Blind Image Deconvolution for Symmetric Blurs by Polynomial Factorization

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ABSTRACT

In image acquisition, the captured image is often the result of the object being convolved with a blur. Deconvolution is necessary to undo the effects of the blur. However, in reality we often know very little of its exact structure, and therefore we have to perform blind deconvolution. Most existing methods are computationally intensive. Here, we show that if the blur is symmetric, we have an efficient algorithm for deconvolution based on polynomial factorization in the z-domain.

1. INTRODUCTION

In imaging using a diffraction-limited system with incoherent light, the object and the image intensities are related by $g = h * f + n$, where f is the object, g is the observed image, h is the intensity impulse response, and n is the additive noise.[1] They are sampled on a regular grid. h essentially blurs the object, and our goal is to recover f given only g. This is therefore a blind deconvolution problem, where we seek to deconvolve g to obtain a close approximation to the object f without an exact knowledge of the degradation h. This problem is especially important for digital photography. Successful post-processing could imply relaxation of manufacturing requirements in camera design.[2]

Without noise, the blind deconvolution problem almost always has a unique solution.[3] We can take the z-transform of the imaging equation and obtain $G(z_1, z_2) = H(z_1, z_2)F(z_1, z_2)$. So, we only need to factorize a bi-variate polynomial into two non-trivial components. This process is unique for higher dimensions up to a scaling factor except for contrived cases. However, there is no known simple scheme for bi-variate polynomial factorization.

2. NEW ALGORITHM

It is reasonable to incorporate the following two assumptions which are consistent with most of the algorithms found in the literature.[4]

1. We know the support size of both f and h.
2. h is a symmetric function (to be defined more precisely in the next section). Although some blurs, such as motion blur, do not fall into this category, many others, such as defocus blur, usually do.

2.1. Algorithm

Let the z-transform of the ith column of h be $h_i(z)$, and that of the ith row be $h'_i(z)$. Similar definitions hold for f and g. Let the size of h be $M \times M$, and that of f be $N \times N$ ($N > M$). We have the following set of equations:

$$\begin{aligned} h_1(z)f_1(z) &= g_1(z) \\ h_2(z)f_1(z) + h_1(z)f_2(z) &= g_2(z) \\ \vdots &= \vdots \\ h_M(z)f_1(z) + \ldots h_1(z)f_M(z) &= g_M(z) \\ \vdots &= \vdots \\ h_M(z)f_N(z) &= g_{M+N-1}(z) \end{aligned} \tag{1}$$

With symmetry, we have $h_j(z) = h_{M+1-j}(z) = h'_j(z) = h'_{M+1-j}(z)$. We can generate another set of equations with f replaced by f' and g by g'. Looking at the first and the last rows of the two sets of equations, we have $h_1(z)f_1(z) = g_1(z)$, $h_1(z)f_N(z) = g_{M+N-1}(z)$, $h_1(z)f'_1(z) = g'_1(z)$, and $h_1(z)f'_N(z) = g'_{M+N-1}(z)$. Assuming that $f_1(z)$, $f_N(z)$, $f'_1(z)$ and $f'_N(z)$ have no common factors, we can readily derive $h_1(z)$ from g by

$$h_1(z) = \mathrm{GCD}\{g_1(z), g_{M+N-1}(z), g'_1(z), g'_{M+N-1}(z)\}, \tag{2}$$

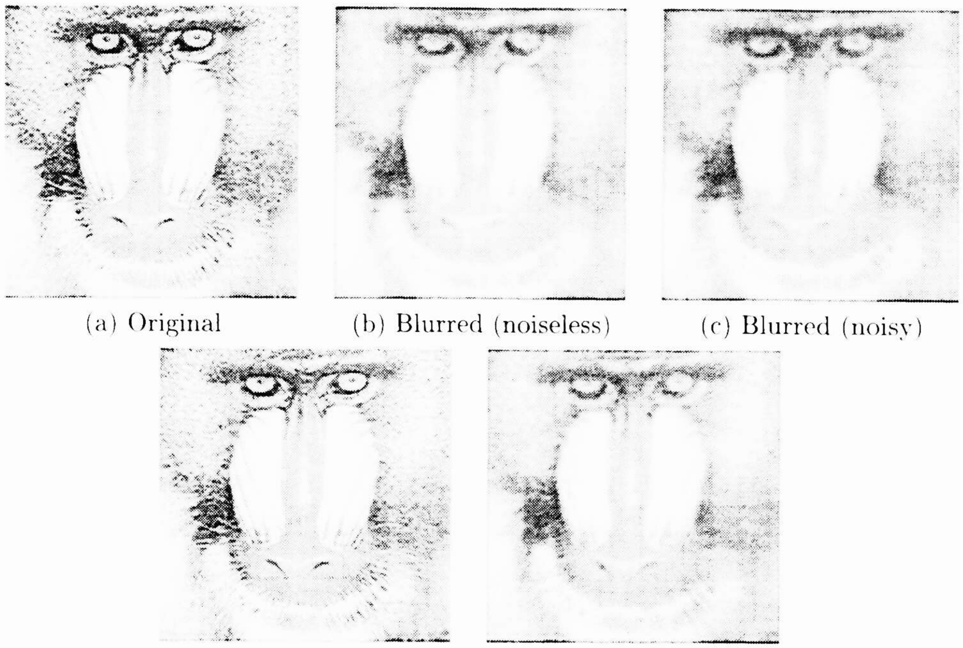
where GCD is the greatest common divisor operator.

To deduce the inner elements of h requires a somewhat different approach. For the second column of h, we have $h_2(z)f_1(z) + h_1(z)f_2(z) = g_2(z)$. From (2), we can calculate the roots of $h_1(z)$, denoted as α_i, by a standard root-finding algorithm. Putting in these roots, we have $h_2(\alpha_i)f_1(\alpha_i) = g_2(\alpha_i)$ for all i, which could be rearranged to give the equation $h_2(\alpha_i) = g_2(\alpha_i)/f_1(\alpha_i)$.

Note that $h_2(z)$ is a polynomial of degree N, and we have $N - 1$ roots α_i. However, the actual number of parameters is only $(N-1)/2$, because of symmetry in $h_2(z)$. Therefore, we have evaluated the polynomial at more points than the number of parameters, and we can readily recover its coefficients. For other columns, it is straightforward to extend the above arguments to give

$$h_k(\alpha_i) = \frac{g_k(\alpha_i) - \sum_{m=2}^{k-1} h_m(\alpha_i)f_{k-m+1}(\alpha_i)}{f_1(\alpha_i)}. \tag{3}$$

Part of the 18th Congress of the International Commission for Optics: Optics for the Next Millennium
San Francisco, California • August 1999 SPIE Vol. 3749 • 0277-786X/99/$10.00



(a) Original (b) Blurred (noiseless) (c) Blurred (noisy)

(d) Restored (noiseless) (e) Restored (noisy)

Figure 1: Simulation results

Because of symmetry, we only have to go through half of the elements of h. We can then apply the conventional image restoration operators, such as Wiener and homomorphic filters, to our blurred image.[5]

2.2. Simulation

We have tested the algorithm on the "baboon" image shown in figure 1(a). A small kernel approximation of a Gaussian blur with size 7×7 and a standard deviation of 3.5 pixels is applied to blur the image. (b) shows the blurred image with no noise added to it, while (c) shows the image corrupted with additive white Gaussian noise, with a signal-to-noise ratio (SNR) of about 80dB. The restored images are shown in (d) and (e) respectively. We can see that in the noiseless case, perfect reconstruction is possible, while for the noisy case, the restoration still shows some improvement in the image quality.

3. SUMMARY AND CONCLUSION

We have demonstrated a blind image deconvolution algorithm for symmetric blurs using polynomial factorization in the z-domain. Simulation results further show that the algorithm is successful even at the presence of a small amount of additive noise. However, currently much emphasis is put on equation (2) to identify the roots correctly. As the noise amount increases, the perturbation of the roots by the additive noise is more likely to cause the GCD operation to identify the wrong roots. We note that the large set of equations in (1) could potentially provide a redundant check to minimize the probability of misidentification, although we have yet to come up with an algorithm to achieve that.

This work is partially supported under the Programmable Digital Camera project by Intel, Hewlett-Packard, Kodak, Canon, and Interval Research.

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