Joint optimization of splitter pattern and image reconstruction for metasurface-based color imaging systems

Haosen Liu, Edmund Y. Lam

Abstract

The metasurface-based splitters have demonstrated great potential in high-resolution, dark-scene, and ultra-fast imaging due to their excellent capabilities of preserving light intensity, attracting increasingly more research interests. This paper investigates the splitter pattern design, which determines the theoretical performance limit of metasurface-based color imaging systems, to guide their further development. We first establish a unified mathematical model for the splitter-based imaging process. This model reveals that the raw image detected by the splitters is degraded by noise, incomplete sampling, and blurring. Based on this, we develop a deep neural network capable of simultaneously tackling multiple image processing tasks to achieve color image reconstruction. We then conduct evaluation experiments for several existing patterns, through which we establish three design principles to guide the optimization of a splitter pattern. Furthermore, by jointly learning the parameters of our proposed imaging model and reconstruction network, we achieve an automatic optimization of a splitter pattern. The experiments demonstrate that the optimized pattern can achieve better quantitative and visual results than the original pattern, validating the effectiveness of this pattern optimization method.

1. Introduction

High-sensitivity image sensors are needed for many challenging applications, e.g., high-resolution, dark-scene, and/or ultra-fast imaging, where the incident light received at each sensor pixel is extremely limited. Conventional color image sensors are generally implemented with color filters [1]. They have an inherent drawback that the color filter will reduce the intensity of the incident light, and thus their light sensitivity cannot meet the requirement of those demanding applications. To tackle this problem, a recent development is to replace the color filter with a metasurface-based color splitter [2,3]. Metasurfaces are flat optical components composed of subwavelength structures [4]. By engineering and arranging these individual structures, metasurfaces are capable of tuning key properties of a light wave (e.g., amplitude and phase) at specific frequencies with subwavelength spatial resolution [5]. Consequently, metasurfaces can act as color splitters that can flexibly divert primary colors (i.e., red, green, and blue) at a single position to specified sensor pixels at different positions. An image sensor with such a metasurface-based splitter is expected to hold a light intensity sensitivity roughly three times higher than that with color filters [6], showing great potential in extremely challenging applications.

The design process of a metasurface-based splitter can be divided into three steps. The first step is to determine the splitter pattern, which involves the arrangement of a set of splitters and their focal points for different colors. Then, the second step is to calculate the phase profile based on the given focal points. Finally, the third step is to optimize the subwavelength structures of metasurfaces to approximate the required phase profile. Up to now, most of the existing works focus their attention on the latter two steps. The relation between focal points and the phase profile can be referred to [7]. The subwavelength structure arrangement can be optimized via various optimization methods, such as gradient descent [6,8], adjoint optimization [9–12] and deep learning methods [13]. However, not much prior work has carefully studied the first step, i.e., the splitter pattern design. As such, the design principles are still not clear, and existing patterns are not well analyzed and evaluated. This is problematic since it is the foundation for the subsequent two steps. In fact, the splitter pattern design determines the theoretical performance limit. As such, here we report for the first time a detailed study on the splitter pattern design in order to further promote the development of splitter-based sensors.

Based on the design philosophy of computational imaging [14], the front-end optics (e.g., the splitter pattern) and the post-processing image reconstruction method should be jointly designed. An efficient
Besides, as shown in Fig. 1(a), a splitter-based imaging system generally consists of a set of splitters that are not unique at different positions. To specify the positions where each type of splitter is placed, we adopt a sampling operator that preserves values at specified positions and replaces values at other positions with 0. As such, the imaging process of splitter-based system can be modeled as

\[ g = \sum_{s \in S} (f \downarrow s \ast w_s) + n, \]  

where \( S \) denotes the set of splitters, \( g \in R^{H\times W} \) denotes the detected raw image, \( f \in R^{H\times W\times c} \) denotes the color image to be detected, \( n \in R^{H\times W} \) denotes the noise, \( \ast \) denotes the transposed convolution, \( \downarrow \) and \( \ast \) denote the sampling operator and parameters of the transposed convolution for the \( s \)-th splitter. Furthermore, it is worth noting that this model has an equivalent form as

\[ g = \sum_{c \in C} ((f \ast k_c) \downarrow c) + n, \]  

where \( C \) denotes the set of colors in the raw image, \( \ast \) denotes the convolution operator illustrated in Fig. 1(c), \( \downarrow \) and \( \ast \) denote the sampling operator and the convolution kernel for the \( c \)-th color in the raw image. Note that this sampling operator specifies the positions where each type of convolution kernel is applied.

Both of these two forms are illustrated in Fig. 1(a). They are mathematically equivalent, but they hold different meanings. The first one focuses on how light information is diverted by each splitter. Parameters \( w_s \) involved in Eq. (1) are directly related to the splitter pattern. Therefore, we implement this form as a splitter network to achieve the joint optimization of the splitter pattern and reconstruction method. By contrast, the second form focuses on how light information is accumulated at each sensor pixel. It can be used to analyze the quality of the raw image captured by detector sensors. More specifically, since Eq. (2) consists of the convolution operator, the sampling operator, and noise variables, it indicates that the detected raw image is blurred among spatial positions and/or color channels, incompletely sampled, and corrupted by noise.

2.2. Reconstruction method

To tackle all of the degradations involved in the splitter-based imaging process and achieve a high-quality recovery of the ground truth color image \( f \), we develop a deep convolutional neural network...
Joint optimization

To further enable the joint optimization of the splitter pattern and the reconstruction network, we implement the imaging model Eq. (1) as a splitter network \( g = \mathcal{E}(\cdot; w) \), substitute \( g \) in the loss function Eq. (3) with this network, and set \( w \) as optimization variables. In this way, the loss function is modified as

\[
\mathcal{L}(\theta, w) = \frac{1}{T} \sum_{t=1}^{T} \| f_t - D(\mathcal{E}(f_t; w); \theta) \|^2.
\]  

(4)

To meet physical requirements, reasonable constraints have to be imposed on parameters \( w \) during the joint optimization. Here, we consider two constraints, i.e., (a) elements of \( w \) should be non-negative and (b) elements of \( w \), for each splitter should sum to 1. To impose these two constraints while avoiding the challenging constrained optimization for networks, we obtain \( w \), by imposing the softmax function \( \tau(\cdot) \) on auxiliary variables \( z \), i.e., \( w = \tau(z) \), since the outputs of a softmax function can naturally meet the above two constraints. Therefore, the final imaging model and the loss function used for joint optimization are

\[
g = \sum_{j \in S} \left( f_j \cdot \tau(z_j) \right) + n,
\]  

(5)

and

\[
\mathcal{L}(\theta, z) = \frac{1}{T} \sum_{t=1}^{T} \| f_t - D(\mathcal{E}(f_t; \tau(z); \theta)) \|^2.
\]  

(6)

By initializing \( z \) with a splitter pattern and optimizing this loss function with the Adam optimizer, we can achieve an automatic optimization of this splitter pattern.

3. Results and analysis

3.1. Splitter pattern evaluation

Splitter patterns are evaluated using the pipeline illustrated in Fig. 3(a). We first generate degraded raw images for each splitter pattern, then estimate the reference image from each raw image using the reconstruction network shown in Fig. 2, and finally evaluate each recovered image. The degraded raw image \( g \) is generated as Eq. (1), where noise \( n \) is assumed to be white Gaussian noise and parameters of transposed convolution \( w \) are determined by the splitter pattern to be evaluated. The five patterns \([2,6,8,37,38]\) involved in evaluation are illustrated in Figs. 3(b)–3(f). For each pattern, a reconstruction network is trained. The reference images used for training are from the dataset DIV2K \([39]\), and those for testing are from the datasets Kodak24 \([40]\) and McMaster \([41]\). During training, Gaussian noise of standard deviations \( \sigma \) ranging from 0 to 30 is added so that the trained model can be applicable to different noise levels. In each training batch, 80 image patches of size 64 \( \times \) 64 are extracted and augmented by flipping horizontally or vertically and rotating 90 degrees. The learning rate is set as \( 10^{-4} \). To quantitatively evaluate the quality of recovered images, we adopt peak signal-to-noise ratio (PSNR) and structure similarity index measure (SSIM) as evaluation indices. All of the average index values of each pattern at each test noise level are provided in Table 1. For visual assessments, several representative results are shown in Figs. 4 and 5. By comparing and analyzing the results of all splitter patterns, we establish three principles as follows:

Avoid using the sub-pixel structure. The sub-pixel structure refers to the structure adopted by Pattern B \([8]\) and Pattern C \([37]\). As illustrated in Figs. 3(c), 3(d), such a structure treats incident light at a set of positions as a whole and splits different colors to its color sub-pixels. This kind of structure is most widely adopted in existing splitter patterns \([3,8–11,37]\). However, as Table 1 shows, evaluation indices achieved by pattern B and pattern C are the worst among the competing patterns in all cases. Also, the visual results shown in Figs. 4 and 5 demonstrate that these two patterns of the sub-pixel structure tend to result in false textures and/or severe blurring effects. The reason behind this phenomenon is that the sub-pixel structure will lead to a severe loss of position information. Since all of the sub-pixels in a single unit share the same position information, it is impossible to recover the lost position information by using the relations among these sub-pixels. Besides patterns using the sub-pixel structure, other patterns are also likely to result in the blurring effect. One example is the result of pattern A \([6]\) in Fig. 4. This is because colors detected by each sensor pixel are collected from different positions. It is inevitable to result in the loss of position to some extent for all splitter patterns. However, different from the case of the sub-pixel structure, the lost position information for other patterns is likely to be recovered by using the

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1 This dataset can be downloaded from the website: https://r0k.us/graphics/kodak/.
Fig. 3. Illustration of the splitter pattern evaluation experiment, where we use (a) the unified pipeline to evaluate (b)–(f) five existing patterns.

Table 1

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Rank</th>
<th>$\sigma = 0$</th>
<th>$\sigma = 5$</th>
<th>$\sigma = 10$</th>
<th>$\sigma = 15$</th>
<th>$\sigma = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSNR</td>
<td>SSIM</td>
<td>PSNR</td>
<td>SSIM</td>
<td>PSNR</td>
<td>SSIM</td>
</tr>
<tr>
<td>Pattern A</td>
<td>2</td>
<td>40.02</td>
<td>0.9764</td>
<td>37.58</td>
<td>0.9569</td>
<td>35.72</td>
</tr>
<tr>
<td>Pattern B</td>
<td>4</td>
<td>32.93</td>
<td>0.9198</td>
<td>32.70</td>
<td>0.9132</td>
<td>32.33</td>
</tr>
<tr>
<td>Pattern C</td>
<td>5</td>
<td>31.53</td>
<td>0.9159</td>
<td>31.37</td>
<td>0.9088</td>
<td>31.08</td>
</tr>
<tr>
<td>Pattern D</td>
<td>1</td>
<td>40.35</td>
<td>0.9773</td>
<td>38.56</td>
<td>0.9650</td>
<td>36.62</td>
</tr>
<tr>
<td>Pattern E</td>
<td>3</td>
<td>35.65</td>
<td>0.9674</td>
<td>34.84</td>
<td>0.9552</td>
<td>33.89</td>
</tr>
</tbody>
</table>

3.2. Joint optimization of splitter pattern and image reconstruction

To jointly optimize the splitter pattern and the reconstruction network, the joint network illustrated in Fig. 6(a) is adopted. The splitter network is implemented based on our proposed differentiable forward imaging model shown in Eq. (5). The reconstruction network, together with the experimental setting including datasets used for training and testing, are just the same as the ones we use for the pattern evaluation experiments. Here, we initialize the learnable parameters $\mathbf{z}$ based on two patterns, i.e., pattern A and pattern D, to test whether the proposed joint optimization method can improve existing patterns. The comparisons between these two patterns and their optimized patterns are provided as follows:

**Optimization of pattern A.** As Table 2 shows, on average, the optimized pattern achieves better performance than the original pattern in terms of both PSNR and SSIM in all cases. The PSNR improvement is up to 0.78 dB. Besides, as Fig. 7 shows, the optimized pattern successfully tackles the blurring problem of the original pattern. By comparing Figs. 3(b) and 6(b), it can be observed that the modification made to pattern A results in a better separation of colors, with less blurring effect.

**Different colors mixed and hard to separate.** To tackle this problem, we propose the use of more splitters to fully split incident light so that color information can be better separated during the imaging process.
of the optimized pattern over the original pattern follows the second design principle we establish above, i.e., colors should be split to a region as small as possible. In Fig. 3(b), a single color at a position is equally split to a set of sensor pixels. By contrast, in Fig. 6(b), a single color at a position is only diverted to a single sensor pixel.

Optimization of pattern D. As Table 3 shows, on average, the optimized pattern achieves better performances than the original pattern in terms of both PSNR and SSIM in all cases. The PSNR improvement is up to 0.75 dB. Besides, as Fig. 8 shows, the optimized pattern is capable of correcting the false color caused by the original pattern. The comparison between Figs. 3(c) and 6(c) indicates that the modification of the optimized pattern over the original pattern follows both the second and third design principles. On the one side, the red color and blue color are diverted to single sensor pixels, respectively. On the other hand, parts of green color are diverted to the neighboring pixel from the incident light that is originally not split.

In conclusion, our proposed joint optimization method successfully improves existing splitter patterns, validating its effectiveness.
Fig. 6. Illustration of the joint optimization experiment, where (a) the joint network is trained to automatically optimize a splitter pattern. (b) Optimized result of pattern A. (c) Optimized result of pattern D. The parameters of these optimized patterns can be found in our publicly available trained models.

Interestingly, although not specifically designed, the improvements of the optimized patterns over the original patterns follow our proposed design principles, which can demonstrate that these principles are reasonable.

### 3.3. More studies

To evaluate our proposed method from more aspects, this subsection provides more studies, including comparison with the filter-based color imaging system, ablation study on the initialization of $z$, and evaluation with the quantitative indicator CIEDE2000. Here, pattern D and its optimized pattern are adopted as representatives to participate in the evaluation. Besides these two, the filter-based imaging system and the other optimized pattern, $z$, of which is randomly initialized, are also evaluated. The comparison results are visualized in Fig. 9 and summarized as follows:

#### Evaluation with CIEDE2000

The CIEDE2000 is an indicator used to measure the color difference between two RGB images. It can be implemented with the MATLAB function `imcolordiff`. As Fig. 9(b) shows, the optimized pattern achieves smaller CIEDE2000 values than pattern D, implying that the optimized pattern can better recover the color information. This conclusion is consistent with our observation from
Fig. 7. Visual results of pattern A and its optimized pattern on zoomed regions of the image 'kodim05' at the noise level $\sigma = 10$.

Fig. 8. Visual results of pattern D and its optimized pattern on zoomed regions of the image 'kodim24' at the noise level $\sigma = 5$.

Fig. 9. Performance comparison among a filter-based system, splitter-based systems with pattern D, and two optimized patterns with different initialization methods. The ‘Optimized’ is initialized with pattern D, while ‘Optimized-R’ is with random initialization. For the indicator CIEDE2000, a smaller value indicates a better performance.

Comparison with filter-based system. To incorporate the filter-based system into comparison, the widely adopted Bayer color filter array [41] is used as a representative. From Fig. 9, it can be observed that pattern D performs better than the Bayer pattern in terms of PSNR, while worse in terms of CIEDE2000. However, the drawback of pattern D is greatly overcome after optimization with our proposed method. Its optimized pattern outperforms the Bayer pattern in terms of both PSNR and CIEDE2000.

Ablation study on the initialization of $z$. In our default setting, $z$ is initialized with existing patterns. To study the effect of initialization, a randomly initialized $z$ is evaluated here. Note that the dimensions of $z$ are kept the same in two initialization methods. From Fig. 9, it can be observed that initializing $z$ with pattern D and with random values achieve similar performances, implying that the initialization of $z$ only has weak effects on our proposed method.

4. Conclusion and outlook

In summary, this work is to tackle the splitter pattern design problem. We first establish a differentiable mathematical model for splitter-based image sensors, showing that the detected raw image is degraded by noise, incomplete sampling, and blurring. Because of this, we develop a deep neural network capable of simultaneously tackling multiple image-processing tasks to achieve image reconstruction. With this reconstruction network, we conduct an evaluation of five existing patterns, through which we establish three splitter pattern design principles. Furthermore, we implement our proposed differentiable model with a splitter network and concatenate it with the reconstruction network so that we can achieve an automatic optimization of a splitter pattern via jointly training these two networks. We apply this method to two existing splitters. In both cases, the optimized pattern achieves better performance than the original pattern, demonstrating the effectiveness of our proposed joint optimization method.

This work can be further extended. Here, we have proposed a method to jointly optimize the splitter pattern and the reconstruction...
network, but there is still a gap between the splitter pattern design and the metasurface design. A more interesting and more challenging extension is to close this gap by directly optimizing the desired phase profiles, or even the subwavelength structures of metasurface-based splitters, together with the reconstruction method. To achieve this, functions mapping from them to splitter patterns or detected raw images have to be established. Also, more constraints should be incorporated based on physical requirements.

CRediT authorship contribution statement

Haosen Liu: Conceptualization, Investigation, Formal analysis, Methodology, Software, Writing – original draft, Writing – review & editing. Edmund Y. Lam: Conceptualization, Funding acquisition, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Our source codes and trained models are publicly available at the linkage https://github.com/haosennn/Splitter.

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