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High-resolution Reconstruction of Human Brain MRI Image based on Local Polynomial Regression

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Abstract—**This paper introduces a new local polynomial regression (LPR)-based high-resolution image reconstruction method for human brain magnetic resonance images. In LPR, the image pixels are modeled locally by a polynomial using least-squares (LS) criterion with a kernel having a certain bandwidth matrix. Steering kernels with local orientation are used in LPR to adapt better to local characteristics of images. Furthermore, a refined intersection of confidence intervals (RICI) adaptive scale selector is adopted to select the scale of the steering kernels. The resulting steering-kernel-based LPR with RICI (SK-LPR-RICI) method is applied to reconstruct a high-resolution brain MRI image from a set of low-resolution MRI images. Simulation results show that the proposed SK-LPR-RICI method can effectively improve the image resolution and peak signal-to-noise ratio.**

Keywords-MRI; image reconstruction; local polynomial regression; adaptive scale selection

I. INTRODUCTION

Magnetic resonance imaging (MRI) is a commonly-used non-invasive imaging technique to visualize internal body tissues. The demands of high-resolution (HR) brain MRI images are mounting in both the neurology and the medicine [1]. However, a difficulty is also increasing for certain standard MRI protocols, such as the diffusion-weighted imaging (DWI) and echo-planar imaging (EPI), to achieve high spatial resolution, because the protocols have to compromise between spatial resolution or temporal resolution and signal-to-noise ratio (SNR) [2]. Moreover, Acquiring HR images, especially in slice-select direction, would result in the reduction of SNR, which is proportional to the main magnetic field strength. Although SNR might be increased by using higher magnetic field scanners, it in return would increase inhomogeneity of magnetic field and hence introduce distortion artifacts into images [3].

To address the problem, various HR image reconstruction techniques have been proposed as post-processing methods to restore an HR image from a series of low-resolution (LR) subpixel-shifted MR images, which can be easily obtained both in a short time and to a relatively large SNR value. Among the reconstruction methods, the iterative back projection (IBP) method, proposed by Irani and Peleg in [4], was the most popular one and also frequently used due to its simplicity and easy implementation. Peled and Yeshurun [5] presented an application of IBP into MRI data, which aimed to reconstruct images of human white matter fiber tract from Diffusion Tensor Imaging (DTI). However, IBP was also known for the low-rate convergence and sensitivity to noise. The IBP method and most HR image reconstruction methods, such as the nonlinear gradient-based regularization method [6], consider the estimated HR image globally and hence it is difficult to suppress the additive noise wile preserving local image details well.

In this paper, a new HR image reconstruction method based on local polynomial regression (LPR) was proposed and applied for reconstruction of HR MRI images. LPR is a flexible and efficient nonparametric regression method [7], and it has received considerable attention because it possesses an excellent spatial adaptation and simple implementation. In LPR, we model each sample in the HR image as a local polynomial with a kernel having a certain bandwidth matrix and solve a series of local least-squares (LS) estimation problem using neighboring samples from the known LR images, instead of the whole image. Since images may vary considerably, it is very crucial to choose a proper local kernel to achieve the best bias-variance tradeoff in estimating the local polynomial coefficients. Generally, the optimal local kernel should be selected to minimize the mean squared error (MSE). In 2D LPR, because the bandwidth $\mathbf{H}$ is a matrix, not only the kernel scale, but also the kernel shape has significant effect on the estimation results. Conventionally, the bandwidth matrix selection for multivariate LPR is simplified by using a symmetric kernel $K_h(\mathbf{u}) = \frac{1}{h^d} K(\mathbf{u}/h)$, where $K(\mathbf{u})$ is a symmetric basis-kernel, h is the kernel scale and d is the dimension of the multivariate LPR (for an image, $d = 2$). As a result, the selection of the scale h is much easier than the selection of a matrix $\mathbf{H}$, and several adaptive scale selection methods were developed, [7] and [8]. Unlike the above methods, Masry [9] considered a more general kernel, which might not be symmetric. More precisely, the basis-kernel $K(\mathbf{u})$ may be of various shapes. But, the optimal scale selection method in [9] is generally not directly usable in practice because its analytical form always involves quantities which are difficult to be estimated.

Recently, Katkovnik *et al.* [10] applied an intersection confidence intervals (ICI) rule to the multivariate LPR problem for selecting the optimal local scale. Given a set of scale parameters in ascending order, the ICI rule determines the optimal scale by comparing the confidence intervals of the estimates with different scales from the scale set. However, the ICI method can only select the scale from the finite scale set. In [11], a new refined ICI (RICI) method was developed for LPR and it can achieve more accurate scale selection results than the conventional ICI method. Furthermore, motivated by the iterative steering kernel regression method of Takeda *et al.* [12], the local kernel shape can be iteratively determined from the local orientation of image data. The locally adaptive steering

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kernel has a better adaptability to local image than the conventional symmetric kernel. Coupled with the RICI scale selection, a fully automatic steering-kernel-based LPR with refined ICI method (SK-LPR-RICI) for restoring images is proposed in this paper.

A practical application of the proposed SK-LPR-RICI method is HR human brain MRI image reconstruction. Because local kernels with orientations are employed, the SK-LPR-RICI HR reconstruction method can better adapt to local orientations in the image. Experimental results on test images and real brain MRI images show that the proposed method has a better SNR and perceptual equality than the conventional IBP method.

II. Methods

A. LPR for Image Processing

An $N_1 \times N_2$ gray-scale image can be viewed as the observations of a 2D function where the samples are taken along the explanatory variable $\mathbf{X}_i = (x_1, x_2)$ at $x_1 = 1,2,\cdots,N_1$ and $x_2 = 1,2,\cdots,N_2$ with Y_i the intensity of the image pixel. We assume the homoscedastic model:

$$Y_i = m(\mathbf{X}_i) + \sigma(\mathbf{X}_i)\varepsilon_i , \quad (1)$$

where $m(\mathbf{X}_i)$ is a smooth function, ε_i is an independent identically distributed (i.i.d.) additive white Gaussian noise (AWGN) with zero mean and unit variance, and $\sigma^2(\mathbf{X}_i)$ is its conditional variance. The problem is to estimate $m(\mathbf{X}_i)$ from the noisy samples Y_i. In LPR, the image pixels can be approximated locally, say at a given point $\mathbf{x}$, by a bivariate polynomial of a certain degree *p*:

$$m(\mathbf{X}:\mathbf{x}) = \sum_{\mathcal{K}=0}^{p} \sum_{k_1+k_2=\mathcal{K}} \beta_{k_1,k_2} \prod_{j=1}^{2} (X_j - x_j)^{k_j} , \quad (2)$$

where $\boldsymbol{\beta} = \{\beta_{k_1,k_2} : k_1 + k_2 = \mathcal{K} \text{ and } \mathcal{K} = 0,\cdots,p\}$ is the vector of coefficients with length.

Since ε_i is i.i.d. and Gaussian distributed, the maximum likelihood (ML) estimation of the coefficient vector $\boldsymbol{\beta}$ at location $\mathbf{x}$ can be obtained by solving the following weighted least-squares regression problem:

$$\hat{\boldsymbol{\beta}}(\mathbf{x};\mathbf{H}) = \arg\min_{\boldsymbol{\beta}} \{J(\mathbf{x};\mathbf{H})\} , \quad (3)$$

where

$$J(\mathbf{x};\mathbf{H}) = \sum_{i=1}^{n} K_{\mathbf{H}}(\mathbf{X}_i - \mathbf{x})[Y_i - \sum_{\mathcal{K}=0}^{p} \sum_{k_1+k_2=\mathcal{K}} \beta_{k_1,k_2} \prod_{j=1}^{2} (X_{i,j} - x_j)^{k_j}]^2 , \quad (4)$$

is a LS cost function and $K_{\mathbf{H}}(\mathbf{X}_i - \mathbf{x}) = \frac{1}{|\mathbf{H}|} K(\mathbf{H}^{-1}(\mathbf{X}_i - \mathbf{x}))$ is a bivariate non-negative kernel function. The bandwidth matrix $\mathbf{H}$ determines the weights of neighboring samples around $\mathbf{x}$ to be used in estimating $\boldsymbol{\beta}(\mathbf{x})$.

The WLS solution to (3) is:

$$\hat{\boldsymbol{\beta}}(\mathbf{x};h) = (\mathbb{X}^T \mathbb{W} \mathbb{X})^{-1} \mathbb{X}^T \mathbb{W} \mathbb{Y} , \quad (5)$$

where

$$\mathbb{X} = \begin{bmatrix} 1 & (\mathbf{X}_1 - \mathbf{x})^T & \text{vech}\{(\mathbf{X}_1 - \mathbf{x})(\mathbf{X}_1 - \mathbf{x})^T\} & \cdots \\ 1 & (\mathbf{X}_2 - \mathbf{x})^T & \text{vech}\{(\mathbf{X}_2 - \mathbf{x})(\mathbf{X}_2 - \mathbf{x})^T\} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ 1 & (\mathbf{X}_n - \mathbf{x})^T & \text{vech}\{(\mathbf{X}_n - \mathbf{x})(\mathbf{X}_n - \mathbf{x})^T\} & \cdots \end{bmatrix}, \mathbb{Y} = [Y_1, Y_2, \cdots, Y_n]^T, \text{ and}$$

$\mathbb{W} = diag\{K_h(\mathbf{X}_1 - \mathbf{x}), \cdots, K_h(\mathbf{X}_n - \mathbf{x})\}$ is the weighting matrix. From the estimated coefficient vector $\hat{\boldsymbol{\beta}}(\mathbf{x};h)$, $m(\mathbf{x})$ can be estimated from $\hat{\beta}_{0,\cdots,0}(\mathbf{x})$.

B. Adaptive Kernel Selection

The Gaussian-type kernel is generally employed in LPR for image processing because it can be transformed to produce a steering kernel. Given a bandwidth matrix $\mathbf{H}$, the Gaussian kernel is calculated as:

$$\mathbf{K}_{\mathbf{H}}(\mathbf{u}) = \frac{1}{2\pi|\mathbf{H}|} \exp\{-\mathbf{u}^T (\mathbf{H}^{-1})^T \mathbf{H}^{-1} \mathbf{u} / 2\} . \quad (6)$$

Since the Gaussian kernel is not of compact support, it is truncated to a sufficient kernel size $N_K \times N_K$ to reduce the arithmetic complexity.

When the bandwidth matrix is chosen as $\mathbf{H} = h\mathbf{I}$, a symmetric Gaussian kernel results, and it is used in most LPR methods for image processing because of its simplicity. For the symmetric basis-kernel, the locally adaptive scale parameter *h* at a point $\mathbf{x}$ will control the amount of smoothing to be performed at that point. When *h* is small, image details such as edges will be preserved. However, possible additive noise may not be removed effectively. On the contrary, a large-scale kernel has better de-noising properties at the expense of possibly blurring of the image details. Therefore, a locally adaptive scale selection method is crucial to LPR in image processing.

The refined intersection of confidence interval (RICI) rule has been shown to be an effective tool for adaptive scale selection [11]. The RICI rule is an empirical method to select the adaptive scale from a set of scales in ascending order. The RICI rule examines a sequence of confidence intervals of the estimates to determine the optimal scale and the corresponding adaptive LPR result. The details of RICI method and its application in LPR are omitted to save space, and more information can be found in our previous work [11].

C. Steering Kernel-based LPR with RICI (SK-LPR-RICI))

Recently, Takeda *et al.* [12] proposed an iterative steering kernel regression (ISKR) method where a steering kernel can adapt to the dominant orientation locally. The ISKR method was shown to have a better performance than the conventional symmetric kernel, especially along image edges. But, the scale selection process in the ISKR method is not automatic and the criterion for selecting smoothing parameter for real images is somewhat subjective. To address this problem, we develop a RICI-based fully data-driven adaptive scale selection method for steering-kernel-based LPR. That is, we propose to construct the local steering kernel $K_h(\mathbf{u}) = \frac{1}{h^2} K(\mathbf{u}/h)$ at each sample $\mathbf{x}$ by a steering basis-kernel $K(\mathbf{u})$ and a scale h. The shape of the basis-kernel $K(\mathbf{u})$ is determined first based on the local dominant orientation estimation around $\mathbf{x}$. Then, the RICI rule is employed to choose the optimal scale parameter h for the steering basis-kernel $K(\mathbf{u})$.

The generation of a steering basis-kernel follows closely the work in [12]. The orientation and shape of the steering kernel is determined using principal components analysis (PCA), and then a steering basis-kernel is obtained by elongating and rotating a symmetric Gaussian kernel so that it can adapt to the local dominant orientation. More details regarding the steering kernel are referred to [12]. Once the locally adaptive steering basis-kernel is determined, the RICI adaptive scale selection method can be used to determine the local adaptive scale parameter from a scale set $\widetilde{\mathbf{H}} = \{h_j \mathbf{I} \mid h_j = h_0 a^j,\ j = 1, \cdots, J\}$, where $a > 1$ is a step factor, $h_0 > 0$ is the base scale, and J is the size of the scale set. More precisely, with different scale h_j, we use the steering kernel $K_{h_j}(\mathbf{X}-\mathbf{x};\mathbf{H}_{S_i}) = \frac{1}{h_j^2} K((\mathbf{X}-\mathbf{x})/h_j;\mathbf{H}_S)$ with a bandwidth matrix $h_j \mathbf{H}_S$ to fit a *p*-order polynomial at $\mathbf{x}$ and obtain a series of smoothed estimates of $\hat{m}(\mathbf{x}; h_j)$. The RICI method is then invoked to determine an adaptive scale h^{opt}. Finally, a LPR using local adaptive scale parameters is performed on the image to yield the final estimate $\hat{m}(\mathbf{x}; h^{opt})$.

In conclusion, the proposed SK-LPR-RICI method can be summarized as follows:

1. *Initial gradient estimation*
   With an initial bandwidth $\mathbf{H}^{(0)} = h^{(0)}\mathbf{I}$, calculate the initial values of $\hat{m}^{(0)}(\mathbf{x};\mathbf{H}^{(0)})$ and its gradient estimation $\hat{\beta}_{0,1}^{(0)}(\mathbf{x};\mathbf{H}^{(0)})$ and $\hat{\beta}_{1,0}^{(0)}(\mathbf{x};\mathbf{H}^{(0)})$.
2. *Steering kernel estimation* (*l-th iteration*)
   $\hat{\beta}_{0,1}^{(l-1)}(\mathbf{x};\mathbf{H}^{(l-1)})$ and $\hat{\beta}_{1,0}^{(l-1)}(\mathbf{x};\mathbf{H}^{(l-1)})$ are used to calculate the steering bandwidth matrix $\mathbf{H}_{S,\mathbf{x}}^{(l)}$ and the steering basis-kernel $K_{\mathbf{x}}^{(l)}(\mathbf{u};\mathbf{H}_{S,\mathbf{x}})$.
3. *Local scale parameter estimation* (*l-th iteration*)
   Obtain a series of LPR results using kernels with the bandwidth matrix $h \cdot \mathbf{H}_{S,\mathbf{x}}^{(l)}$, where h belonging to the scale set $\widetilde{\mathbf{H}}$. Determine the optimal scale $h^{opt,(l)}$ using the RICI rule.
4. *LPR with the estimated steering kernel (l-th iteration)*
   A LPR is performed with the adaptive bandwidth $\mathbf{H}^{(l)} = h^{opt,(l)}\mathbf{H}_{S,\mathbf{x}}^{(l)}$ to obtain $\hat{m}^{(l)}(\mathbf{x};\mathbf{H}^{(l)})$, along with its partial derivatives $\hat{\beta}_{1,0}^{(l)}(\mathbf{x};h^{opt})$ and $\hat{\beta}_{0,1}^{(l)}(\mathbf{x};h^{opt})$.

III. RESULTS AND DISCUSSION

In this section, we first used the standard test image *Lena* to show that the SK-LPR-RICI method can achieve better SNR results than the IBP method under different noise variances. Then, a test image *Phantom* was adopted to illustrate the better performance of SK-LPR-RICI in edge-preserving. At last, an example of human brain MRI image reconstruction was given.

A. Test Image: Lena

The standard test image *Lena* was used here because it contains a good mixture of details, flat area, and texture. A set of four LR (128×128) images with size were generated from the original HR (256×256) *Lena* by decimating it by a factor of 2 both horizontally and vertically and then with a shift of zero pixels, one pixel to right, one pixel to bottom, and one pixel to right and bottom, respectively. Gaussian noise of zero mean was added to the LR images with different values of variance (σ^2=5, 25, 100). Three different image reconstruction methods were evaluated and they were the IBP, SK-LPR-RICI, and LPR-RICI, which was similar to SK-LPR-RICI but employed symmetric circular kernels. In LPR-based methods, Gaussian kernels were used, and the scale set for RICI was chosen as $\widetilde{\mathbf{H}} = \{h_j : \mid h_j = 1/4, 1/2, 1, 2, 4\}$. The kernel size parameter N_K was set to 11. We estimated the smooth function $\hat{m}(\mathbf{X}_i)$ using linear regression with $p = 1$. The threshold parameter κ used for the RICI-based methods was selected as 1.96. The parameter settings were used in other experiments in this paper as well.

The peak signal-to-noise ratio (PSNR) was used to evaluate the reconstruction results. The PSNR values listed in Table I were averages of 100 independent Monte-Carlo realizations. One realization was illustrated in Figure 1, and we can see that all these image reconstruction methods achieved satisfactory results. By comparing the PSNR values in Table I, we can see that SK-LPR-RICI had the highest PSNR, while IBP had the lowest PSNR. The good performances of the LPR-based methods were mainly due to the adaptive-kernel-supported LS regression procedures for each pixel, instead of the whole image. Because SK-LPR-RICI used steering kernels, it obtained better result than LPR-RICI with symmetric kernels.

B. Test Image: Phantom

Next, a test image *Phantom* was adopted and it was good at providing homogenerous edges for observation. The original LR images and reconstructed HR images were presented in Figure 2, and some specific areas of the HR images, which were indicated with red rectangular boxes, were enlarged for better visualization. It shows that the edges were well reconstructed and preserved in the SK-LPR-RICI method. On the other hand, the edges reconstructed by the IBP method were blurred to some extent.

C. Human Brain MRI Image

Lastly, we applied the proposed SK-LPR-SICI method on real human brain data. Due to an inherent characteristic of MRI modality, the HR reconstruction post-processing method could only be employed in the slice-select direction [13]. So in this case we set the slice thickness in LR images wide enough to acquire reference HR image with the same slice width as the reconstructed HR image, which was intended to evaluate the improvement of the reconstruction method. We aim to use several LR images to reconstruct an HR image whose quality is as close as that of the reference HR image. The resolution in LR slices was set to be 1 mm, and the slice thickness was 4 mm. The first set of LR slices included 28 slices. The second set of LR slices was slightly shifted up in the slice-select direction by 1 mm. The third and fourth sets were shifted in the direction by 2 mm and 3 mm, respectively. Then, in the first set we extracted one column of every slice in the same index to obtain the first LR image. Repeating the procedures, other three LR MRI images were obtained. Using the four LR images, HR MRI images were reconstructed using different methods, as shown in Figure 3. It can be seen that the SK-LPR-RICI method enhanced the resolution effectively while preserved the detailed information satisfactorily. In addition, compared with the reference HR image, the SK-LPR-RICI method can smooth out undesired additive noise.

IV. CONCLUSION

In this paper, a novel HR image reconstruction method based on LPR was introduced. Coupled with the RICI adaptive scale selection method and the steering kernel estimation method, the proposed SK-LPR-RICI method can efficiently remove noise and satisfactorily retain the image details, making it a useful technique in HR image reconstruction. The method was applied to reconstruct human brain MRI images and the result showed that the resolution in the slice-select direction of MRI data was improved and additive noise was removed.

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TABLE I
PSNR COMPARISONS BETWEEN HR IMAGE RECONSTRUCTION METHODS UNDER DIFFERENT NOISE VARIANCE

Noise Variance	IBP	LPR-RICI	SK-LPR-RICI
σ^2=5	45.38	47.36	48.65
σ^2=25	37.97	39.57	43.09
σ^2=100	33.26	35.67	40.07

Figure 1. Comparisons between different HR reconstruction methods for the test image *Lena*: (a) LR noisy images, (b) IBP result, (c) LPR-RICI result, (d) SK-LPR-RICI result.

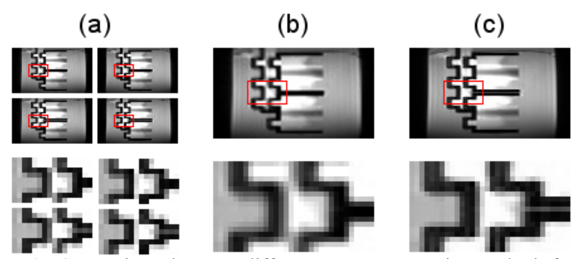

Figure 2. Comparisons between different HR reconstruction methods for the test image *Phantom*: (a) LR noisy images, (b) IBP result, (c) SK-LPR-RICI result.

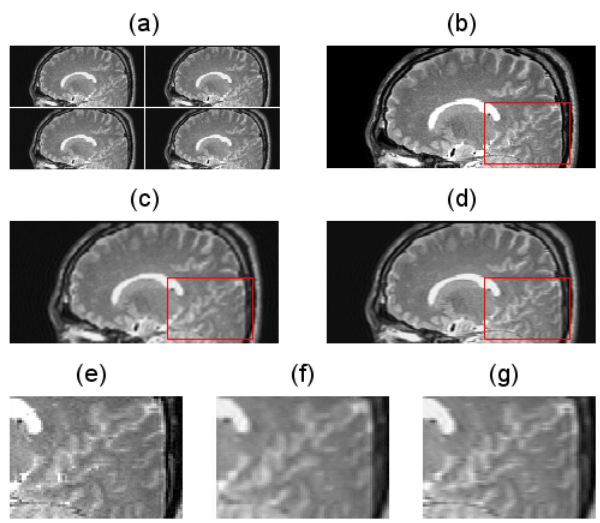

Figure 3. Comparisons between different HR reconstruction methods for real human brain MRI image: (a) LR MRI images, (b) reference HR image, (c) IBP method, (d) SK-LPR-RICI method, (e) enlarged images of the reference HR image, (f) enlarged images of the IBP result, (g) enlarged images of the SK-LPR-RICI result.