

MULTILEVEL FAST MULTIPOLE ALGORITHM FOR SOLVING COMBINED FIELD INTEGRAL EQUATION OF ELECTROMAGNETIC SCATTERING †

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Key Terms

Fast multipole, multilevel algorithm, integral equation, method of moments, electromagnetic scattering.

Abstract

The fast multipole method (FMM) has been implemented to speed up the matrix-vector multiply when an iterative method is used to solve combined field integral equation (CFIE). FMM reduces the complexity from $O(N^2)$ to $O(N^{1.5})$. With a multilevel fast multipole algorithm (MLFMA), it is further reduced to $O(N\log N)$. A 110,592 unknown problem can be solved within 24 hours on a SUN Sparc10.

1. Introduction

The electromagnetic (EM) field scattering by three-dimensional (3D) arbitrarily shaped conductor can be obtained by finding the solution of an integral equation where the unknown function is the induced current distribution. The integral equation is discretized into a matrix equation by the method of moments (MOM). The resultant matrix equation

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is then solved by Gaussian elimination, which requires $O(N^3)$ floating-point operations if Gaussian elimination is used to solve N linear equations, or $O(N^2)$ operations per iteration if the conjugate gradient (CG) method is used.

The fast multipole method (FMM) [1-9] speeds up the matrix-vector multiply in the conjugate gradient (CG) method when it is used to solve the matrix equation iteratively. In this paper, FMM is applied to solve the electromagnetic scattering from three dimensional arbitrary shape conducting bodies. The electric field integral equation (EFIE), magnetic field integral equation (MFIE), and combined field integral equation (CFIE) are considered. FMM formula for CFIE has been derived, which reduces the complexity of a matrix-vector multiply from $O(N^2)$ to $O(N^{1.5})$, where N is the number of unknowns. With a nonnested method, using the ray-propagation fast multipole algorithm (RPFMA) [5,7], the cost of a FMM matrix-vector multiply is further reduced to $O(N^{4/3})$. We have implemented a multilevel fast multipole algorithm (MLFMA), whose complexity is reduced to $O(N \log N)$. This algorithm also requires less memory ($O(N \log N)$), and hence, larger problems can be solved on a small computer.

2. The Combined Field Integral Equation (CFIE)

Practical electromagnetic problems are often three-dimensional and involve arbitrary geometry. The arbitrary surface is described by dividing it into a number of connected patches which are mathematically described as parametric quadratic surfaces [7,10]. For conducting objects, the electric field integral equation (EFIE) is given by

$$\hat{t} \cdot \int_S \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dS' = \frac{4\pi i}{k\eta} \hat{t} \cdot \mathbf{E}^i(\mathbf{r}), \quad (1)$$

for \mathbf{r} on surface S , where \hat{t} is any unit tangent vector on S , and

$$\overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') = \left(\overline{\mathbf{I}} - \frac{1}{k^2} \nabla \nabla' \right) g(\mathbf{r}, \mathbf{r}'), \quad (2)$$

$$g(\mathbf{r}, \mathbf{r}') = \frac{e^{ikR}}{R}, \quad R = |\mathbf{r} - \mathbf{r}'|. \quad (3)$$

For closed conducting objects, the magnetic field integral equation (MFIE) is given by

$$2\pi\mathbf{J}(\mathbf{r}) - \hat{n} \times \nabla \times \int_S dS' g(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}') = 4\pi\hat{n} \times \mathbf{H}^i(\mathbf{r}) \quad (4)$$

for \mathbf{r} approaches to S from outside, where \hat{n} is an outwardly directed normal. MFIE (4) can also be written as

$$2\pi\hat{t} \cdot \mathbf{J}(\mathbf{r}) - \hat{t} \cdot \hat{n} \times \nabla \times \int_S dS' g(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}') = 4\pi\hat{t} \cdot \hat{n} \times \mathbf{H}^i(\mathbf{r}). \quad (5)$$

In general, either EFIE or MFIE can be used for closed conducting objects, but it can be corrupted by the presence of homogeneous solutions corresponding to interior cavity modes. A remedy is to solve combined field integral equation (CFIE) [11]. It has been proven that the CFIE eliminates the effect of these resonances [11]. CFIE for closed conducting objects is simply a linear combination of EFIE (1) and MFIE (5) and is of the form:

$$\alpha\text{EFIE} + (1 - \alpha)\frac{i}{k}\text{MFIE}. \quad (6)$$

The combination parameter α ranges from 0 to 1 and can be chosen to be any value within this range. It is found $\alpha = 0.2$ to be an overall good choice.

To apply the method of moments (MOM) to the CFIE, the unknown current $\mathbf{J}(\mathbf{r})$ is first expanded in an appropriately chosen set of basis functions $\{\mathbf{j}_i\}$. Then, it is multiplied by a set of N testing functions $\{\mathbf{t}_j\}$ and the inner products are taken:

$$\sum_{i=1}^N A_{ji} a_i = F_j, \quad j = 1, 2, \dots, N, \quad (7)$$

where

$$A_{ji} = \alpha \int_S dS \mathbf{t}_j(\mathbf{r}) \cdot \int_S dS' \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{j}_i(\mathbf{r}') \\ + (1 - \alpha) \left[\frac{2\pi i}{k} \int_S dS \mathbf{t}_j(\mathbf{r}) \cdot \mathbf{j}_i(\mathbf{r}) - \frac{i}{k} \int_S dS \mathbf{t}_j(\mathbf{r}) \cdot \hat{n} \times \nabla \times \int_S dS' g(\mathbf{r}, \mathbf{r}') \mathbf{j}_i(\mathbf{r}') \right]. \quad (8)$$

Consequently, the integral equations are approximated by matrix equations (7) using the method of moments (MOM). The specially designed basis functions for subdomains which contain surface curvature are the generalized “rooftop” basis functions with a constant charge density and without line charge accumulation on the surfaces [10].

3. The Fast Multipole Method (FMM)

The FMM idea is first to divide the subscatterers into groups. Then, addition theorem is used to translate the scattered field of different scattering centers within a group into a single center (aggregation). Hence, the number of scattering centers is reduced. Similarly, for each group, the field scattered by all the other group centers can be first “received” by the group center, and then “redistributed” to the subscatterers belonging to the group (disaggregation).

Letting \mathbf{r}_j and \mathbf{r}_i be the field point and source point, respectively, we have

$$\mathbf{r}_{ji} = \mathbf{r}_j - \mathbf{r}_i = \mathbf{r}_j - \mathbf{r}_m + \mathbf{r}_m - \mathbf{r}_{m'} + \mathbf{r}_{m'} - \mathbf{r}_i = \mathbf{r}_{jm} + \mathbf{r}_{mm'} - \mathbf{r}_{im'}, \quad (9)$$

where \mathbf{r}_m and $\mathbf{r}_{m'}$ are the centers of the m -th and m' -th groups, which \mathbf{r}_j and \mathbf{r}_i are belong to, respectively. Then, using the addition theorem [2,12] we derive a formula to calculate the matrix elements in non-nearby group pairs [7,9],

$$A_{ji} = \frac{ik}{4\pi} \int d^2\hat{k} \mathbf{V}_{fm_j}(\hat{k}) \cdot \alpha_{mm'}(\hat{k} \cdot \hat{r}_{mm'}) \mathbf{V}_{sm'i}^*(\hat{k}), \quad (10)$$

where

$$\mathbf{V}_{fm_j}(\hat{k}) = \alpha \int_S dS e^{i\mathbf{k} \cdot \mathbf{r}_{jm}} (\bar{\mathbf{I}} - \hat{k}\hat{k}) \cdot \mathbf{t}_j(\mathbf{r}_{jm}) - (1 - \alpha)\hat{k} \times \int_S dS e^{i\mathbf{k} \cdot \mathbf{r}_{jm}} \mathbf{t}_j(\mathbf{r}_{jm}) \times \hat{n}, \quad (11)$$

$$\mathbf{V}_{sm'i}(\hat{k}) = \int_S dS e^{i\mathbf{k} \cdot \mathbf{r}_{im'}} (\bar{\mathbf{I}} - \hat{k}\hat{k}) \cdot \mathbf{j}_i(\mathbf{r}_{im'}). \quad (12)$$

$$\alpha_{mm'}(\hat{r}_{mm'} \cdot \hat{k}) = \sum_{l=0}^L i^l (2l+1) h_l^{(1)}(kr_{mm'}) P_l(\hat{r}_{mm'} \cdot \hat{k}). \quad (13)$$

When we use an iterative method to solve the matrix equations (7), a matrix-vector multiply is to be computed in each iteration. Then, we can write the matrix-vector multiply as

$$\sum_{i=1}^N A_{ji} a_i = \sum_{m' \in B_m} \sum_{i \in G_{m'}} A_{ji} a_i + \frac{ik}{4\pi} \int d^2 \hat{k} \mathbf{V}_{fmj}(\hat{k}) \cdot \sum_{m' \notin B_m} \alpha_{mm'}(\hat{k} \cdot \hat{r}_{mm}) \sum_{i \in G_{m'}} \mathbf{V}_{sm'i}^*(\hat{k}) a_i \quad (14)$$

for $j \in G_m$, where G_m denotes all elements in the m -th group, and B_m denotes all nearby groups of the m -th group (including itself). The first term in (14) is the contribution from the nearby groups, and the second term is the far interaction calculated by FMM. It has been proven that the computation cost using (14) with 2-level FMM is of order $O(N^{1.5})$ [2,7]. Numerical simulations also show that the complexity is of order $O(N^{1.5})$ [9].

4. Multilevel Fast Multipole Algorithm (MLFMA)

To implement a multilevel fast multipole algorithm (MLFMA), the entire object is first enclosed in a large cube, which is partitioned into eight smaller cubes. Each subcube is then recursively subdivided into smaller cubes until the edge length of the finest cube is about half of a wavelength. Cubes at all levels are indexed. At the finest level, we find the cube in which each basis function resides by comparing the coordinates of the center of the basis function with the center of cube. We further find nonempty cubes by sorting. Only nonempty cubes are recorded using tree-structured data at all levels [13,14]. Thus, the computational cost depends only on the nonempty cubes.

The basic algorithm for matrix-vector multiply is broken down into two sweeps [15]: the first sweep consists of constructing outer multipole expansions for each nonempty cube at all levels. The second sweep consists of constructing local multipole expansions contributed from the well-separated cubes at all levels. When the cube becomes larger as one progresses from the finest level to the coarsest level, the number of multipole expansions should increase. In the first sweep, the outer multipole expansions are computed at the finest level, and then the expansions for larger cube are obtained using shifting and interpolation. Let $\mathbf{r}_{m'_l}$ and $\mathbf{r}_{m'_{l-1}}$ be the cube centers at level l and $l-1$, respectively; then the outer multipole

expansions for coarser level $l - 1$ should be

$$\mathbf{V}_{sm'_{l-1}i}(\hat{\mathbf{k}}) = e^{i\mathbf{k}\cdot\mathbf{r}'_{m'_l m'_{l-1}}} \mathbf{V}_{sm'_i}(\hat{\mathbf{k}}). \quad (15)$$

But $\mathbf{V}_{sm'_i}(\hat{\mathbf{k}})$ has only K_l values, and we need K_{l-1} values of $\mathbf{V}_{sm'_{l-1}i}(\hat{\mathbf{k}})$. Therefore we will interpolate $\mathbf{V}_{sm'_i}(\hat{\mathbf{k}})$ to K_{l-1} values first. That is

$$\mathbf{V}_{sm'_{l-1}i}(\hat{\mathbf{k}}_{(l-1)n'}) = e^{i\mathbf{k}_{(l-1)n'}\cdot\mathbf{r}'_{m'_l m'_{l-1}}} \sum_{n=1}^{K_l} W_{n'n} \mathbf{V}_{sm'_i}(\hat{\mathbf{k}}_{ln}), \quad (16)$$

where interpolation matrix W is a sparse matrix.

At the coarsest level, the local multipole expansions contributed from well-separated cubes are calculated using the second part of (14). At the second sweep, the local expansions for smaller cubes include the contributions from parent cube using shifting and antepolation [16], and from the well-separated cubes at this level but not well-separated at the parent level. If the local multipole expansions received by a cube center at level $l - 1$ is $\mathbf{B}(\hat{\mathbf{k}})$, then the contribution from all well-separated cubes can be written as

$$I = \int d^2\hat{\mathbf{k}} \mathbf{V}_{fm_{l-1}j}(\hat{\mathbf{k}}) \cdot \mathbf{B}(\hat{\mathbf{k}}) = \sum_{n'=1}^{K_{l-1}} w_{n'} \mathbf{V}_{fm_{l-1}j}(\hat{\mathbf{k}}_{(l-1)n'}) \cdot \mathbf{B}(\hat{\mathbf{k}}_{(l-1)n'}), \quad (17)$$

where $w_{n'}$ is the weighting function. Substituting the interpolation expression for $\mathbf{V}_{fm_{l-1}j}(\hat{\mathbf{k}}_{(l-1)n'})$ into (17), and changing the order of two summations lead to

$$I = \sum_{n=1}^{K_l} w_n \mathbf{V}_{fm_{l-1}j}(\hat{\mathbf{k}}_{ln}) \cdot \sum_{n'=1}^{K_{l-1}} W_{n'n} \mathbf{B}(\hat{\mathbf{k}}_{(l-1)n'}) e^{i\mathbf{k}_{(l-1)n'}\cdot\mathbf{r}'_{m'_l m'_{l-1}}} w_{n'} / w_n. \quad (18)$$

The above operation is called antepolation [16]. At the finest level, the contributions from non-well-separated cubes are calculated directly. Since only nonempty cubes are considered, the complexity for MLFMA is reduced to $O(N\log N)$, and the memory requirements

for MLFMA are of order $O(N\log N)$. Dembart and Yip [4,8] have implemented MLFMA using radiation function, interpolation and filtering, which has a complexity $O(N\log^2 N)$.

5. Numerical Results

In Figure 1, we plot the normalized residual norm in CG method as functions of the number of iterations for EFIE, MFIE, and CFIE. It is found that CFIE converges much faster than EFIE and MFIE. EFIE and MFIE may not converge or converge too slowly at specific frequencies corresponding to the eigenvalues of the interior resonator problem, and give a wrong solution. At all frequencies, CFIE has a unique solution, and converges faster than EFIE and MFIE if the matrices are solved by an iterative method since the matrix for CFIE has smaller condition number than those for EFIE and MFIE.

Figure 2 shows the validation of the numerical results from EFIE, MFIE, and CFIE with 2-level FMM against the Mie series solution of the bistatic RCS of a conducting sphere of radius 1m at frequency of 0.72 GHz for the parallel polarization. 9,408 unknowns are used. The solution of CFIE with FMM agrees with Mie series very well, and much better than the solutions of EFIE and MFIE.

Figure 3 shows the CPU time per iteration and memory requirements for MLFMA as functions of the number of unknowns for using disk (partially out of core) and without using disk (in core), respectively. The numerical simulations are performed on a SUN Sparc10 workstation, and for calculating the bistatic RCS of a conducting sphere by solving CFIE. About 7.6 unknowns per wavelength are used. It is found that simulations using disk needs CPU time about 50% more than that without using disk. On a workstation with 128 MB core memory, about 30,000 unknowns problem can be solved in core. Using 300 MB of a disk, we can solve about 110,000 unknown problem on the workstation. In Figure 4, we plot the normalized bistatic RCS of a conducting sphere of radius 1m at 2.4 GHz (the diameter is 16λ). A total of 110,592 unknowns with 6-level FMM is used. It takes 24 hours on the workstation. A good agreement between our result and Mie series is observed.

Figure 5 shows the monostatic RCS for $2.5 \times 2.5 \times 3.75 \lambda^3$ open cavity as functions of observed angles. Numerical results are obtained by using LUD and CG with 3-level FMM.

The experimental measurement by Naval Air Warfare Center is also given for comparison. There is a good agreement between these results for both HH and VV polarizations.

Figure 6 shows the bistatic RCS for 15λ conducting cube as functions of bistatic angles. Our numerical results are compared with numerical results using high frequency solution [18,19] and dual surface MFIE (DMFIE) [18,19]. All results agree with each other very well. We use 132,300 unknowns (7 unknowns per wavelength) and 6-level FMM. It takes 35 hours on the SUN Sparc10 workstation (about 10 Mflops). In [19], iterative solver is used to solve the full matrix from DMFIE. In that case, they use 4.9 unknowns per wavelength and two-fold symmetry to reduce the number of unknowns to 16,428. It needs 2.16 GB of memory for the matrix alone, and takes 174 minutes of CPU time on a nCube 2 hypercube (about 3.6 Gflops). It is clear that MLFMA needs much less memory and takes much shorter CPU time than the standard matrix solver.

6. Conclusions

The fast multipole method (FMM) has been implemented to speed up the matrix-vector multiply when an iterative method is used to solve EFIE, MFIE, and CFIE. At all frequencies, CFIE has a unique solution, and converges faster than EFIE and MFIE. The FMM approach reduces the complexity of a matrix-vector multiply from $O(N^2)$ to $O(N^{1.5})$. With a multilevel fast multipole algorithm (MLFMA), the complexity is reduced to $O(N\log N)$. The FMM also requires less memory, and hence, can solve a larger problem on a small computer. About 110,000 unknown problem is solved within 24 hours on a SUN Sparc10 workstation.

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Figure Captions

Figure 1. Comparison of the convergence of solutions of EFIE, MFIE, and CFIE for a conducting sphere of radius 1 m at 0.72 GHz for parallel polarization (9,048 unknowns, 7.2 unknowns per wavelength).

Figure 2. Validations of EFIE, MFIE, and CFIE with 2-level FMM against the Mie series solution of the bistatic RCS of a conducting sphere of radius 1 m at 0.72 GHz for parallel polarization. The RCS is normalized by πa^2 .

Figure 3. CPU time per iteration (a) and memory requirements (b) as functions of number of unknowns for MLFMA.

Figure 4. Validations of CFIE with MLFMA against the Mie series of the bistatic RCS of a metallic sphere of radius 1m at 2.4 GHz for VV polarization. 110,592 unknowns with 6-level FMM are used. The RCS is normalized by πa^2 .

Figure 5. Monostatic RCS for $2.5 \times 2.5 \times 3.75 \lambda^3$ open cavity. Solid curve, LUD; dashed curve, Measured by Naval Air Warfare Center [17]; points, MLFMA. (a): HH polarization; (b): VV polarization.

Figure 6. Bistatic RCS for 15λ cube. Solid curve, MLFMA with 6 levels and 132,300 unknowns; dashed curve, HF solution [18,19]; points, DMFIE [19]. (a): in E -plane; (b): in H -plane.

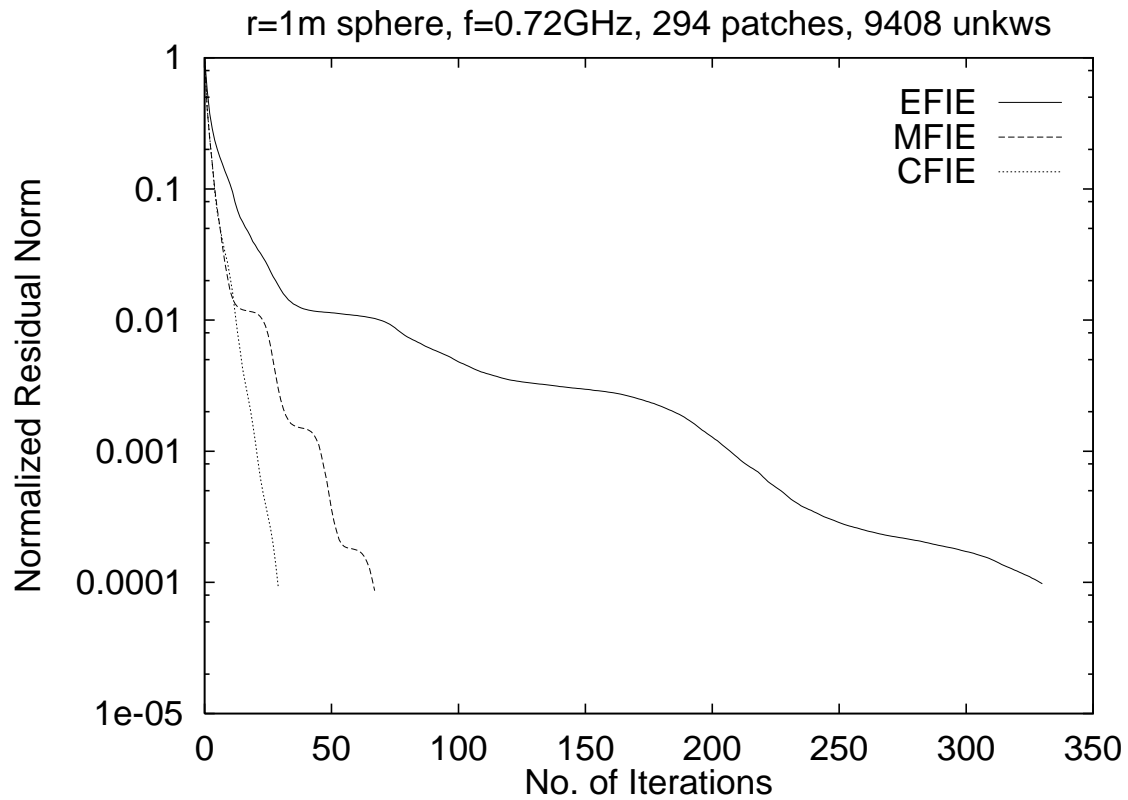


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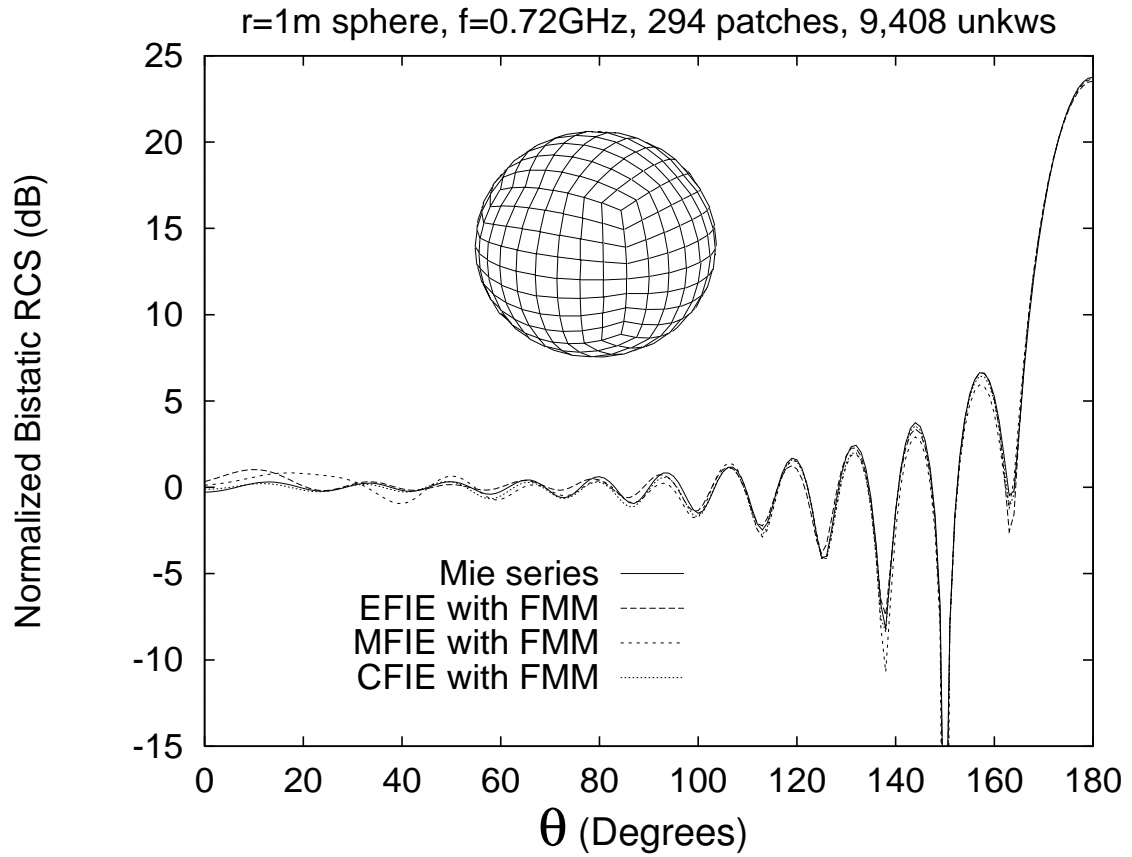


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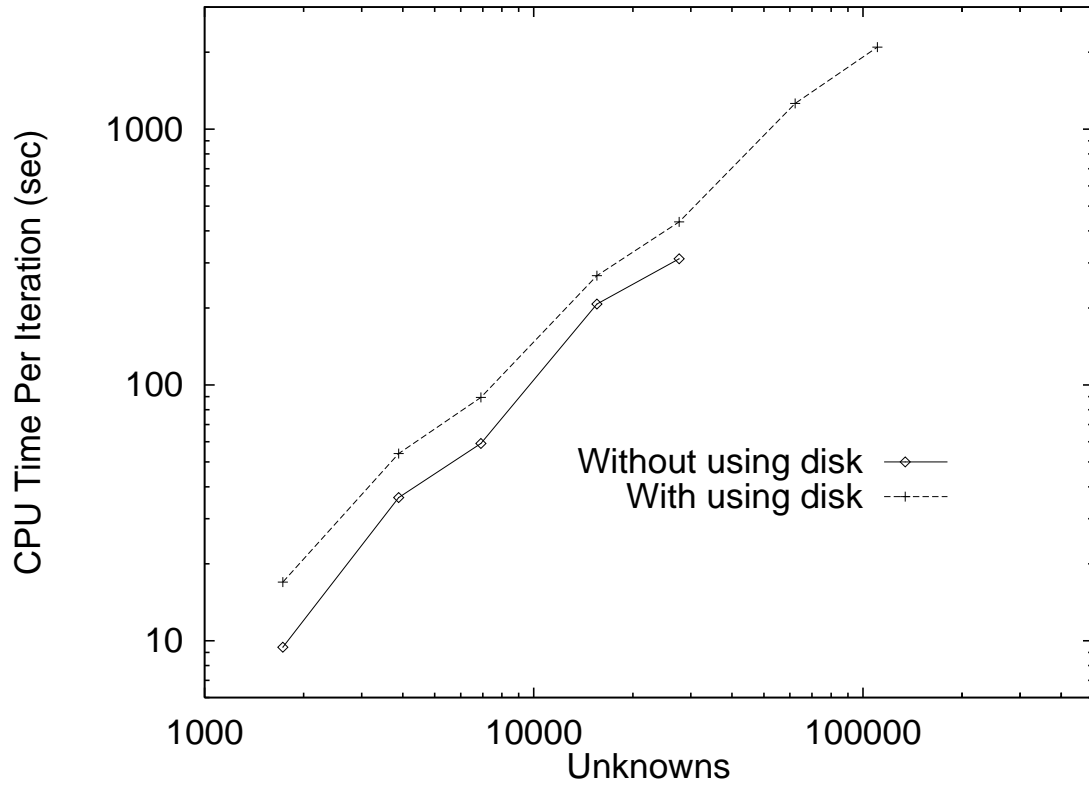


Figure 3a. CPU time per iteration (a) and memory requirements (b) as functions of number of unknowns for MLFMA.

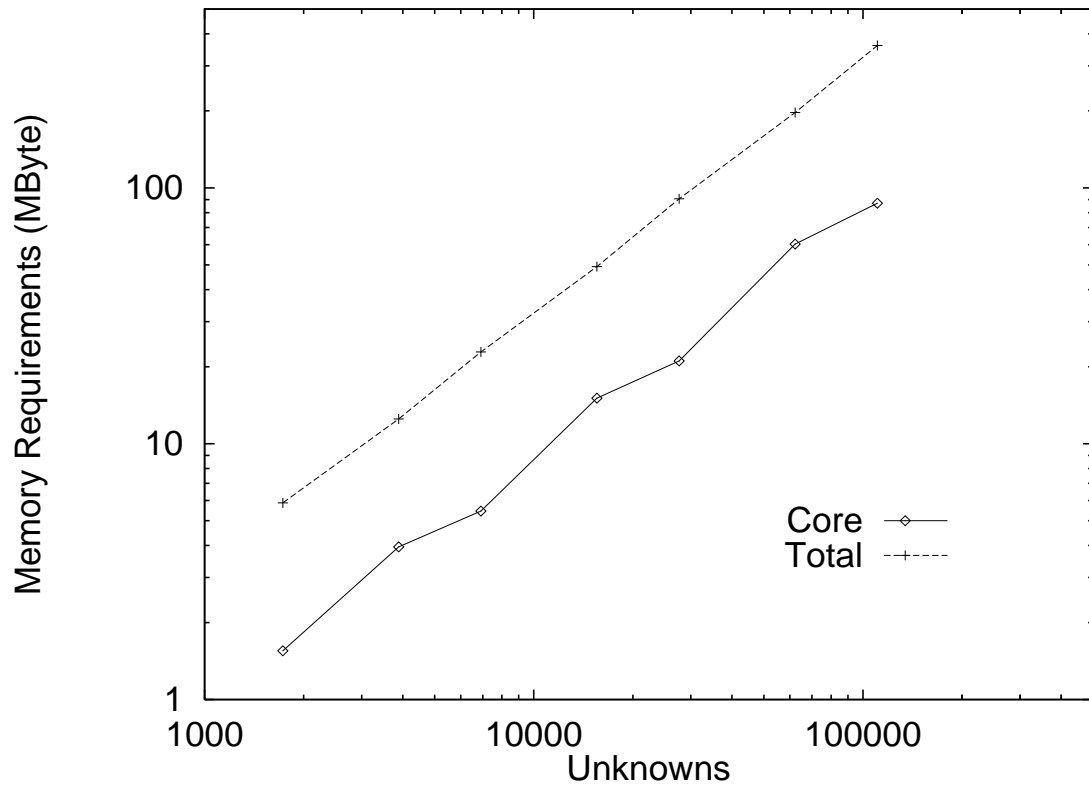


Figure 3b.

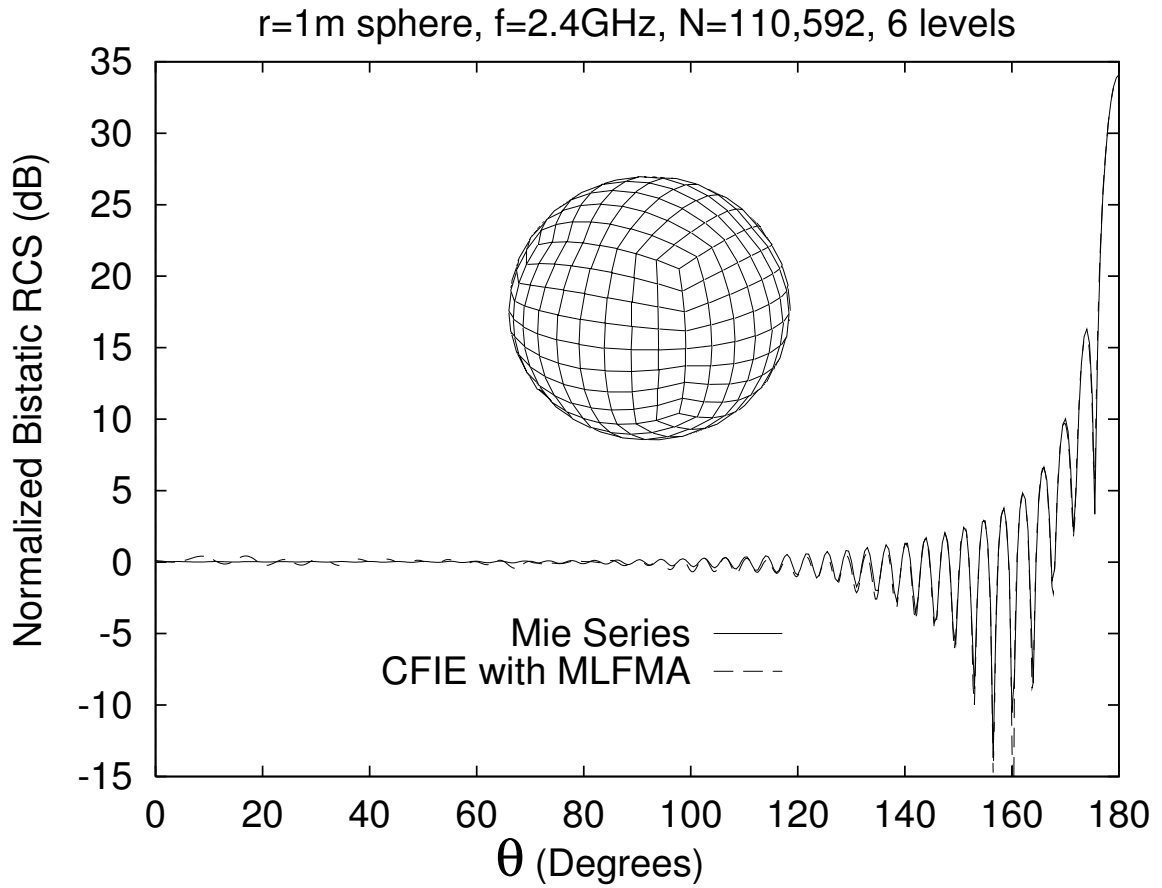


Figure 4. Validations of CFIE with MLFMA against the Mie series of the bistatic RCS of a metallic sphere of radius 1m at 2.4 GHz for VV polarization. 110,592 unknowns with 6-level FMM are used. The RCS is normalized by πa^2 .

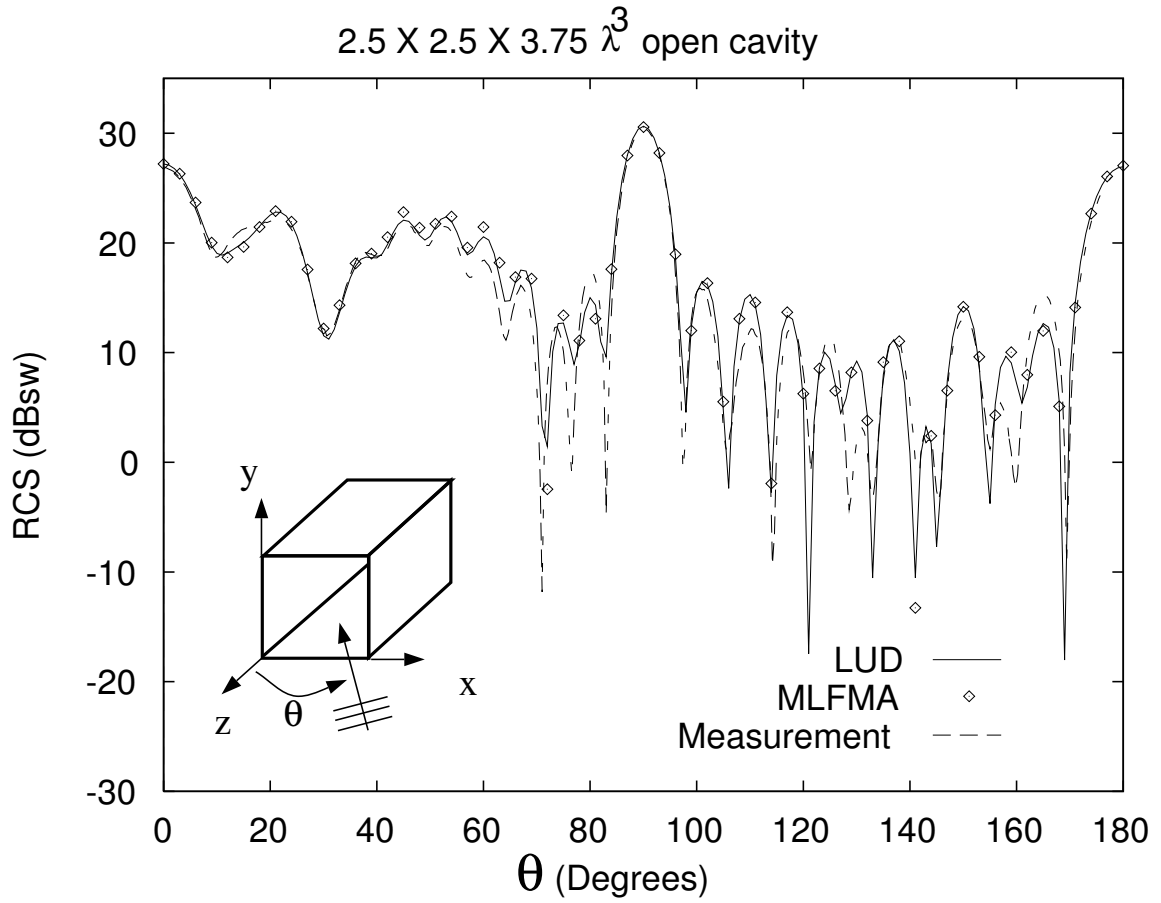


Figure 5. Monostatic RCS for $2.5 \times 2.5 \times 3.75 \lambda^3$ open cavity. Solid curve, LUD; dashed curve, Measured by Naval Air Warfare Center [17]; points, MLFMA. (a): HH polarization.

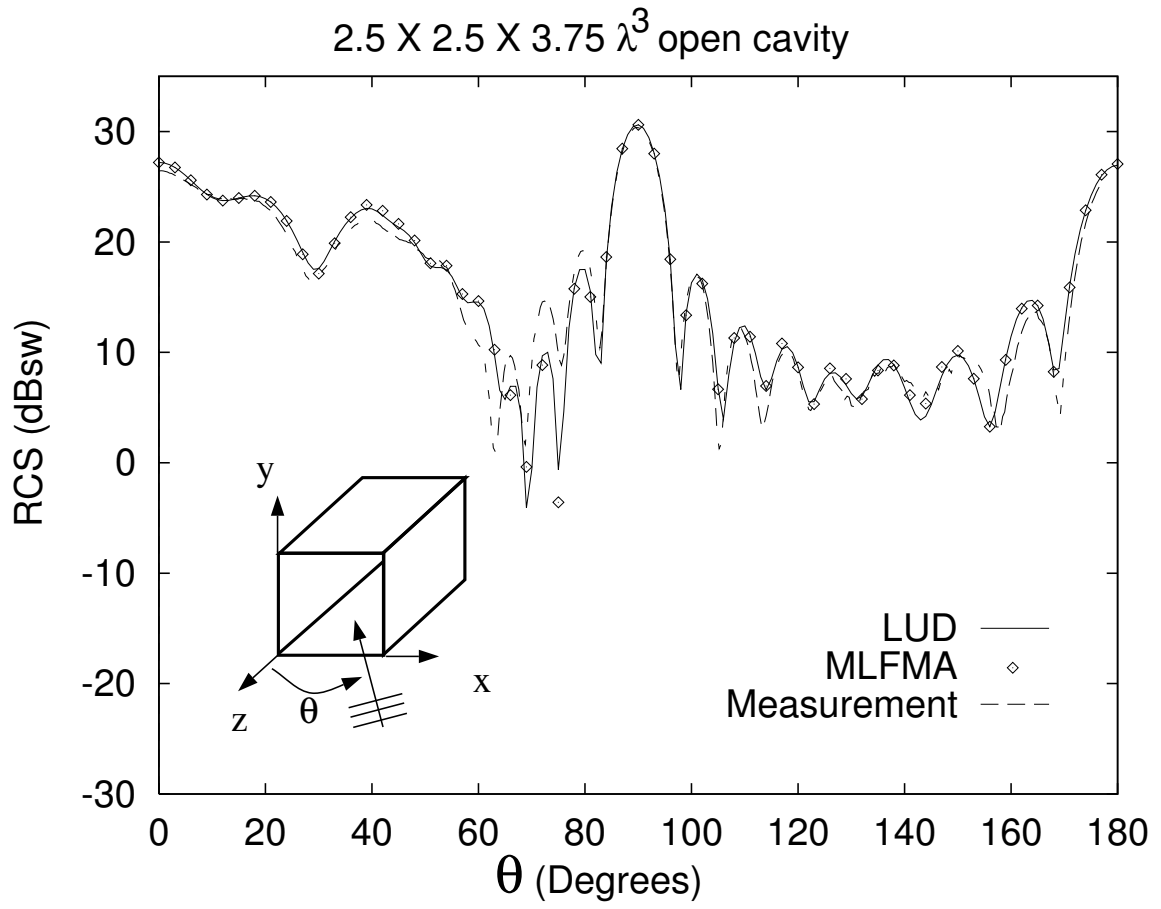


Figure 5b. (b): VV polarization.

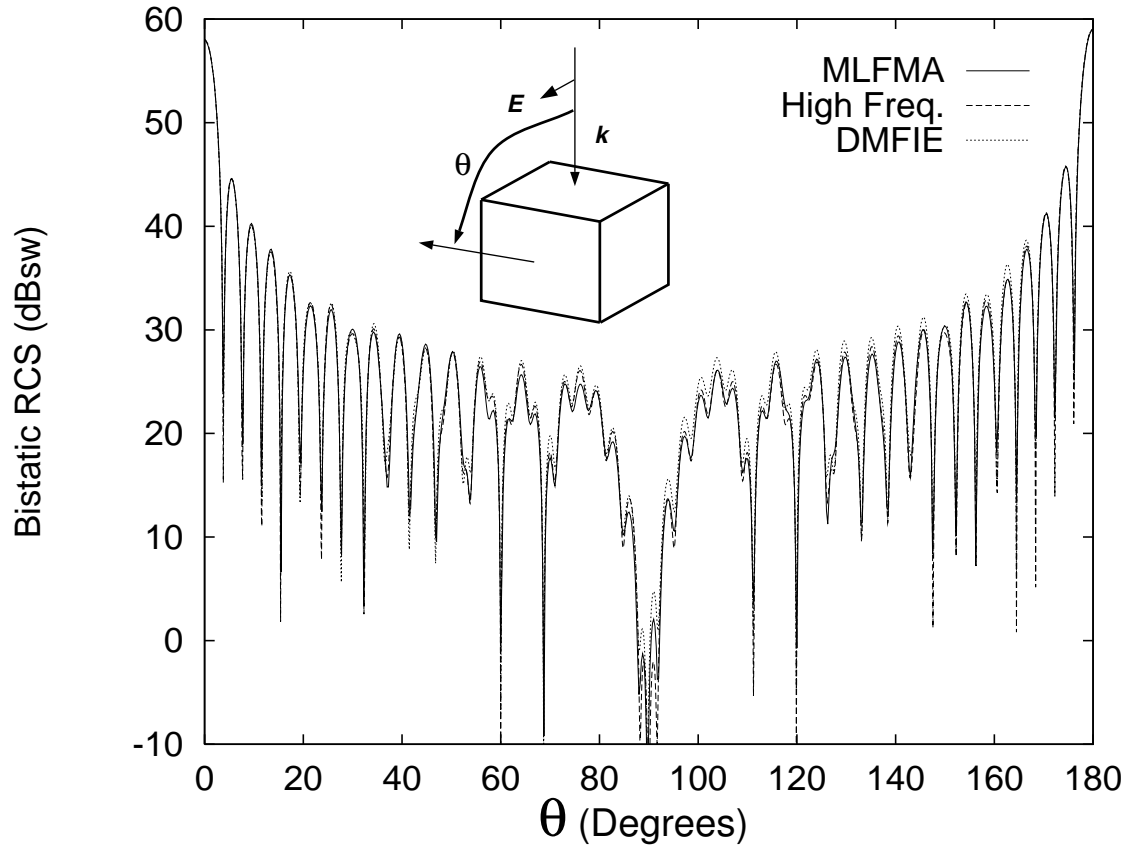


Figure 6a. Bistatic RCS for 15λ cube. Solid curve, MLFMA with 6 levels and 132,300 unknowns; dashed curve, HF solution [18,19]; points, DMFIE [19]. (a): in E -plane.

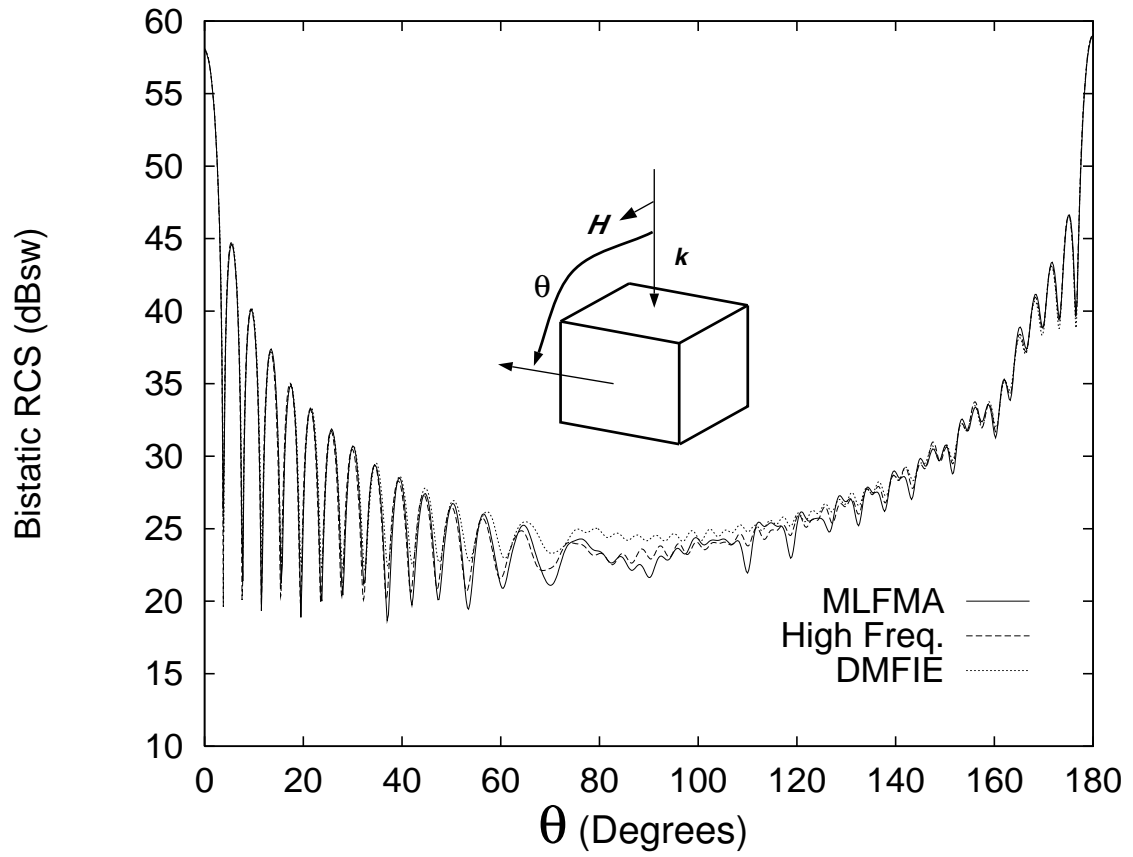


Figure 6b. (b): in H -plane.