SECTIONAL IMAGE RECONSTRUCTION IN OPTICAL SCANNING HOLOGRAPHY USING COMPRESSED SENSING

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ABSTRACT
Optical scanning holography is a form of digital holographic system, which allows us to capture a three-dimensional (3D) object in the two-dimensional (2D) hologram. A post-processing step known as sectional image reconstruction can then be applied to reconstruct a 2D plane of the object. In this paper, we show that the Fourier transform of the hologram corresponds to samples along a semi-spherical surface of the Fourier transform of the 3D object. Since the Fourier basis satisfies the uniform uncertainty principle, we can view the hologram capture as a form of compressed sensing, and reconstruction of the sectional images can then be accomplished by solving a total variation minimization problem.

Index Terms— Image reconstruction, holography, sectional image, compressed sensing

1. INTRODUCTION

Three-dimensional (3D) imaging of small objects, especially dynamic ones, is very desirable for scientists to study the microscopic world. Currently, confocal microscopy is commonly applied for this task. It provides two-dimensional (2D) images with high lateral resolutions but only a shallow depth, due to the inherent physics involved in the imaging process. To achieve a volumetric capture, it needs sequential acquisition at different axial planes. This is, however, a slow process and is unsuitable for moving objects.

In contrast, digital holographic imaging — though currently delivering images with a lower lateral resolution — can capture the entire 3D volume during a single $x$-$y$ scan. The data need to be “interpreted,” i.e., processed to obtain the individual 2D sections. This is called sectional image reconstruction, or simply, sectioning. High-resolution volume imagery of dynamic objects is therefore possible with holographic imaging.

In this paper we focus on a particular form of digital holographic imaging known as optical scanning holography (OSH), which has been shown to give better than $1\mu m$ lateral resolution with fluorescent specimens [1]. We analyze the imaging process and show that the hologram is composed of samples in the 3D spatial frequency domain. There are two major results. First is that the holographic data are projections of the object on the 3D Fourier basis. This obeys the uniform uncertainty principle (UUP) and is incoherent with the canonical basis [2]. Hence, we can apply compressed sensing to design and analyze the sectional image reconstruction. Second, the samples fall on a semi-spherical surface in the high frequency vicinity. This provides an alternative way to sample the 3D spatial frequency domain, compared with using radial lines in computed tomography [3] and spiral trajectory in magnetic resonance imaging.

2. COMPRESSED SENSING THEORY

Compressed sensing considers the problem of reconstructing a sparse signal (or an image) from incomplete linear measurements [4]. To achieve an accurate reconstruction, an operation matrix representing the data capture system satisfies a sufficient condition referred to as the UUP [4]. The property essentially guarantees that columns in the operation matrix are almost orthogonal rather than globally perfectly orthogonal. Then, every set of column vectors with cardinality less than the number of nonzero entries in the original signal approximately operates like an orthonormal system. Therefore, an encoding map by the operation is of an isometry, i.e., the distance of the signal can be preserved by the orthonormal encoding system.

As a special case of the operation matrix, discrete Fourier transform has been analyzed in the compressed sensing literature and its performance on a reconstruction with incomplete frequency information is well discussed [3, 5]. The basis is incoherent with the canonical basis and satisfies the UUP. It is then possible to reconstruct the signal from a few measurements of its frequency domain by solving an $\ell_1$ minimization problem. For the measurements based on Fourier basis, they are not required to be sampled at random.
3. OSH IMAGING SCHEME

The OSH system is depicted in Fig. 1 [6, 7]. A laser source is split into two beams by a beamsplitter (BS1). One beam works as a reference light and the other scans an object \( g(x, y; z) \). In what follows, we also denote a two-dimensional section of the object at depth \( z \) as \( g(x, y; z) \). The holograms we measure by OSH system are complex, and hence we often separate them into real and imaginary parts for display. The operating principle of OSH is described at length in [6, 7, 8].

For the purposes below, it is important to state that the optical transfer function (OTF) for the system is

\[
H(k_x, k_y; z) = \exp\left\{-\frac{jz}{2k_0}(k_x^2 + k_y^2)\right\},
\]

where \( k_x \) and \( k_y \) are the spatial frequency coordinates. The propagation vector of a plane wave is \( \vec{k} \), and \( k_0 \) denotes the magnitude of this vector, i.e.

\[
k_0 = |\vec{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2}.
\]

The symbol \( k_z \) denotes the spatial frequency along the \( z \)-axis.

4. COMPRESSIVE OPTICAL SCANNING HOLOGRAPHY

We can apply the paraxial approximation [9] on \( k_z \). This gives

\[
k_z = (k_0^2 - k_x^2 - k_y^2)^{1/2} \approx k_0 - \frac{k_x^2 + k_y^2}{2k_0}.
\]

Substituting the approximation into Eq. (1), we can represent the OTF as

\[
H(k_x, k_y; z) \approx \exp\left\{-jz(k_0 - k_z)\right\} = \exp\left\{jz(k_z - k_0)\right\}.
\]

The OTF is multiplied with the Fourier transform of an object \( g(x, y; z) \) to yield the frequency domain of the digital hologram for that particular section. Summing over all depths, we can describe the Fourier transform of the digital hologram as

\[
Q(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left|g(x, y; z)\right|^2 \exp\left\{-jz(k_x x + k_y y - z(k_z - k_0))\right\} \, dx \, dy \, dz.
\]

5. METHODOLOGY

We turn our attention to look at the reconstruction problem. Given \( H(k_x, k_y; z) \) and \( h(x, y; z) = \mathcal{F}^{-1}_{xy}\{H(k_x, k_y; z)\} = -\frac{jk_0}{2\pi} \exp\left\{\frac{jk_0(x^2 + y^2)}{2z}\right\} \)}

We use \( \mathcal{F}_{xy}\{\cdot\} \) to emphasize it is a 2D Fourier transform on the \( x-y \) plane. Writing it as a double integral, we can then describe the image formation with a triple integral, i.e.

\[
Q(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left|g(x, y; z)\right|^2 \times \exp\left\{-jz(k_x x + k_y y - z(k_z - k_0))\right\} \, dx \, dy \, dz.
\]
[10], an OSH hologram can be expressed with respect to a convolution [11, 12], i.e.
\[ q(x, y) = \int_{-\infty}^{\infty} g(x, y; z) * h(x, y; z) \, dz. \]

Let \( g(x, y; z) = |g(x, y; z)|^2 \). Then
\[ q(x, y) = \int_{-\infty}^{\infty} \tilde{g}(x, y; z) * h(x, y; z) \, dz. \]

We discretize the depth \( z \) into \( z_1, z_2, \ldots, z_n \) and use a lexicographical ordering of \( q(x, y) \) and \( \tilde{g}(x, y; z_i) \) into \( q \) and \( \tilde{g}_i \). Making use of a multiplication to replace the convolution in Eq. (9), \( h(x, y; z_i) \) is reorganized into a matrix, \( \tilde{H}_i \). The hologram, further, is
\[ q = \sum_{i=1}^{n} \tilde{H}_i \tilde{g}_i = \tilde{H}g, \]
where \( \tilde{H} = [\tilde{H}_1 \cdots \tilde{H}_n] \) and \( g = [\tilde{g}_1^T \cdots \tilde{g}_n^T]^T \). It is an inverse problem to reconstruct \( g \) from a measurement vector \( q \). In this paper, we solve it by using a total variation (TV) minimization method, which preserves the finite-difference in the solution, and is shown to deliver good performance in compressed sensing [2]. The variational problem is of the form
\[ J_{TV}(g) = \|q - \tilde{H}g\|^2 + \alpha \|g\|_{TV}, \]
where \( \alpha \) is the regularization parameter. The notation \( \| \cdot \|_{TV} \) stands for the total variation norm. Its 2D form is given by
\[ \|v(x, y)\|_{TV} = \int \int |\nabla v(x, y)| \, dx \, dy, \]
where \( \nabla v(x, y) = (\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}) \) is the gradient of \( v(x, y) \) and \( |\cdot| \) denotes the Euclidean norm, i.e., \( |(x, y)| = \sqrt{x^2 + y^2} \).

Note further that we need to maintain the nonnegativity of the solution. Hence, the overall equation should be
\[ J_{TV}(g) = \|q - \tilde{H}g\|^2 + \alpha \|g\|_{TV}, \text{ such that } g \geq 0. \]
This can be solved with the primal-dual Newton method using a gradient projection in the iterations [13].

6. EXPERIMENTS

An experiment is conducted to demonstrate the performance of the reconstruction method. We use an object with two sections, which has been tested with different reconstruction schemes in the past [11, 14, 15]. The two sections are at different \( z \) positions, and each contains a bar. We simulate its scanning by an OSH system, and obtain a complex digital hologram as our data.

We aim to reconstruct sectional images at \( z_1 \) and \( z_2 \) from the hologram. Fig. 3(a) shows the simulated object, where two sections are located at \( z_1 = 14\text{mm} \) and \( z_2 = 15\text{mm} \) away from the OSH scanner. Shown in Fig. 3(b) are the real parts of the Fresnel zone plates (FZPs) [7] of a point source at \( z_1 \) and \( z_2 \) respectively. The real and imaginary parts of the complex hologram are shown in Fig. 4. Fig. 4(a) is the sine hologram, which is also known as the real hologram, and (b) is the cosine hologram, also called the imaginary hologram.

We apply the proposed method to reconstruct the sectional images in the hologram. As shown in Fig. 5(a) and Fig. 5(b), the two sections are well separated with sharp edges. Defocus noise [7] has little effect on the reconstructed sections. We can compare them with the reconstructed sections by the Tikhonov regularization shown in Fig. 6 [11]. We then apply the TV minimization into the reconstruction of sectional images from the undersampled spatial frequency data, and show that our algorithm can preserve edges and suppress defocus noise us-

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Fig. 3: The object and FZPs in the experiment. Shown in (a) is the object, in which two elements (i.e., rectangular bars) are at \( z_1 \) and \( z_2 \) positions, respectively. Shown in (b) are the real parts of the FZPs of a point source at \( z_1 \) and \( z_2 \).

Fig. 4: Holograms of the object in the experiment. Sine hologram is the real part and cosine hologram is the imaginary part.

Fig. 5: Reconstruction of sectional images at \( z_1 \) and \( z_2 \) from the hologram. The focused intensity in each section is recovered nonuniformly. The corners and edges are recovered with high intensity, but the middle parts have lower intensity.

7. CONCLUSION

In this paper we demonstrate the connection between OSH imaging and compressed sensing. We then apply the TV minimization into the reconstruction of sectional images from the undersampled spatial frequency data, and show that our algorithm can preserve edges and suppress defocus noise us-
Fig. 5: The reconstructed sections by the TV minimization method. (a) is the section at $z_1 = 14$mm and (b) at $z_2 = 15$mm.

Fig. 6: The reconstructed sections by the Tikhonov regularization method. (a) is the section at $z_1 = 14$mm and (b) at $z_2 = 15$mm.

8. REFERENCES


