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Application of Tikhonov regularization to super-resolution reconstruction of brain MRI image

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Abstract. This paper presents an image super-resolution method that enhances spatial resolution of MRI images in the slice-select direction. The algorithm employs Tikhonov regularization, using a standard model of general imaging process and then reformulating the reconstruction as a regularized minimization problem. Our experimental result shows improvements in both signal-to-noise ratio and visual quality.

Key words: Super-resolution Reconstruction, MRI, Tikhonov Regularization.

1 Introduction

Magnetic resonance imaging (MRI) is a non-invasive diagnostic technique to produce computerized images of internal body tissues. Its major goal is to maximize the image spatial resolution so as to provide accurate information for investigators. In this sense, only the improvement of in-plane resolution is not enough. To obtain high-resolution (HR) data in 3-D, the spatial resolution of the standard MRI protocol does not suffice [1], such as diffusion-weighted imaging (DWI) and echo-planar imaging (EPI). That is because acquiring HR images, especially in slice-select direction, would result in the reduction in signal-to-noise ratio (SNR). SNR is proportional to the main magnetic field strength. The decrease in SNR might be obviated by the usage of higher magnetic field scanners, but the corresponding changes in both T1 and T2 would further reduce the gain in SNR at the ultrahigh-field-strength MRI [2]. Moreover, higher magnetic strength would both increase inhomogeneity and introduce distortion artifacts into images. Therefore, super-resolution technique, as a post-processing method, has been introduced to enhance the resolution of MRI images.

Image super-resolution reconstruction is to restore a HR image from several low-resolution (LR) images taken from the same scene, but slightly different view point. Although these images may be translated, blurred, rotated, or corrupted with noise, they can be useful to provide different information for a HR image. Recently the method has been used for the improvement of MRI image

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quality. This application attracted a number of super-resolution reconstruction algorithms to be used in it. Irani and Peleg [3] introduced the Iterative Back Projection (IBP) into the reconstruction. Peled and Yeshurun [4] first presented an implementation of IBP into MRI data, which aimed to reconstruct images of human white matter fiber tract from Diffusion Tensor Imaging (DTI). IBP, because of its simplicity and easy implementation, was frequently utilized in the early development of super-resolution. But the algorithm was also known for its low rate of convergence and sensitivity to noise. Then Hsu *et al.* [5] proposed the application of wavelet-based Projection onto Convex Sets (POCS) super-resolution into cardiovascular MRI images. It extracted information from the non-stationary effect of heart and blood vessels in the successive images to reconstruct a HR image. As for stationary objects, such as the brain, it is not effective to perform a good reconstruction. But requirements of high quality brain MRI images are increasing now in both the neurology and the medicine. Therefore, we take Tikhonov regularization into this application to obtain a brain MRI image super-resolution reconstruction.

Tikhonov regularization is the most commonly used method in the regularization to ill-conditioned problems. In these problems, even small changes in input can result in wild oscillations in the approximation of a solution. In general, image restoration is ill-conditioned and difficult to find out a unique solution directly. Such is the case in MRI image super-resolution reconstruction. To achieve a reasonable solution, we use Tikhonov regularization to reformulate the problem as a regularized unconstrained minimization problem. Then it has a unique minimizer, i.e., the reasonable solution for the reconstruction problem.

In this work we take use of Tikhonov regularization into brain MRI image super-resolution reconstruction. This technique is utilized on a simulation and real brain MRI images to demonstrate the applicability and performance of the super-resolution reconstruction in MRI.

2 Methods

2.1 Observation Model

To perform a super-resolution reconstruction, the first step is to formulate an observation model that relates the original HR image to the acquired LR images. We consider the following problem:

$$Y = Hf + n\,, \tag{1}$$

where Y and f are vectors of length m representing the acquired LR images and the original HR image respectively, and H is an $m \times m$ linear operator that characterizes the degrading process. n is the vector of length m that represents the additive Gaussian white noise contaminating the measurement. So the problem is to determine the HR image given acquired LR images with the degradation matrix and the additive noise.

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2.2 Tikhonov regularization

Generally, the super-resolution reconstruction is an ill-posed problem because of an insufficient number of LR images and ill-conditioned degrading operator. It is necessary to rely on a regularization to stabilize the inversion of ill-posed problem. Here we take use of Tikhonov regularization for the reconstruction. Through the regularization, the problem (1) is replaced by the problem of seeking an estimate x to minimize the Lagrangian:

$$\min_f \left[\|Y - Hf\|_2^2 + \alpha \|Cf\|_2^2 \right] , \tag{2}$$

where the operator C is generally a high-pass filter, and $\| \cdot \|$ represents L_2 norm. The first term measures the fidelity of the solution to the data while the second term manages the smoothness of the solution. α denotes the Lagrange multiplier, commonly referred to as the regularization factor, that controls the tradeoff between the two terms. C is often chosen as the Laplacian operator to smooth the solution. So the minimizer of (2) expressed as normal equations is:

$$H^T Y = (H^T H - \alpha C^T C) f \, . \tag{3}$$

where H and C are block Toeplitz matrices. Equation (3) can be solved by Conjugate Gradient (CG), because it is more advantageous than others to solve large, sparse and symmetric positive definite linear system.

2.3 Signal-to-noise Ratio (SNR)

When it comes to a method for digital image restoration, one should ensure that SNR is not compromised. Because super-resolution reconstruction belongs to the category, it is necessary to take SNR into account. In general, SNR is defined as the ratio of the mean pixel values to the standard deviation of the pixel values outside an interest. For the purpose of demonstrating the improvement of the method, this definition is still satisfactory.

3 Results

3.1 Simulation

A high-SNR, HR (64×64, 1 mm in-plane) image is shifted and contracted so as to create 4 images (32×32). The first two images are acquired with 0- and 1-mm shift in the phase-encode direction, and copies of the two images are shifted by 1 mm in the frequency-encode direction to get the other two images. Then Gaussian white noise is added to the four LR images to give them a SNR of 25dB. A part of one of the LR shifted images with additive noise is shown in Fig. 1A.

The reconstructed image after Tikhonov regularization is shown in Fig. 1B. The original HR image is shown for comparison in Fig. 1C. During the simulation, SNR in LR images has been chosen from 5dB to 25dB. All reconstructed images show the corpus callosum clearer than LR images and at the same time, decrease the impact of noise effectively.

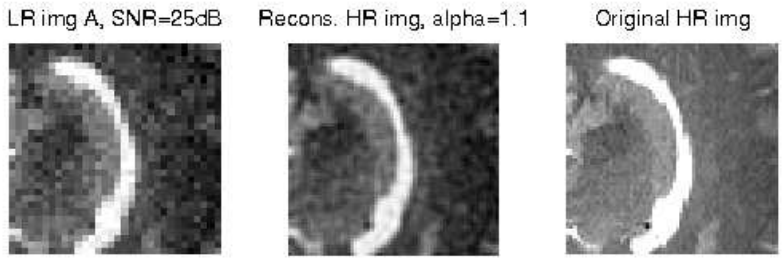

Fig. 1. The reconstructed HR image

3.2 Brain Imaging

We perform super-resolution reconstruction on human brain data. Owing to an inherent characteristic of MRI modality, the super-resolution postprocessing method could only be employed in the slice-select direction [9]. So in this case we set the slice thickness in LR images wide enough to acquire HR data with the same slice width as the reconstructed image. It is intended to analyze the improvement of the super-resolution reconstruction method. Our goal is to achieve a result whose image quality is as close as that of HR data, based on acquired LR images. The resolution in LR slices is set to be 1 mm, and the slice thickness is 4 mm. The first set of LR slices will include 28 slices. The second set of LR slices is slightly shifted up in the slice-select direction by 1 mm. The third and fourth set is shifted in the direction by 2 mm and 3 mm, respectively. Then in the first set, we extract one column of every slice in the same index to obtain the first LR image. Taking use of the same procedure, we achieve the other three LR images. They make up of the four LR images for a reconstruction.

One of LR images is shown in Fig. 2A. Figure 2B is the reconstructed HR image and Figure 2C is a HR image for a comparison. As the comparison shows, the reconstructed image contains clearer information thanks to the high resolution. So the improvement in the result is obvious. SNR is 11.53dB and 8.01dB in reconstructed HR image and original HR image, respectively.

4 Discussion and Conclusion

In this work, we investigate the possibility of using Tikhonov regularization to perform super-resolution reconstruction in human brain MRI images. Simulation result shows Tikhonov regularization is effective to be used in the super-resolution reconstruction. It is amenable to noise, to a large extent. The range of original SNR in LR images is from 5dB to 25dB in the simulation. The reconstructed image is always able to provide more readable information than noisy LR images. In the real data experiment, these LR images are extracted from the four sets of LR slices. The reconstructed image in the slice-select direction

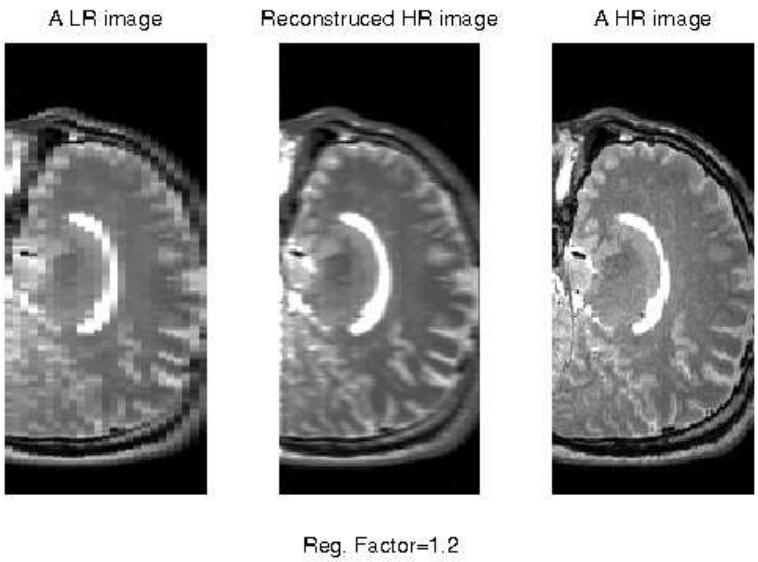

Fig. 2. The reconstructed HR image

presents clearer details than LR image because of the higher resolution, and gives acceptable SNR values.

To conclude, Tikhonov regularization is applicable to super-resolution reconstruction of brain MRI images. It works well to improve the resolution in the slice-select direction of MRI data without the loss of SNR.

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References

1. Greenspan H., Oz G., Kiryati N. and Peled S., MRI inter-slice reconstruction using super resolution. Magn Reson Imaging (2002);20:437-46
2. Eyal C., Siuyan L., Noga A., Amos F., Daniel F. Resolution enhancement in MRI Magn Reson Imaging (2006);24:133-54
3. Irani, M. and Peleg, S.: Motion analysis for image enhancement: resolution, occlusion, and transparency. J Vis Comm Image Rep (1993);4:324–335
4. Peled, S. and Yeshurun, Y.: Superresolution in MRI : Application to human white matter fiber tract visualization by diffusion tensor imaging. Magn Reson Med (2001);45(1):29–35
5. Hsu, J.T. et al.: Application of wavelet-based POCS super-resolution for cardiovascular MRI image enhancement. Proc. of ICIG'04 (2004);00:524–575

6. Katsaggelos, A.K.(ed.): Digital Image Restoration Sprintger Series in Information Sciences, Vol.23. Springer-Verlag (2003)
7. Banham, M.R. and Katsaggelos, A.K.: Digital Image Restoration Sig Proc Mag (1997);14(4):24–41
8. Saad Y.: Iterative methods for sparse linear systems (1st edn) PWS Publ., Company, Boston, Mass. (1996)
9. Peeters R. et al.: The use of super-resolution techniques to reduce slice thickness in functional MRI. Int J of Imag Sys Tech (2004);14(3):131–138