p)) \quad (1)$$

Where  $x, y = [-N/2 : N/2 - 1]$  is image pixel and  $u, v = [-1/2 : 1/2]$  is sampling position in  $k$ -space.  $d_p$  is the density compensation function. It can be seen from (1) that for a given  $(u_p, v_p)$ , if two pixels  $(x_1, y_1)$  and  $(x_2, y_2)$  have the same phase such that  $2\pi(x_1u_p + y_1v_p) = 2\pi(x_2u_p + y_2v_p)$ , the data  $s_p$  has the same contribution to the two pixel locations. Also, if we define  $C_p = xu_p + yv_p$ , because of the periodic property of the complex exponential function,  $C_p$  is also a periodic function with unity period and we can concentrate on the main phase band  $[0, 2\pi)$ , corresponding to the fractional part of  $C_p$  in  $[0, 1)$ , which we denote as  $\langle C_p \rangle$ . If  $C_p$  is negative, we add an integer to it to bring it to between 0 and 1, and  $\langle C_p \rangle$  then is the resulting fractional part. Thus, each pixel  $(x, y)$  has a one-to-one correspondence to  $\langle C_p \rangle$ .

It is noted that if some pixels have the same  $\langle C_p \rangle$ , the acquired raw data in  $k$ -space has the same contribution to these pixels. Thus, all the pixels can be classified into groups, where each group is labeled by a different representative value and pixels in the same group receive the same contribution. If the contributions of the given data to all the groups are known, for a pixel, we only need to know which group it belongs to. Consequently, (1) is calculated only once for each group instead of for each pixel in the direct Fourier transform method. Therefore, the tradeoff between the reduction of computer loads and loss of image quality lies in the number of groups. Instead of requiring an exact value of  $\langle C_p \rangle$  before two pixels can be classified into the same group, we use the Lloyd-Max quantization algorithm to classify all the pixels into only  $M$  groups where  $M \ll N^2$ . The group boundaries are calculated by quantizing the interval  $[0, 1)$  into  $M$  bins in the least squares sense of quantization error [6]. The representative value of each group is the centroid of the corresponding group, which is the optimal point to give the lowest quality loss.

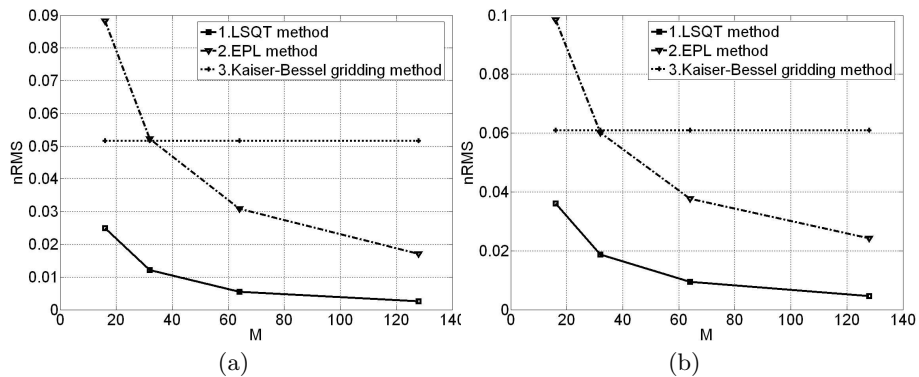
Therefore, for the  $k$ -space data  $s$  with length  $L$ , we can pre-compute a least squares quantization table (LSQT)  $Q$  of size  $M \times L$ . The construction of table

can be accomplished off-line and reused for the same  $k$ -space trajectory, being independent of the object being imaged. After loading the table, when a data arrives, the contributions of the data to all the groups can be computed. Then for a pixel, we use binary-searching algorithm to map it to the corresponding group and set contribution directly. Considering the symmetric and periodic properties of  $\langle C_p \rangle$ , only pixels where  $x \geq 0$  need to undergo mapping [3].

### 3 Results on Adaptability and Reusability

Our experiments were performed with the Shepp and Logan phantom [7] with three different sizes,  $N = 64, 128$  and  $256$ . Spiral trajectory used in [5] was applied to generate  $k$ -space data from phantoms. The size of  $k$ -space data is 13,392. Then there are three cases of image reconstruction:  $L > N \times N$ ,  $L \approx N \times N$  and  $L < N \times N$ .  $M$ , the number of groups takes the values 16, 32, 64 and 128 as in [5].

Similar to that in [5], the reconstructed image obtained with the direct Fourier transform is used as a standard. The normalized Root Mean Square (nRMS) error is calculated after reconstructed images are scaled to a range of gray levels [0,255] and the Maximum Absolute Difference (MAD) is normalized by dividing by 255.

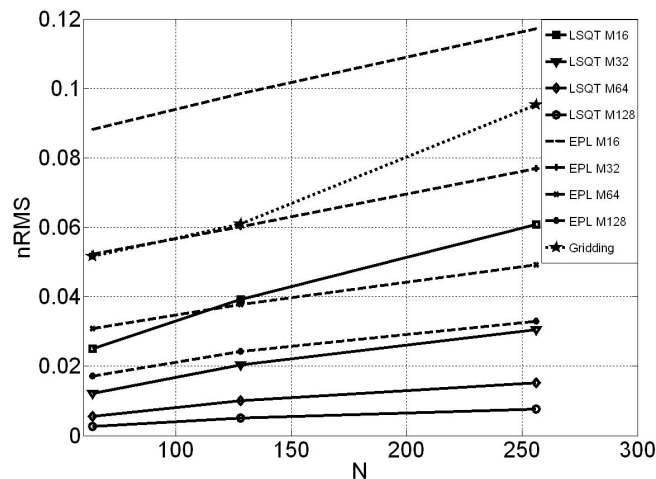


**Fig. 1.** (a) and (b) The nRMS of different methods against the number of groups for reconstructing  $N=64$  and  $N=128$  phantoms, respectively.

Fig. 1 (a) and (b) shows the nRMS of different methods against the number of groups for reconstructing  $N = 64$  and  $N = 128$  phantoms, respectively. The performance of reconstructing  $N = 256$  image from data acquired using this spiral trajectory can be found in [5]. For these cases, we can draw the same conclusion that the LSQT method can give more accurate reconstruction results than EPL and Kaiser-Bessel gridding methods when  $M$  takes an appropriate value. Moreover, the LSQT method even when  $M = 16$  performs better than

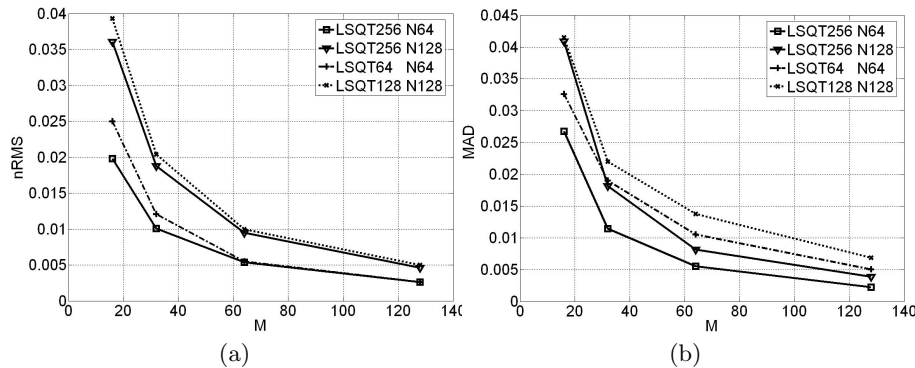
the Kaiser-Bessel gridding. The Kaiser-Bessel gridding used here is the same as that used in [5].

Next we will explore the adaptability of different methods when facing different reconstructing cases, from oversampling to undersampling. Fig. 2 shows the nRMS against the size of reconstructed phantoms for the LSQT, EPL and gridding methods with different  $M$ , where the solid lines are for the LSQT, dashed lines for the EPL and dotted line for the Kaiser-Bessel gridding. Theoretically, the nRMS error will become larger with increasing reconstructed image size while the number of  $k$ -space data is fixed, because the reconstruction case varies from  $L > N \times N$  to  $L < N \times N$ . We can see from this figure that, the nRMS curves for the LSQT are flatter than those of EPL and Kaiser-Bessel gridding methods with the same  $M$  in general. It means compared to other two methods, the LSQT has better adaptability for the different reconstruction cases. The reasons are because the actual distribution of  $\langle C_p \rangle$  is taken into account and Lloyd-Max quantization algorithm is used.



**Fig. 2.** The nRMS against the size of reconstructed Shepp and Logan phantoms for different methods with different  $M$ .

As we described above, because constructing LSQT is only dependent on the  $k$ -space sampling positions and image pixel positions, the LSQT constructed for a given trajectory can be reused for reconstructing different images with the same size. Moreover, another advantage of LSQT method is that the LSQT constructed in the case of  $L < N \times N$  can be reused in the case of  $L \approx N \times N$  and  $L > N \times N$  where  $L$  is fixed. Fig. 3 (a) and (b) show the nRMS and MAD of LSQT method against the number of group for reconstructing different size of phantoms with different LSQTs. Here, LSQT256 N64 and LSQT256 N128 mean reconstructing a  $64 \times 64$  or  $128 \times 128$  phantom respectively but with a LSQT constructed for reconstructing a  $256 \times 256$  phantom from the same



**Fig. 3.** (a) and (b) The nRMS and MAD of LSQT method against the number of groups for reconstructing different size of phantoms with different LSQT's.

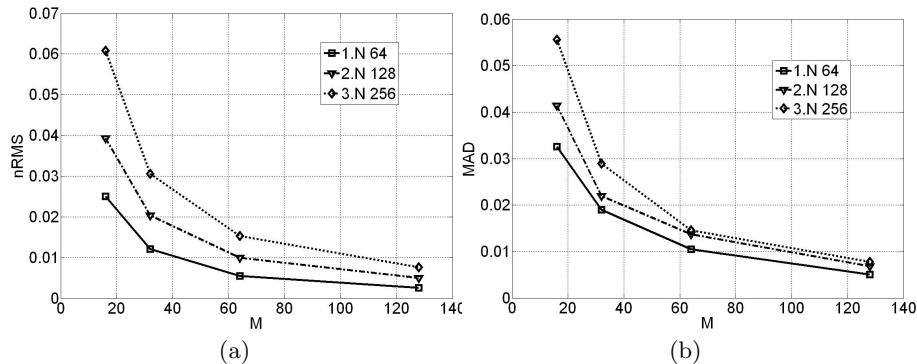
$k$ -space data similarly. LSQT64 N64 and LSQT128 N128 mean reconstruct a  $64 \times 64$  or  $128 \times 128$  phantom with their own LSQTs. It can be seen that reconstructing  $64 \times 64$  and  $128 \times 128$  phantoms with LSQT256 performs better than with their own LSQTs in nRMS and MAD. The reason is that each element of LSQT256 is obtained from  $256 \times 256$   $\langle C_p \rangle$ 's of interval  $[0, 1)$  while each element of LSQT64/128 is obtained from  $64 \times 64$  or  $128 \times 128$   $\langle C_p \rangle$ 's of interval  $[0, 1)$ . Obviously, the representative values stored in LSQT256 are more precise than those stored in LSQT64/128 when  $\langle C_p \rangle$ 's have similar distribution. This advantage means the LSQT method can be reused not only for reconstructing different images with the same size, but for reconstructing different images of reduced size.

The selection of the number of group controls the quantization precision and the size of LSQT, and hence the reconstruction accuracy and required memory. Therefore, the selection of  $M$  is flexible in trading off required memory against reconstruction error in our method. It means the LSQT method can be customized for the particular application by selecting a predetermined number of groups that corresponds to the desired required memory. Additionally, the selection of  $M$  avoids having the user choose more specific parameters of reconstruction, like kernel width or neighborhood definition like other methods [2], [8]. Fig. 4 illustrates the effect of the number of groups on the nRMS and MAD for reconstructing different size of Shepp and Logan phantoms when the size of  $k$ -space is fixed. The figure suggests that when  $L > N \times N$ ,  $M$  can take a small value while when  $L < N \times N$ ,  $M$  should take a larger value to retain a substantial reduction in reconstruction error.

## 4 Conclusion

In this paper, The LSQT method proposed recently by the authors is explored in further detail in its adaptability, reusability and choice of the number of

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**Fig. 4.** (a) and (b) The nRMS and MAD against the number of groups for reconstructing different size of Shepp and Logan phantoms when the size of  $k$ -space is fixed.

groups. The experimental results show further that LSQT performs better in reconstruction accuracy than EPL and Kaiser-Bessel gridding methods and has better adaptability than EPL when facing different reconstruction cases. Moreover, LSQT can be reused for reconstructing different images of reduced size.

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