Reducing the acquisition time of optical scanning holography by compressed sensing

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Abstract: We propose a compressed sensing approach to improve the acquisition time of optical scanning holography with a spiral trajectory. A two-sectional object is reconstructed with high fidelity by at least 5× speed improvement.

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1. Introduction
Optical scanning holography (OSH) is a digital holographic technique that records and reconstructs the three-dimensional (3D) object by heterodyne generation [1]. The system illuminates and encodes the object directly with a time-varying Fresnel zone plate (FZP), rather than encoding object information with a reference beam as in conventional holographic imaging setup. This unique approach permits imaging in an incoherent mode [2], which is particularly useful for fluorescence-stained biological samples. However, OSH requires two-dimensional (2D) raster scanning in order to capture a 3D object, which is slow compared to conventional camera-based holographic setup. Also, the resulting holographic data are highly redundant, as the large FZP pattern illuminates the same coordinates multiple times during the back-and-forth scanning [1]. Doing a full rectangular scan is therefore a waste of acquisition time, especially when the 3D object is a group of tiny scattered fluorescence beads (ℓ0-sparse) or a few highly opaque cells with sharp edges (TV-sparse). In this paper, we propose a compressed sensing (CS) approach to reduce the acquisition time by scanning the object with a spiral trajectory. This is related to compressive holography which undersamples the holographic data with a binary mask [3]. With a sparsity constraint that matches the object prior knowledge, such as total-variation (TV) regularization presented in this paper, the hologram can be decoded without losing much image quality.

2. Theory
Assuming the 3D object consists of finite number of thin sections, the holographic imaging by OSH is given by

\[
I(x, y) = \sum_i h(x, y, z_i) \ast O(x, y, z_i) + n(x, y),
\]

where \(I(x, y)\) is the measured hologram, \(O(x, y, z_i)\) is the sectional object at depth \(z_i\), \(h(x, y, z_i)\) is the FZP at depth \(z_i\), and \(n(x, y)\) is the imaging noise [4]. The inverse problem relies on depth-dependency of the FZP to separate and reconstruct different sections of objects (\(N\) pixels per section) from a hologram (\(N\) pixels). This is ill-defined as there are multiple times the unknowns over the constraints, unless we have prior information about the object itself. For example, if the object is piecewise-linear, a TV regularization term is added to the objective function to further constrain the problem [4], i.e.

\[
\{\hat{O}_i\} = \arg \min_{\{O_i\}} \frac{1}{2} \left\| I - \sum_i h_i \ast O_i \right\|_2^2 + \alpha \sum_i \left\| \hat{O}_i \right\|_{TV},
\]

where \(\hat{O}_i\) is the reconstructed object at depth \(z_i\), and \(\alpha\) is the penalty factor of the TV constraint.

For most OSH applications, the size of the FZP \(h_i\) is comparable to that of the object \(O_i\). This fulfills the redundancy requirement of CS, as each object voxel information is dispersed to multiple hologram pixels so that the image quality is insensitive to undersampling of holographic data. The sparsity requirement of CS has been thoroughly demonstrated...
by the TV regularization term, which concentrates the extraction of information around the edges of the object. The inverse problem now becomes

$$\{\hat{O}_i\} = \arg \min_{\{O_i\}} \frac{1}{2} \left\| I_S - B_S \left( \sum h_i * \hat{O}_i \right) \right\|_2^2 + \alpha \sum \| \hat{O}_i \|_{TV},$$

(3)

where the function $B_S(I)$ sub-samples $M$ out of $N$ pixels from the hologram using the binary mask $S$. To fulfill incoherency requirement of CS, a uniform random subsampling mask for $S$ is optimal for successful reconstruction. In practice, the mask $S$ must be a continuous scanning trajectory, as the scanning hardware (e.g. translation stage, galvanometer) forbids random access of coordinates at high speed. Here, we propose using a spiral scanning path to fulfill both the hardware constraint and incoherency requirement. To be specific, we choose holographic pixels along the Archimedean spiral which spans over the whole object, i.e.

$$S(x_j, y_j) = \begin{cases} 
1 & \text{if } (x_j, y_j) = \left( R \frac{2\pi p}{\theta_j} \cos \theta_j, R \frac{2\pi p}{\theta_j} \sin \theta_j \right) \\
0 & \text{otherwise},
\end{cases}$$

(4)

where $\theta_j$ is the turn angle in polar coordinate system, $p$ is the number of revolutions of the spiral trajectory, $R$ is the radius of the circular field-of-view.

3. Simulation and results

To evaluate the effectiveness of compressed sensing at different levels of image complexity, we generate a virtual barcode pattern with alternating depths and scan it with a spiral path to obtain a OSH hologram. Sparsity level is measured as $K/N$, where $K$ is the sum of the perimeters of the barcode pattern, whereas compression ratio is equal to $M/N$, where $M$ is the number of pixels along the spiral path. We employ gradient-based reconstruction with TV penalty (TV AL3) [5] with default parameters to reconstruct the sample. The reconstruction result is compared to the ground truth to produce structural similarity index (SSIM) score, as shown in Figure 1. We found that for sparsity level at around or below 20%, the image quality can be maintained above 0.9 at reasonable compression level ($M/N \leq 0.2$). More than that, exact reconstruction is assured at 1% sparsity level if almost half of the holographic pixels are sampled ($M/N \geq 0.4$), but it is less likely so in practice due to noise.

We also apply CS on actual OSH data acquired by Kim et al. [4]. As there is no ground truth object for comparison, we produce the SSIM score by comparing the construction result with that at 100% sub-sampling ratio, i.e. recording all pixels inside the field-of-view. Again our study reveals that the object is recognizable (SSIM = 0.68) at 5% compression ratio. This implies a 20× reduction in acquisition time. The SSIM score attains just below 90% at 20% compression ratio (Table 1), which means the compressed image is almost the same quality as the image reconstructed from raw data. Figure 2 shows the respective undersampling trajectory and the reconstructed sectional images.
Table 1: SSIM score for Kim’s hologram.

<table>
<thead>
<tr>
<th>Number of spiral revolutions $p$</th>
<th>10</th>
<th>40</th>
<th>90</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression ratio $M/N$</td>
<td>5%</td>
<td>20%</td>
<td>42%</td>
<td>100%</td>
</tr>
<tr>
<td>Scanning speed up</td>
<td>20$\times$</td>
<td>5$\times$</td>
<td>2.3$\times$</td>
<td>1$\times$</td>
</tr>
<tr>
<td>SSIM</td>
<td>0.68</td>
<td>0.86</td>
<td>0.99</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Fig. 2: Compressed sensing on actual OSH hologram. The top row illustrates the hologram data (real part) along the spiral path; the undefined pixels are displayed as black color. The corresponding number of spiral revolutions $p$ are shown in Table 1. The reconstructed image in the bottom row shows the top layer in red and bottom layer in blue.

4. Conclusions

We propose to speed up the scan rate of OSH by undersampling the hologram with a spiral scanning trajectory. By choosing a TV-sparse constraint which matches the piecewise-linear geometry of the object, the two-sectional image is reconstructed with high fidelity. We also demonstrates this compressed sensing approach to actual holographic data, which shows that the scan rate can be improved by at least 5 times without losing much image quality.

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References