

# Sparse Reconstruction of Complex Signals in Compressed Sensing Terahertz Imaging

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**Abstract:** In reconstructing complex signals, many existing methods apply regularization on the magnitude only. We show that by adding control on the phase, the quality of the reconstruction can be improved. This is demonstrated in a compressed sensing terahertz imaging system.

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## 1. Introduction

Over the past few years, advances in terahertz technology have inspired wide interest in terahertz imaging for numerous valuable applications, such as security screening, chemical detection, medical imaging, and quality control [1–3]. However, most existing terahertz imaging systems suffer from slow acquisition rate because of their raster-scanning mechanism. For example, one of the fastest raster-scan terahertz imaging systems in existence still needs about six minutes to acquire a  $400 \times 400$  pixel image [4]. This limitation seriously restricts the applicability of terahertz imaging, such as the case where serial acquisition of image data is required.

Recently, compressed sensing (CS) provides researchers a powerful tool to improve imaging efficiency. In [5], Chan et al. design a single-pixel terahertz imaging setup based on CS theory, with the structure shown in Fig. 1. The basic principle of this system can be modeled as a conventional compressive sampling process: In matrix notation,  $\mathbf{y} = \Phi \mathbf{x}$ , where  $\mathbf{y} \in \mathcal{C}^M$  is a column vector of measurements and  $\mathbf{x}$  is an  $N \times N$  complex-valued image with pixels ordered in an  $N^2 \times 1$  vector, sampled by the measurement matrix  $\Phi \in \mathcal{R}^{M \times N}$ . Taking the object mask in Fig. 2 as an example, with sparse amplitude, they reconstruct the object from the incomplete measurements, i. e.,  $M < N^2$ , by

$$\text{minimize } \|\mathbf{x}\|_1 \quad \text{subject to} \quad \|\Phi \mathbf{x} - \mathbf{y}\|_2 \leq \epsilon, \quad (1)$$

with  $\epsilon$  the tolerance to be defined. However, just as the conventional algorithms, this reconstruction process does not exploit any other information aside from sparsity of the amplitude image. Since pulsed terahertz imaging systems are well known for providing spectroscopic phase information [1, 5], in this paper, we take the prior knowledge of the phase into account and attempt to improve the CS reconstruction quality.

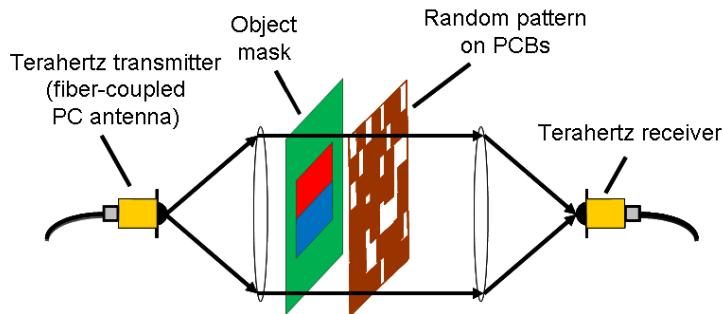


Fig. 1. A schematic of the single-pixel, pulsed terahertz imaging system in [5].

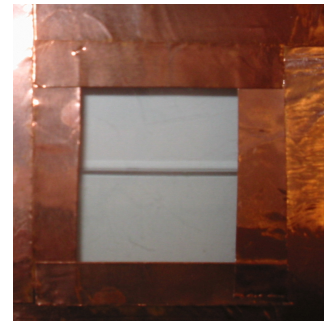


Fig. 2. The rectangular object mask.

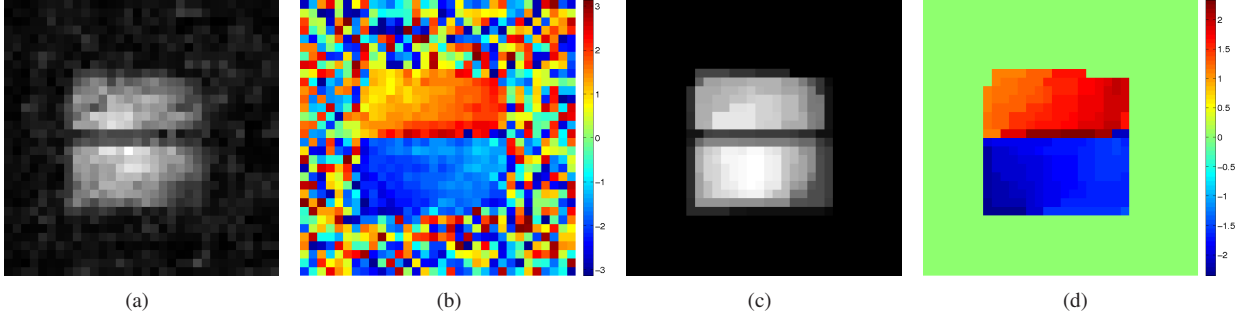


Fig. 3. CS Reconstruction results using 400 complex-valued measurements: (a) the amplitude and (b) the phase images reconstructed by solving the optimization problem with the SPGL1 algorithm [7] as in [5]; (c) the image amplitude and (d) the phase reconstructed with our approach.

## 2. Problem formulation

As mentioned above, we can readily obtain the phase information from pulsed terahertz imaging systems. Generally, the phase map of a terahertz image should be piecewise smooth, i.e., the phase distribution in a small lattice should vary smoothly. However, this kind of knowledge about the smoothness cannot be directly used as a regularizer. Note especially that signal phase is undefined (or unstructured) when the corresponding magnitude is very small. As shown in Fig. 3 (a) and (b), which are the reconstructed magnitude and phase of a complex signal [5], when the signal is very weak outside the rectangular region, the reconstructed phase is random and carries no physical information. We can hardly distinguish the rectangular region from the phase map.

Let  $\angle(x_i)$  and  $\overline{\angle(x_i)}$  be the phase of the  $i$ -th pixel and the mean value of a neighborhood with the  $i$ -th pixel as the center (similar to a Markov random-field [6]), respectively. They are considered to be  $\in [-\pi, \pi)$ , computed by

$$\angle(x_i) = \begin{cases} -j \log \frac{x_i}{|x_i|} & \text{if } |x_i| \geq T \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

Here  $T$  denotes a given threshold for separating the regions containing signal and noise only. With this, we impose an additional smoothness constraint on the phase image for CS reconstruction, i.e.,

$$\left\| \angle(\mathbf{x}) - \overline{\angle(\mathbf{x})} \right\|_2 \equiv \left( \sum_{i=1}^{N^2} \left[ \angle(x_i) - \overline{\angle(x_i)} \right]^2 \right)^{1/2} \leq \sigma. \quad (3)$$

Our reconstruction algorithm for a CS terahertz imaging system can be interpreted as an optimization given by

$$\begin{aligned} & \text{minimize} && \|\Psi \mathbf{x}\|_1 \\ & \text{subject to} && \|\Phi \mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ & && \left\| \angle(\mathbf{x}) - \overline{\angle(\mathbf{x})} \right\|_2 \leq \sigma \end{aligned} \quad (4)$$

where  $\Psi$  represents a sparsifying transform, i.e., an operator mapping a vector of signal to a sparse or compressible data. For example, finite-differences can be used as a sparsifying transform. To solve Eq. 4, we employ the nonlinear conjugate gradient method combined with backtracking line search. In addition, we use a sigmoid function (e.g., the logistic function) to approximate the threshold operation in Eq. 2.

## 3. Numerical experiments

To demonstrate the reconstruction quality, we use the same experimental data as the second experiment in [5]. The test object is a transparent plastic rectangular plate embedded in an opaque screen (see Fig. 2). The thickness of the upper and lower halves of the rectangular pattern are different. According to the transmitting geometry implied in Fig. 1, the thickness difference  $\Delta d$  can be estimated from the phase retardance  $\Delta\varphi$  with

$$\Delta\varphi = 2\pi \frac{\Delta d}{\lambda} (n_{\text{obj}} - n_{\text{air}}), \quad (5)$$

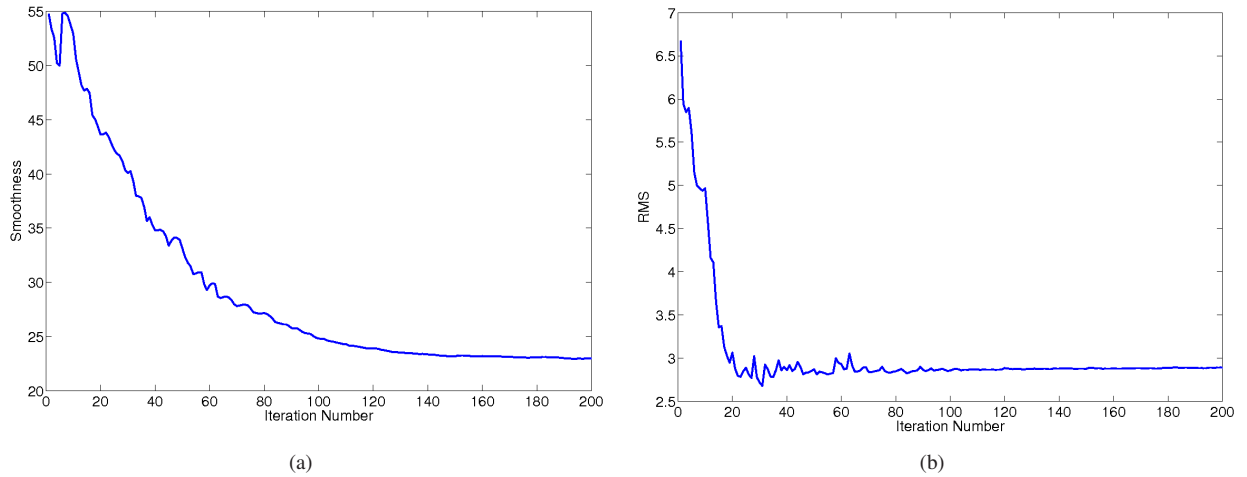


Fig. 4. Reconstruction quality measurements of our method: (a) the smoothness, (b) the fidelity.

where  $\lambda$  is the wavelength of the illumination light, and  $n_{\text{obj}}$  and  $n_{\text{air}}$  are the refractive indices of object and air.

In the experiments, we use 400 measurements for recovering a  $32 \times 32$  image with total variation as the sparsifying transform operation in Eq. 4. Fig. 3 (a) and (b) show the amplitude and phase images based on sparsity of the image amplitude only (i. e., without the constraint given by Eq. 3), while Fig. 3 (c) and (d) correspond to the results with our proposed algorithm. From the reconstruction results, we can see that the reconstruction quality with our algorithm is significantly better on both the amplitude and phase images. For instance, both the amplitude and phase show sharp contrast, and noticeable artifacts in the background of the amplitude image are suppressed. Due to the introduction of the smoothness constraint on the phase, the reconstructed phase image becomes closer to the realistic object (Fig. 2) and smoother in the regions with different thickness. In addition, to further illustrate the reconstruction performance of our algorithm, we provide two quality measurements versus iteration number in Fig. 4, where the smoothness is measured by Eq. 3 and the fidelity is estimated by root mean square (RMS), that is,  $\frac{1}{\sqrt{M}} \|\Phi \mathbf{x} - \mathbf{y}\|_2$ .

#### 4. Conclusion

In this paper, we propose an effective sparse reconstruction method for complex signals with control of the phase. As seen from the experimental results, our algorithm can significantly improve the reconstruction quality both visually and numerically. Although the experiments are conducted on the single-pixel, pulsed terahertz imaging system shown in Fig. 1, the method can be readily extended to other similar imaging modalities. This work was supported in part by the Research Grants Council of the Hong Kong Special Administrative Region, China under Projects HKU 713906 and 713408. Wai Lam Chan and Daniel M. Mittleman acknowledge partial support from the National Science Foundation and from the Air Force Office of Scientific Research through the CONTACT program.

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