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Quantifying Particle Volumes with Photon-counting Digital Holography

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Abstract: We propose a photon-counting digital holography method to accurately reconstruct particle volumes in low light conditions. Our approach integrates a Poisson digital holography model into an unrolled network, overcoming the challenges of shot noise and low signal-to-noise ratios.

1. Motivation

Digital holography (DH) allows registering and retrieving 3D information based on the interference between the object and reference beams. However, the quality of reconstructed images can be greatly deteriorated by various disturbances, such as photon shot noise. This type of noise is particularly problematic in low light and/or limited exposure conditions, where the number of photons captured by the sensor is low. The statistical randomness of photon arrival creates a signal-dependent noise that can severely corrupt interference fringes and result in relatively low signal-to-noise ratios. To address this issue, we propose a photon counting DH method for holographic imaging under high shot noise disturbance. Our approach maximizes the log likelihood of photon counts by integrating a Poisson digital holography model into an unrolled network. We demonstrate the effectiveness of our method by reconstructing 3D particle volumetric information under these challenging conditions.

2. Poisson digital holography

The 3D particles volume is sliced into L sections along the axial direction with complex transmittance $\vartheta_\ell(p,q) = A_\ell(p,q)\exp(j\phi_\ell(p,q))$ at each layer. Supposing that the hologram created at the sensor plane is partitioned into K unit space area (i.e., $\mathbf{I} = [i_1, i_2, \ldots, i_K]$), where the light exposure remains consistently constant throughout the entire exposure duration, the intensity pattern formed at the image sensor plane is represented using matrix notation as [1]

$$\mathbf{I} = \frac{1}{L}\left\|\sum_{\ell=1}^{L} h_\ell * \vartheta_\ell\right\|^2 = \mathsf{H}\mathbf{x}, \tag{1}$$

where $*$ stands for convolution, and $\mathbf{x} \in \mathbb{C}^M$ is a vector corresponding to the flattened version of the segmented 3D object field with $M = LK$. Each column of $\mathsf{H} \in \mathbb{C}^{K\times M}$ is a discretized representation of the impulse response of the free-space propagation kernel h_ℓ over a distance ℓ, which is estimated by the Fresnel approximation [2] as

$$h_\ell(p,q) = \frac{1}{j\lambda\ell} e^{j\frac{2\pi}{\lambda}} e^{j\frac{\pi}{\lambda\ell}(p^2+q^2)}. \tag{2}$$

In low light conditions, the hologram recorded by the photon-counting detector follows the Poisson distribution [3] and its likelihood is given by

$$p(\mathbf{y} \mid \mathbf{x}) = \prod_{k=1}^{K} \frac{[\alpha\mathsf{H}\mathbf{x}]_k^{y_k} e^{-[\alpha\mathsf{H}\mathbf{x}]_k}}{y_k!}, \tag{3}$$

where $[\cdot]_k$ denotes the k-th element of the vector and the scalar α represents the photon level. Our aim is to retrieve the object light field information from the observation. It can be solved by finding the *maximum a posteriori* (MAP) estimate, which is the solution of the optimization problem

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \left[\alpha\mathbf{1}^T\mathsf{H}\mathbf{x} - \mathbf{y}^T\log(\alpha\mathsf{H}\mathbf{x}) - \log p(\mathbf{x})\right], \tag{4}$$

where $p(\mathbf{x})$ is the object prior constraint. By introducing two auxiliary variables, we change Eq. 4 to a constrained optimization problem, and solve it with an unrolled deep network that maps the iterative process of the traditional algorithm (i.e., ADMM [4]) into a deep network structure.

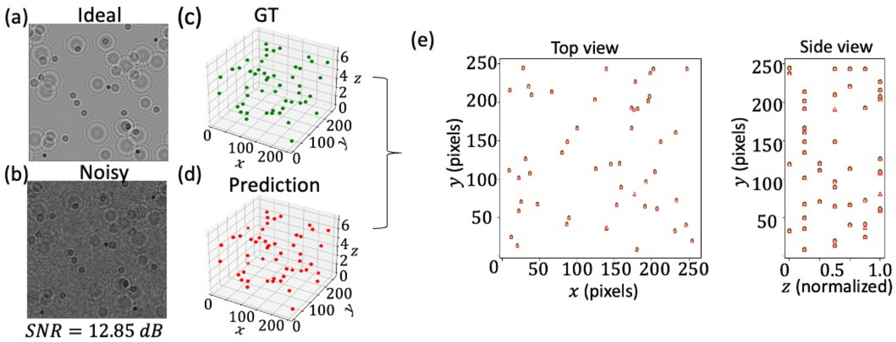

Fig. 1. Prediction example. (a) Hologram under ideal conditions. (b) Hologram with severe shot noise. (c) Ground truth 3D particles volume. (d) Predicted 3D particles volume. (e) 2D projection comparison between ground truth and model inference.

Table 1. Performance comparison between FISTA [5] and our method of 3D particles volumetric prediction with hologarms under different signal-to-noise ratios.

Input SNR(dB)	FISTA [5]		Our method	
	Recall (%)	Precision (%)	Recall (%)	Precision (%)
26.31	99.78	98.86	99.78	98.89
19.41	50.00	99.78	99.77	100.00
12.98	5.35	1.60	96.22	98.89

3. Results and conclusion

We consider a 3D volume in front of the sensor with pixel size of 3.45 µm, illuminated by the planar wave (660 nm). Particles with diameter of 20 µm randomly spread in the volume from 1 cm to 5 cm along the axial direction. The synthetic hologram with severe shot noise contamination (i.e., SNR = 12.85 dB) is shown in Fig. 1(b). As indicated in the 2D projection (Fig. 1(e)), our proposed method successfully reconstructs the particle distribution with a competitive prediction accuracy. Table. 1 presents a quantitative comparison between our method and the state-of-the-art FASTA deconvolution method [5] on holograms affected by different levels of shot noise disturbance. As the SNR of input hologram decreases, FISTA's performance drastically deteriorates, while our method consistently provides reliable particle volumetric information. Even under extreme conditions (i.e., SNR = 12.98 dB), FISTA fails to provide useful information, whereas our method still achieves robust predictions on particle locations with a recall of 96.22 % and precision of 98.89 %. Furthermore, our method offers an advantage over FISTA, which is an iterative Compressed Sensing (CS) method, by avoiding the need for hundreds of iterations. This contributes to a faster inference and more efficient computation after the network training phase. Specifically, for a single reconstruction, our method requires only 0.2 seconds, while FISTA takes 18 seconds, resulting in a time-saving of 90 times. In summary, the proposed photon-counting DH method offers a solution for accurately quantifying particle volumes under low signal-to-noise ratios with severe shot-noise disturbance.

4. Acknowledgement

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