Bayesian Reconstruction in Optical Scanning Holography

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Abstract: Optical Scanning Holography (OSH) is a technique that scans 3D objects onto a 2D hologram. Here, we analyze the OSH object reconstruction process from a Bayesian perspective and provide an appropriate solution.

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1. Introduction

Techniques to recover 3D information from 2D-recorded data are gaining popularity lately. Among such techniques, Optical Scanning Holography (OSH) has facilitated various applications in a wide range of fields [1, 2]. In OSH, 3D information of objects is scanned and recorded on a 2D hologram. Hence it is crucial to reconstruct sectional images, i.e. layer-by-layer images, after the scanning process. The most recent approach used in the reconstruction is by solving an inverse problem [3].

Existing studies find solutions of such inverse problems by using various image restoration methods [3, 4, 5]. In this paper, we analyze the inverse problem from a Bayesian perspective. The Bayesian analysis, with its ability to exploit prior knowledge on the nature of OSH, is promising to improve the reconstruction.

2. Problem Formulation

A schematic diagram of OSH is shown in Fig. 1. The detailed architecture and theory of OSH system could be found in [6]. Briefly speaking, the $N \times N$ pixellated 2D hologram $R(x, y)$ is the integration of the object intensity $U(x, y; z_i)$

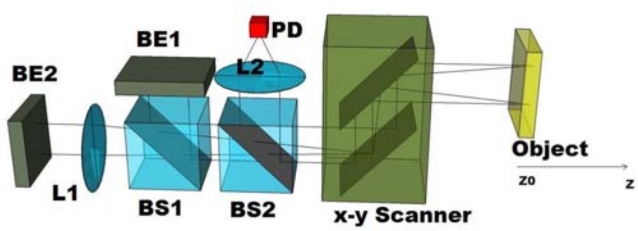

Fig. 1. OSH system. BE1, BE2, beam expanders; L1, L2, lens; BS1, BS2, beam splitters; PD, photodetector

at a depth z_i (depth is discretized into n levels), convolved with the Fresnel zone plate [3] $m(x, y; z_i)$ at that depth, for all the depths concerned, with additive noise $V(x, y)$, i. e.

$$R(x,y) = \sum_{i=1}^{n} U(x,y;z_i) * m(x,y;z_i) + V(x,y). \tag{1}$$

By using the inverse imaging approach [3], Eq. (1) could be rewritten as

$$\mathbf{r} = \sum_{i=1}^{n} M_i \mathbf{u}_i + \mathbf{v}, \tag{2}$$

where $\mathbf{u}_i$, of size $N^2 \times 1$, stands for the raster-scanned $U(x, y; z_i)$ in Eq. (1); M_i, of size $N^2 \times N^2$, is the convolution matrix developed from $m(x, y; z_i)$; $\mathbf{r}$, of size $N^2 \times 1$, is the lexicographical version of $R(x, y)$; $\mathbf{v}$, of size $N^2 \times 1$, represents the lexicographically ordered noise. To further simplify Eq. (2), we let $M = [M_1 \;\; M_2 \;\; \ldots \;\; M_n]$ and $\mathbf{u} = [\mathbf{u}_1^T \;\; \mathbf{u}_2^T \;\; \ldots \;\; \mathbf{u}_n^T]^T$. Then we have a linear relationship in imaging, i. e.

$$\mathbf{r} = M\mathbf{u} + \mathbf{v}, \tag{3}$$

where $\mathbf{r}$, given by the hologram obtained in OSH, and M, given by the system, are known while $\mathbf{u}$ is to be found.

3. Bayesian Analysis

An introduction to Bayesian interpretation of generic image restoration could be found in [7]. As for OSH, we assume that the elements of **r**, **u**, and **v** in Eq. (3) are represented by random variables r, u, and v, respectively, and that $\mathcal{P}(r|u,v)$ denotes the probability density for hologram r, given the true object u and the noise v. Existing studies normally view the noise in OSH hologram as Gaussian random noise [3]. We thus assume that the variance of v is α. Then we have

$$\mathcal{P}(r|u,v) = \mathcal{P}(r|u,\alpha) = \left(\frac{1}{\sqrt{2\pi\alpha}}\right)^{N^2} \exp\left(-\frac{\|r - Mu\|^2}{2\alpha}\right). \tag{4}$$

On the other hand, we assume that the true object u itself has a Gaussian prior probability density with the variance β so that

$$\mathcal{P}(u|\beta) = \left(\frac{1}{\sqrt{2\pi\beta}}\right)^{N^2 n} \exp\left(-\frac{\|u\|^2}{2\beta}\right). \tag{5}$$

According to Bayesian statistics, given $\mathcal{P}(u|\beta)$, the prior probability density for u, and $\mathcal{P}(r|u,\alpha)$, the measurement probability density for r, we can obtain the posterior probability density for u, i. e. $\mathcal{P}(u|r,\alpha,\beta) \propto \mathcal{P}(r|u,\alpha)\mathcal{P}(u|\beta) \propto \exp[-(\|r - Mu\|^2)/(2\alpha) - (\|u\|^2)/(2\beta)]$. Hence, the maximum *a posteriori* (MAP) estimate of u, or the Bayesian reconstruction of sectional OSH images, is the solution

$$\hat{u} = \text{arg min} \left(\|r - Mu\|^2 + \frac{\alpha}{\beta}\|u\|^2\right). \tag{6}$$

4. Results and Conclusion

To test the functionality of the Bayesian reconstruction model, we construct a two-section object as shown in Fig. 2. The hologram of the object is shown in Fig. 3. Assuming that the variances of the object and the noise are our *a priori* knowledge, we use a conjugate gradient method provided in [8] to find the minimizer of Eq. (6) and to obtain the reconstructed sectional images, as shown in Fig. 4. We also consider some possible improvements on this model. An instance is that the Gaussian prior of u might be revised to an assumption better suited with specific OSH objects, say a Bernoulli prior in this binomial scenario. Noise characteristic is also a possible place for further amendment. In conclusion, Bayesian analysis should result to more precise reconstructions along the advancement of computational algorithms.

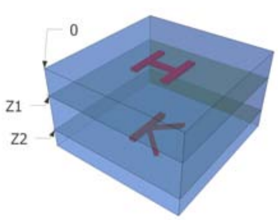

Fig. 2. The object with "H" and "K" on two sections at z_1 and z_2.

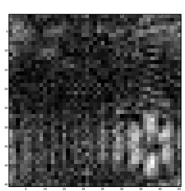

Fig. 3. The real part of the hologram.

(a) "H" at z_1

(b) "K" at z_2

Fig. 4. Reconstruction results

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