

Robust ptychographic imaging with low-discrepancy sequences

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ABSTRACT

Ptychography is a widely-used imaging technique that enables high-resolution, large field-of-view reconstruction via a scanning process. The design of the scanning pattern is critical, as it directly affects both reconstruction quality and computational efficiency. In this paper, we propose a set of new sampling patterns based on low-discrepancy sequences. This non-periodic configuration promotes a relatively uniform distribution of scanning positions and reduces the number of points located far from measured locations. Unlike the common non-periodic scan patterns such as Fermat spiral pattern, which is deterministic but lacks extensibility and exhibits subtle periodicity, low-discrepancy sequences offer provably uniform coverage, support progressive sampling without a predefined number of points, and produce a flatter power spectral density. Consequently, the proposed patterns exhibit enhanced robustness against common experimental artifacts, such as noise and scanning drift. In simulation experiments, we evaluate the performance of low-discrepancy scanning patterns from multiple perspectives, including spatial distribution, power spectral density, and robustness under drifting conditions. Optical experiments further validate the effectiveness and practical utility of our method.

1. Introduction

With the development of coherent light sources, coherent diffractive imaging (CDI) systems can achieve high-resolution imaging for non-periodic objects [1]. As a lensless imaging technique, the resolution of CDI is primarily limited by the wavelength of the light source. CDI has the potential to achieve atomic resolution when using electron beam, making it popular in material analysis, biological imaging [2], and other fields. Among such imaging systems, ptychography [3] is a commonly used approach that combines conventional CDI with a scanning process. Ptychography offers a large field-of-view, and its reconstruction algorithms converge more easily with the prior knowledge of the scanning positions [4].

A key factor for successful reconstruction in ptychography is the overlap region between two successive scanning points [5]. The overlap ratio should be set to an appropriate value to achieve the best image quality. The design of scanning patterns and overlap regions is important for the final reconstruction results. In conventional ptychog-

raphy, a raster scan is used to capture diffraction patterns, which may introduce grid artifacts into the final reconstruction [6], especially for near-field ptychography. Although in real experiments, it is difficult to achieve a perfect grid that would reduce such artifacts, there is still room for designing high-efficiency scanning patterns. Concentric patterns [7] and Fermat spiral patterns [8] have been proposed to achieve higher-quality imaging. Another method for sampling pattern collection, called “fly-scan”, improves data acquisition speed by continuously moving the object [9]. This means the diffraction patterns are the continuous integration from parts of the object. Due to the continuous image capturing process, the required frame rate of the imaging sensor is higher than that in conventional ptychography. Similar approaches have also been used in multiple plane phase retrieval [10,11], which can achieve scanning along the propagation direction [12,13].

If the prior knowledge of the target object is available, it is possible to design specialized sampling patterns for such applications. Luo and Andersson [14] demonstrated a method for designing the under-sampling patterns for microscopy. A proper sampling pattern can be

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generated using a simplified matching pursuit algorithm based on the frequency structure, enabling comparable imaging quality with fewer diffraction patterns and reduced scanning time in ptychography. Lin et al. [15] proposed a two-stage method that first detects the boundary of the object and then designs sampling patterns along the edges to avoid empty backgrounds. With additional prior information about the target object, the efficiency of the ptychographic imaging process can be improved. Deep learning approaches [16] leverage data-driven priors as constraints in the solution space, which can lead to faster convergence. Similarly, the use of physical masks in the imaging systems, such as sensor mask in [17], introduces a known modulation prior that improves the condition of the imaging problem and yields higher-quality reconstructions with fewer measurements.

When designing sampling patterns, most existing studies consider factors such as frequency information, power spectral density, and scanning trajectories [18]. However, the field of view (FOV) of these patterns has received little attention [8]. Their FOV is often much smaller than that of the raster scan strategy [15]. These designs are based on the assumption that valid information mainly exists in the central region, with only a few scanning positions located near the edge of the entire view [19]. This assumption does not always hold in real applications. For biological imaging, when observing tissue with ptychography, cells may appear anywhere in the whole FOV. Hence in this paper, we consider a general sampling pattern design based on the premise that valid information may exist at any point, even near the boundary of the entire view.

In numerical integration and sampling theory, for a given point set, its discrepancy is defined as the maximum of the absolute difference between the proportion of points falling in the region and the region's volume over all admissible regions in the space. A low-discrepancy sequence is generated to drive this discrepancy down to the optimal value. Unlike purely random point sets which only offer probabilistic guarantees and exhibit clustering and voids, low-discrepancy sequence ensures rigorous, worst-case bounds on the sampling discrepancy, which provides provably uniform coverage. It is also different from stratified sampling. While stratification partitions the space into fixed cells, it suffers from the curse of dimensionality and typically requires a pre-defined number of points. Adding more points later breaks the stratified structure and destroys uniformity. Inspired by the theory of low-discrepancy sampling [20], we explore efficient sampling pattern designs for ptychography. While existing non-periodic pattern such as the Fermat spiral trajectory provides improved overlap uniformity, the number of its points needs to be known in advance to determine the space occupied by each point, making them non-extensible [8]. For progressive sampling

strategies such as adaptive scanning via deep reinforcement learning, they can achieve atomic resolution and adapts the trajectory on-the-fly, but it does not guarantee a flat power spectrum which may lead to artifacts in real conditions [21]. Low-discrepancy sequences overcome these limitations. They provide provably uniform coverage, support progressive sampling, and generate a flat power spectrum that minimizes aliasing artifacts. These sequences, such as the Sobol sequence, have been successfully applied in electron backscatter diffraction to reduce the number of scan points while preserving statistical distributions [22], but they are seldom considered in ptychography for scanning pattern designs. Based on this idea, we aim to distribute the scanning areas as uniformly as possible without periodicity or symmetry, thereby acquiring information of the target object from more diverse locations in both spatial and frequency domains. For Fourier ptychography, raster scan is performed in the Fourier domain and low-discrepancy sequences may also be used to help with the sampling in the frequency domain. In this paper, we mainly consider real-space scanning with low-discrepancy sequences.

2. Method

2.1. Imaging model with general sampling pattern

For ptychographic imaging, diffraction patterns are captured by sequentially recording the far-field wavefront of the interaction between the probe and object at different positions [23]. The sample-probe interaction is conventionally modeled as a complex multiplicative relationship, and the validity of this approximation is analyzed in [24]. The mathematical model of a typical transmissive ptychography system is given as

$$I_j(\mathbf{v}) = |F[P(\mathbf{u}) \cdot O_j(\mathbf{u})]|^2, j = 1, 2, \dots, N, \quad (1)$$

where j denotes the j^{th} scanning process, $P(\mathbf{u})$ is the probe function and $O_j(\mathbf{u})$ is the illuminated region of the target object in the current scan, $\mathbf{u}$ and $\mathbf{v}$ represent the coordinates in real space and reciprocal space respectively, F denotes the 2D Fourier transform, and $I_j(\mathbf{v})$ is the captured diffraction pattern. For near-field ptychography, F refers to the Fresnel propagation.

During data acquisition, the probe and sample undergo relative translational scanning to ensure strong localized illumination of distinct sample regions at each measurement position [25]. The scanning protocol must fully cover the sample's region of interest, and should have approximately 60–70% overlap between adjacent probe positions to achieve high-quality reconstructions [26]. A more intuitive

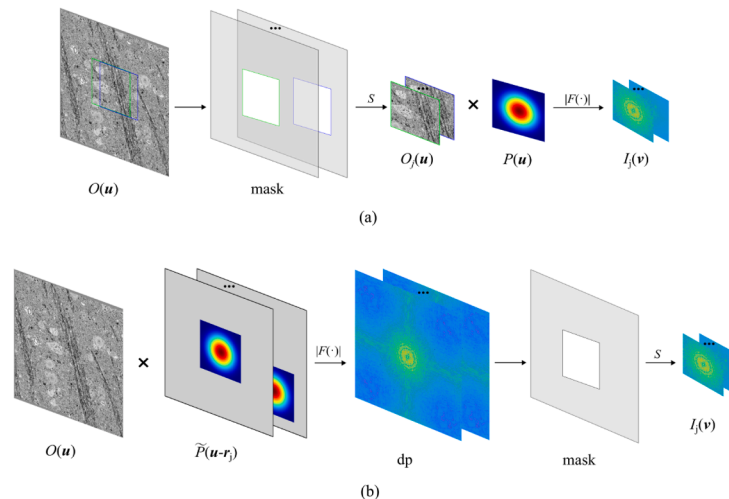


Fig. 1. Different expressions of the ptychographic imaging scheme, dp: diffraction pattern. (a) object-based scanning scheme; (b) probe-based scanning scheme.

explanation is provided in Fig. 1. In real experiments, the probe is usually stationary, while the object moves according to the designed scanning patterns. As illustrated in Fig. 1(a), a sampling operator is used to extract different areas of the whole object according to the scanning pattern, then these regions are illuminated by the probe and finally the imaging sensor is used to capture the far-field diffraction patterns. The corresponding mathematical expression is given in Eq. 1, which is a general forward model for the measured intensity. However, in this expression, the scanning patterns are coupled with the object, making it difficult to analyze the effects of these patterns individually. Besides, the relationship between the scan pattern and the probe remains unclear, leaving room for designing more efficient scanning patterns with respect to the properties of the probe and diffraction pattern.

In this paper, we mainly explore the effects of scanning patterns, which determine the exact locations of all the scans. Let $\mathbf{r}_j$ be the location of the j^{th} scan, the imaging model can be rewritten as

$$\sqrt{I_j(v)} = S(|F[\tilde{P}(\mathbf{u} - \mathbf{r}_j) \cdot O(\mathbf{u})]|), j = 1, 2, \dots, N, \quad (2)$$

where $\tilde{P}$ is the padded probe pattern, and S is a sampling matrix that extracts the region of interest (ROI) to make the dimension of the diffraction patterns compatible with the size of the imaging sensor.

In order to evaluate the performance of the imaging system, the object should be regarded as the input, rather than a component of the imaging system. As shown in Fig. 1(b), the illuminating probe is padded with zero to make the size identical to the object, and its location in the padded region is determined by the scanning pattern. The object is now illuminated by the padded probes and the wavefronts are propagated to the far-field. Finally, if the resolution of the imaging sensor is $M_s \times N_s$, then a sampling matrix S is used to crop the $M_s \times N_s$ regions at the center of the diffraction patterns which are finally captured. We adopt this probe-based scanning scheme in our theoretical analysis. Eq. 2 is the discrete imaging model with a padded probe. Eq. 1 and Eq. 2 describe the same imaging process, but the scanning pattern is coupled with the object in Eq. 1 and with the probe in Eq. 2 respectively, which is the main difference between these two equations. With Eq. 2, the object can be treated as the input. The design of the scanning pattern is related to the predefined scanning probe, which is the main focus of this paper.

For simplicity, we vectorize all variables and omit the coordinates in Eq. 2. The imaging model is now given as

$$\mathbf{i}_j = S|F(\mathbf{p}_j \circ \mathbf{o})|, j = 1, 2, \dots, N, \quad (3)$$

where $\mathbf{i}_j, \mathbf{p}_j, \mathbf{o}$ are the vectorized forms of $\sqrt{I_j(v)}, \tilde{P}(\mathbf{u} - \mathbf{r}_j), O(\mathbf{u})$, F is the 2D Fourier matrix and S is the sampling matrix. As shown in Fig. 1(b), the sampling matrix is related to a binary mask which can extract the central region of the diffraction patterns. Compared with Fig. 1(a) whose sampling is done before the illumination of the probe, the emitted wavefront is almost the same and the difference is only the spatial location. Due to the property of Fourier transform, spatial shift does not change the magnitude of the Fourier coefficients, then the same sampling matrix S can be used for all scanning positions.

If we regard $\mathbf{o}$ as the input and $\mathbf{i}_j$ as the output of this imaging model, we can see that $\mathbf{p}_j$ is the most important component in this system. We now analyze the formulation of $\mathbf{p}_j$. Let D_j be the impulse matrix at the j^{th} scanning position, then

$$\mathbf{p}_j = P * D_j, \quad (4)$$

where $*$ denotes the convolution operation, and $\mathbf{p}_j$ is the vectorized form of P_j . As shown in Fig. 2(a), the final illumination wavefront in the j^{th} scan is the combination of the scanning position and the probe. Finally, by gathering all the scanning positions, we can obtain the complete dataset consisting of all diffraction patterns

$$\mathbf{i} = S|F(\mathbf{p} \circ \mathbf{o})|, \quad (5)$$

where

$$\mathbf{i} = [\mathbf{i}_1 \quad \mathbf{i}_2 \quad \dots \quad \mathbf{i}_N], \quad (6)$$

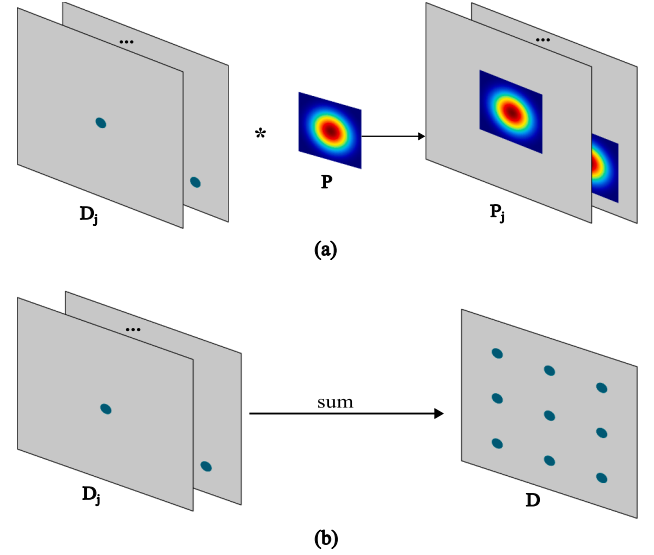


Fig. 2. The effects of probe and scanning pattern in ptychography. (a) extended probe generation process. (b) overall scanning pattern in ptychography.

$$\mathbf{p} = [\mathbf{p}_1 \quad \mathbf{p}_2 \quad \dots \quad \mathbf{p}_N]. \quad (7)$$

For the complete set of scanning position D shown in the right panel of Fig. 2(b), it can be expressed as

$$D = D_1 + D_2 + \dots + D_N, \quad (8)$$

and $P * D$ describes the wavefront distribution in the object plane during the scanning process. In the following sections, we mainly explore the design of matrix D .

2.2. Sampling with low-discrepancy sequences

The design of the scanning pattern is analogous to a rendering problem in 2D plane. For this kind of problem that relies on sampling, uniformly distributed samples lead to better coverage theoretically [27]. Efficiently generating uniformly distributed random numbers in high-dimensional spaces is a common requirement in many related problems. For example, in Monte Carlo integration for light transmission simulation, sampling is ubiquitous, and the quality of the sample distribution directly affects the convergence rate of the integral [28]. In multiple-plane phase retrieval, wavefront propagation with unordered distances can also improve convergence [29,30]. Compared to ordinary pseudo-random numbers, low-discrepancy sequences are widely used in computer graphics, since they can achieve more uniform coverage in high-dimensional spaces and possess additional properties that benefit sampling-based algorithms.

To achieve a scanning process that is uniformly distributed, we consider low-discrepancy sequences. Unlike regular grid sampling or spiral patterns, low-discrepancy sequences avoid periodic or symmetric structures which effectively reduce aliasing artifacts. For purely random sequence, it provides only probabilistic uniformity and is prone to clustering and voids. Its unpredictability is undesirable for ptychography, which can directly affect the reconstructed resolution or introduce artifacts. Low-discrepancy sequences provide relatively uniform coverage per sampling point and support progressive, incremental sampling without requiring a pre-determined resolution. Consequently, the imaging process becomes more robust to noise, since most pixels are illuminated by a sufficient number of scans. Moreover, low-discrepancy sequences ensure that no large gaps exist in the scanning region. The distance from any point to the nearest scan location is averaged, leading to fewer points being far from measured positions. This property makes the scanning process less sensitive to scanning drifts. In this paper, we

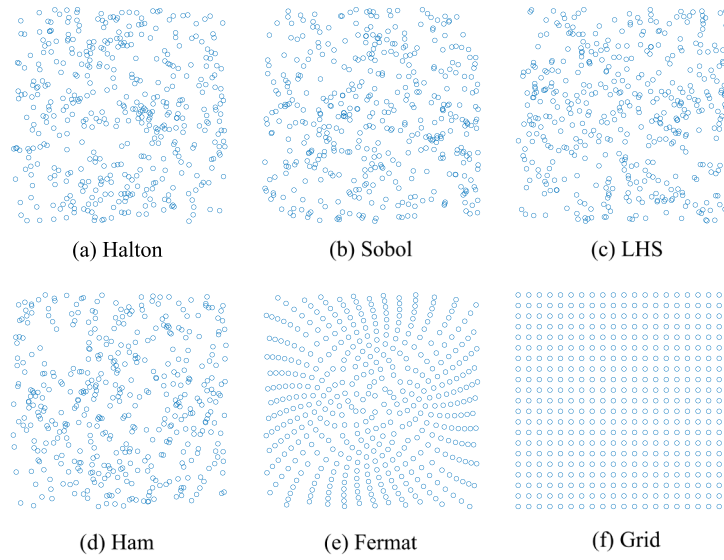


Fig. 3. Diagrams of different kinds of the scanning patterns.

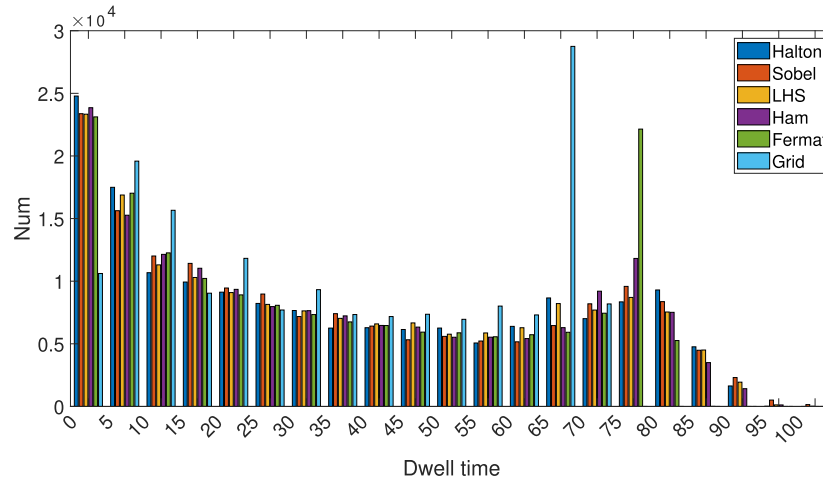


Fig. 4. Dwell time distributions of all the pixels in the scanning plane. x-axis: dwell time, bins of 5 units; y-axis: number of pixels. More dwell time means the current position is scanned more frequently.

explore four kinds of low-discrepancy sequences: Halton sequence, Sobol sequence, Latin hypercube sequence, and Hammersley sequence.

Halton sequence [31] is a deterministic low-discrepancy sequence that uses a different prime base for each dimension. The j^{th} sample in dimension i is constructed by writing the integer j in base p_i , reversing the digits, and interpreting the result as a fraction in $[0, 1)$. It can generate an infinite number of samples progressively and is computationally inexpensive, but the asymptotic discrepancy constant increases more than exponentially with dimension.

Sobol sequence [32] uses base 2 for all dimensions, and each dimension is generated using a different direction number, then they are chosen to ensure low discrepancy across the entire multidimensional space. Base-2 construction allows for fast generation using bit-wise operations. It is also deterministic and extensible, but may not maintain related theoretical properties when sampling from non-uniform multivariate distributions.

Latin hypercube sequence (LHS) [33] is a stratified technique that ensures each dimension is evenly covered. The range of each input variable is divided into N equal-probability intervals, and one sample is placed randomly within each interval. The samples for different dimensions are then randomly paired to form the final N -point sample set. It guarantees that the projection onto each individual dimension is uni-

formly distributed, generally providing better space coverage than pure random sampling. However, it is difficult to add new samples after generation and the entire sequence must be regenerated.

The Hammersley sequence (Ham) [34] is a variant of the Halton sequence designed for fixed-size sample sets. It uses only one-dimensional van der Corput sequences for the last $s - 1$ dimensions, while the first coordinate is simply the sample index j divided by the total number of points N . It requires the total number of samples to be specified in advance, and the distribution of the partial set is generally not uniform.

To generate scanning patterns using these sequences, we set the number of dimensions to 2, and the initial 2D sequences are scaled based on the size of the object. As shown in Fig. 3, all the scanning patterns have the same number of scanning points.

It is worth emphasizing the conceptual difference between low-discrepancy sequences and the Fermat spiral. Fermat spiral is a geometric construction that places points along a spiral trajectory; it is non-periodic and provides good spatial coverage, but its point set is fixed once the total number of points N is chosen. Moreover, the Fermat spiral exhibits subtle long-range correlations in the frequency domain, which can affect high-frequency information recovery. In contrast, low-discrepancy sequences are inherently extensible. The first $M < N$ points of a sequence already form a valid low-discrepancy set. This property

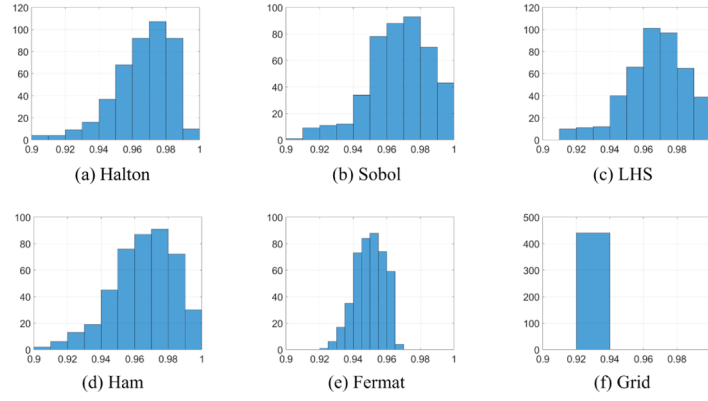


Fig. 5. Distribution for the lower bound of overlap ratio of different scanning patterns, x-axis: O_r , y-axis: number of scanning points.

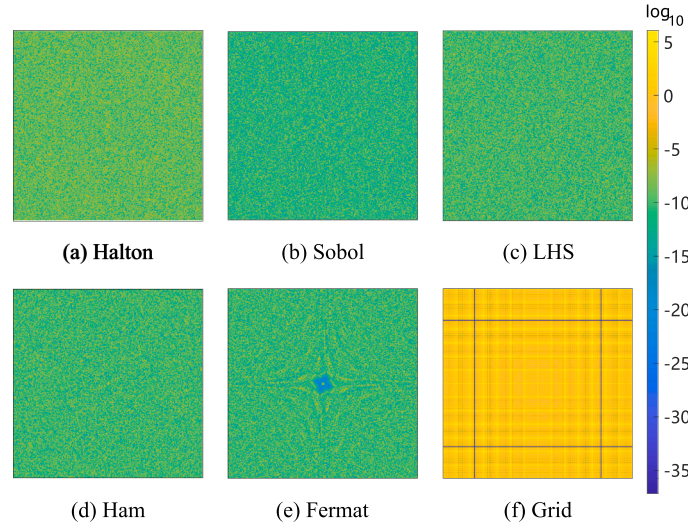


Fig. 6. Amplitude information in the frequency domain of different scanning patterns, (a)–(d) correspond to low-discrepancy sequences, which exhibit no peaks in the central region.

enables progressive refinement of the scanning pattern without re-designing. Furthermore, the theoretical guarantee of low discrepancy ensures asymptotically optimal uniformity, which directly translates to robust performances under noise and drift conditions. The definitions of these sequences are given in [Appendix A](#).

3. Numerical analysis and experiments

In this section, we evaluate the performances of low-discrepancy sequences from multiple perspectives. For all scanning patterns, the total number of scanning points is the same, and the boundaries of these patterns are set to be identical by adjusting the step sizes.

3.1. Evaluation metrics and results in spatial and frequency domains

In this part, we evaluate the scanning patterns in both spacial and frequency domains, including the dwell time, overlap ratio, frequency map, and power spectrum density. In the following analysis, we distinguish four kinds of uniformity relevant to scanning pattern design. The spatial uniformity of the scan positions is the fundamental property of the point set, characterized by its discrepancy, which has been discussed in [Section 2.2](#). The dwell-time uniformity and overlap uniformity are derived metrics that depend on both the scan positions and the probe size. They are determined by the distributions of these two metrics reflecting the redundancy of illumination in real space. The Fourier-domain

uniformity describes how evenly the sampling pattern distributes energy across spatial frequencies, which directly affects aliasing and reconstruction fidelity.

Dwell time W_i refers to the number of times a pixel $C(x, y)$ is scanned, which can be defined as

$$W_i(C, P_s, R_p) = \sum_{n=1}^{|P_s|} \mathbf{1}_{\sqrt{[(x-x_n)^2 + (y-y_n)^2]} - R_p \leq 0}, \quad \forall (x_n, y_n) \in P_s, \quad (9)$$

where $\mathbf{1}_{(\cdot)}$ is the indicator function, P_s is the point set of scanning pattern, and R_p is the radius of the circular probe function. Dwell time reveals the overall scanning coverage across the entire plane. Since both the values and the overall distribution of the dwell time depend on the total number of scan points, the probe size, and the size of the scanning region, we make them identical among different scanning patterns. We calculate the dwell time for all pixels under different types of scanning patterns, and the statistical results are shown in [Fig. 4](#).

For all patterns, the size of the probe, the field-of-view, and the number of scanning points are the same, then the mean of the dwell time should be identical with all the cases. In simulation experiment, the probe is characterized by a circular binary amplitude mask with a radius of 64 pixels, and its phase is generated using Zernike polynomials. The grid pattern has 21×21 points with a step size of 14 pixels. Since all the scanning patterns have the same FOV (406×406 pixels) and the same number of scanning points (441), the average density of the dwell

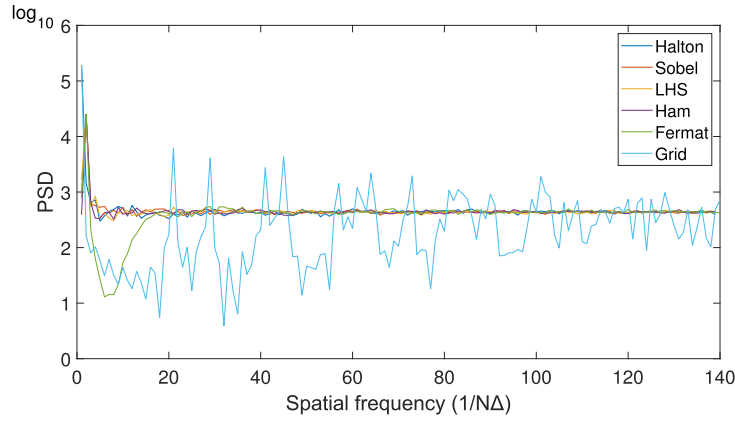


Fig. 7. Power spectral density (PSD) of different kinds of scanning patterns, Δ refers to the detector's pixel size. Low-discrepancy sequences exhibit lower variance for the high spatial frequency region.

time for all pixels are the same as

$$(64 \times 2)^2 / 4 \times \pi \times 441 / (406)^2 \approx 34, \quad (10)$$

which means each pixel is encoded in 34 individual diffraction patterns on average. We also calculate the variance of dwell time for different scanning patterns, and the results are 808 for Halton sequence, 818 for Sobol sequence, 790 for Latin hypercube sequence, 789 for Hammersley sequence, 770 for Fermat spiral pattern and 572 for grid pattern. Compared to other patterns, the low-discrepancy sequences yield the highest variance, showing greater non-uniformity in dwell time, which can lead to variations in signal-to-noise ratio and overlap ratio. The dwell-time uniformity refers to the histogram of per-pixel illumination counts, while the spatial uniformity of the scanning positions characterizes how evenly the probe covers the entire field of view. Low-discrepancy sequences are specifically designed to minimize the spatial discrepancy. For low-discrepancy sequences, the broader dwell-time distribution can be regarded as a trade-off for breaking periodicity and symmetry, which will further be discussed by the analysis in Fourier domain.

Another important parameter of these diffraction patterns with respect to a specific size of the probe is the overlap ratio, whose lower bound O_r can be calculated as [35]

$$O_r = \frac{2}{\pi} \cos^{-1} \left(\frac{d_s}{2r_p} \right) - \frac{d_s}{2\pi r_p^2} \sqrt{4r_p^2 - d_s^2}, \quad (11)$$

where d_s is the minimum distance between the current scan position with another position in the scanning pattern, and r_p is the radius of the probe. This equation only considers overlap region between two scanning points, so the real overlap ratio should be greater than or equal to this value.

We calculate the values of O_r for all the scanning points in different scanning patterns, and the frequency distributions are shown in Fig. 5. Although high dwell time may contribute redundant information without proportional gain, it is important for achieving a high overlap ratio with better convergence. The mean overlap ratio remains above 90% for all scanning patterns under the conditions of this study. Since the grid pattern is periodic, O_r is a constant for all scanning points. The Fermat spiral is less symmetric and the histogram distribution is confined to a narrow interval. The histograms of low-discrepancy sequences have broader distributions, and they have the lowest minimum values, which may lead to localized degradation with low overlap ratio. According to the empirical guidelines in [5], an overlap ratio exceeding 60% is generally considered reasonable for achieving reliable reconstructions in typical ptychographic setups. If the value of r_p is known, we can calculate the maximum allowed distance d_{sm} according to Eq. 11. Finally, we

can obtain the constraint as: for any point in the scanning patterns, there must be another scanning point whose distance is smaller than d_{sm} .

Since the scanning patterns are generated from discrete pixel coordinates with specific construction methods, their periodicity and symmetry can be evaluated in the frequency domain. We calculate the frequency maps of low-discrepancy sequences, grid pattern, and Fermat spiral pattern via Fourier transform, and the results are shown in Fig. 6. Qualitatively, we can find that low-discrepancy patterns exhibit the least fluctuations in the frequency domain, while the grid pattern shows significant peaks at several specific positions. For quantitative evaluation, we calculate the power spectral density, and the results are presented in Fig. 7. Lower variance implies that high-frequency object information is encoded with less aliasing, leading to faster convergence and sharper reconstructions.

Specifically, for the high spatial frequency region ($\geq 5/N\Delta$), we calculate the variance of each scanning pattern. The resulting variances are 1449 for Halton sequence, 815 for Sobol sequence, 839 for Latin hypercube sequence, 830 for Hammersley sequence, 1053 for Fermat spiral pattern and 645440 for grid pattern. It can be observed that the Sobol sequence, Latin hypercube sequence, and Hammersley sequence exhibit lower variance than conventional patterns including Fermat spiral and grid pattern. The significantly lower variance in the high-frequency region of the power spectral density indicates that these low-discrepancy sequences exhibit reduced periodic structures. This is consistent with their construction as aperiodic sequences. The flatter power spectrum also suggests less directional anisotropy, which can be interpreted as reduced symmetry in the scanning point distribution. They are more able to preserve the high-frequency information of the target object. Such high-frequency components are closely related to object textures and crucial for imaging quality enhancement.

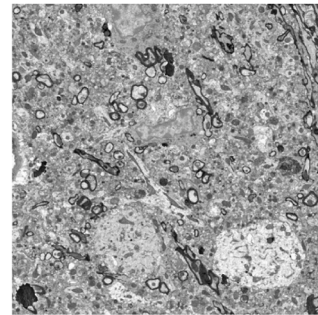


Fig. 8. Axon labeled image in the ventral lateral geniculate nucleus.

3.2. Simulation results

In this section, we evaluate the ptychographic imaging results with different kinds of scanning patterns. The ground truth image is complex-valued whose amplitude is a constant matrix, and its phase is the axon-labeled image of the ventral lateral geniculate nucleus [36], as shown in Fig. 8. To ensure that the number of scanning points is consistent across different patterns, we compute a common multiple of the basic structural units and set the total number of points to 441 for all scanning patterns. The number of scanning points refers to the number of measured diffraction patterns acquired during the scan. The ground truth image has a size of 406×406 pixels. This is related to the number of pixels on the reconstruction grid in the object domain, which controls the spatial resolution of the reconstructed image. If the scanning positions fall at sub-pixel locations that require interpolation, which is the case for most non-grid sequences, we handle this by interpolating the probe function to the required offset using Fourier-based interpolation [4]. This approach is standard in ptychographic reconstruction and ensures that the sub-pixel positions of the scanning pattern are represented in the forward model with Fourier shift theorem.

For fair comparison, we adjust the step sizes of each pattern so that the global overlap ratio is approximately the same for all patterns with similar FOV. This guarantees that any performance difference arises from the spatial arrangement, is not from a systematic under or over lap. The dwell time analysis further confirms that the total exposure per pixel is comparable across patterns.

The far-field diffraction pattern is simulated using the Fraunhofer diffraction equation, which corresponds to the Fourier magnitude of the wavefront in the object plane. The size of the diffraction pattern is identical to the probe, which is 128×128 . After generating the noise-free diffraction patterns, we add Poisson noise with a total flux of 10^4 photons. No scanning drift is added in this experiment. Finally, we apply the ePIE algorithm [37] with the same parameters ($\alpha = \beta = 0.01$) to reconstruct the images for all scanning patterns.

The imaging results are shown in Fig. 9. Quantitative results over the whole field of view are summarized in Table 1, which are averaged over 100 individual experiments for each pattern. Since the scanning points of the Sobol sequence are more uniformly distributed and exhibit lower high-frequency fluctuations, the imaging process becomes more robust to noise.

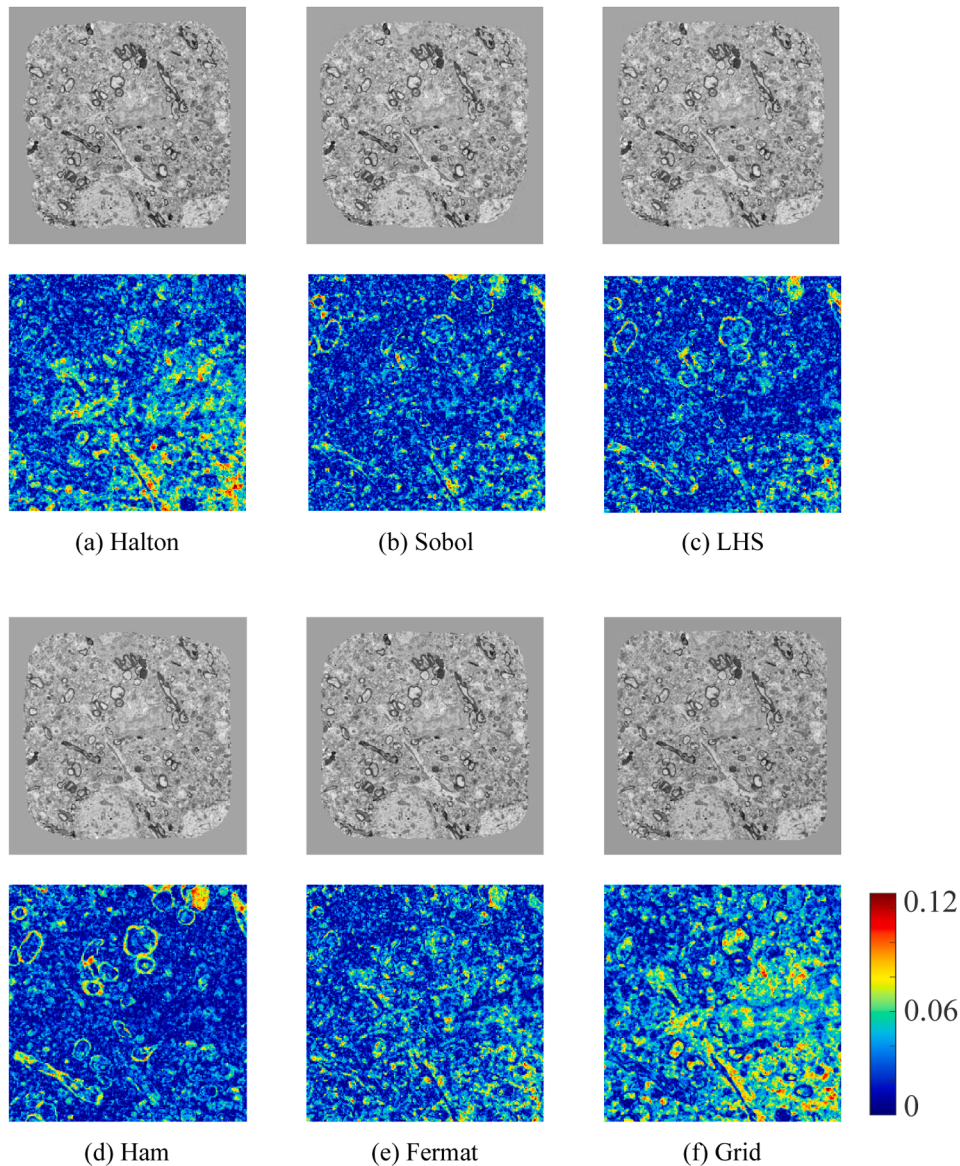


Fig. 9. Phase information (top rows) and error map (bottom rows) of all the imaging results with different scanning patterns.

Table 1
Quantitative evaluations of all the imaging results with Poisson noise (phase information).

	PSNR (dB)	SSIM
Halton	26.31	0.983
Sobol	29.13	0.986
LHS	28.92	0.985
Ham	28.84	0.984
Fermat	27.44	0.978
Grid	23.47	0.971

Table 2
Quantitative evaluations of all the imaging results with different drift levels (phase information).

Drift	PSNR (dB)			SSIM		
	0	± 1	± 2	0	± 1	± 2
Halton	37.33	18.78	16.33	0.9986	0.5786	0.1488
Sobol	40.03	20.53	17.44	0.9991	0.6169	0.1521
LHS	38.14	19.40	17.01	0.9986	0.5912	0.1514
Ham	39.00	20.19	17.32	0.9988	0.6016	0.1546
Fermat	38.98	19.79	16.77	0.9988	0.5943	0.1492
Grid	38.37	19.37	15.99	0.9987	0.5867	0.1421

We also consider a case in real experiment where random drifts exist in the scanning positions [38]. For different scanning patterns, we apply the same drift levels of 0 pixel, ± 1 pixel, and ± 2 pixels randomly to the scanning process under noise-free conditions. For each drift level, the same pixel drifts are added across all scanning patterns. The imaging results are shown in Fig. 10, and the zoomed-in views around the cell

boundary are presented in Fig. 11. Quantitative results over the entire field of view are summarized in Table 2, which are averaged over 100 independent experiments for each pattern. It can be observed that under different drift levels, the Sobol sequence achieves the highest PSNR and SSIM in most cases, indicating that it offers enhanced robustness

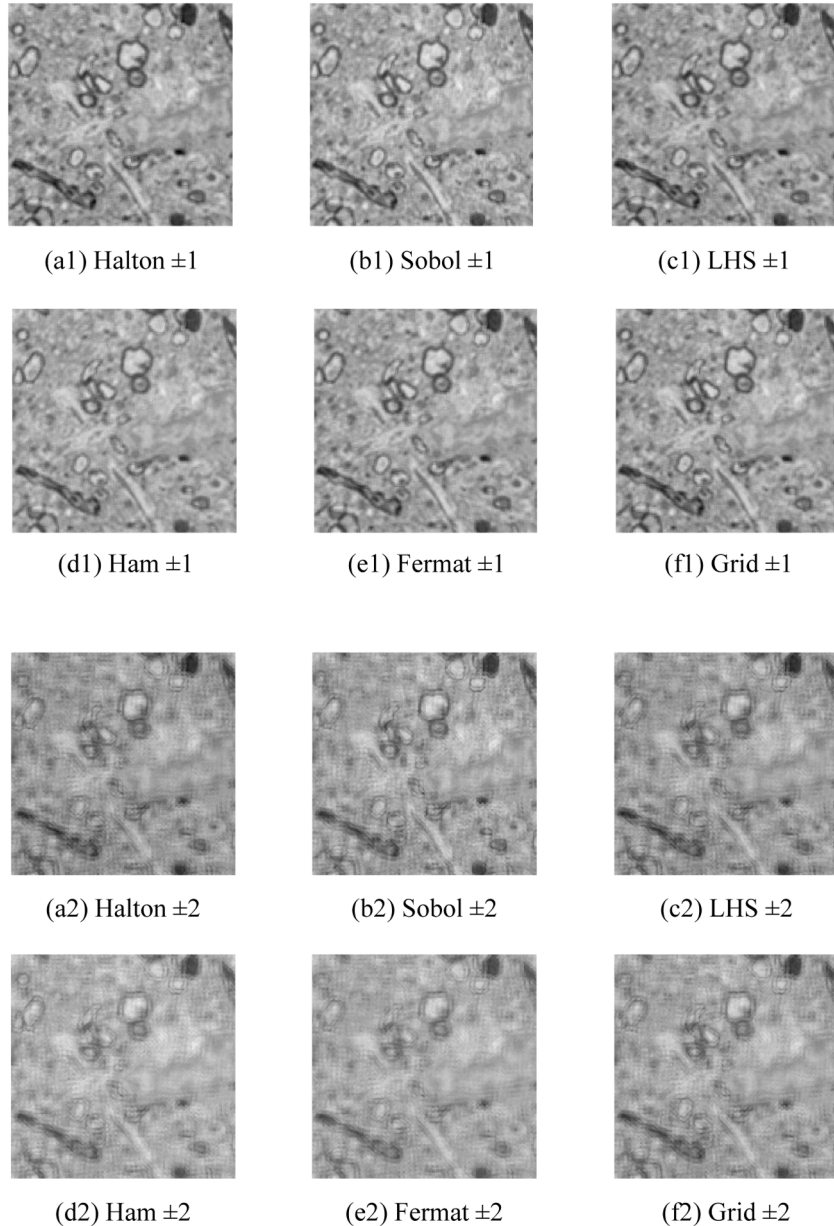


Fig. 10. Phase information of all the imaging results with ± 1 and ± 2 random drifts.

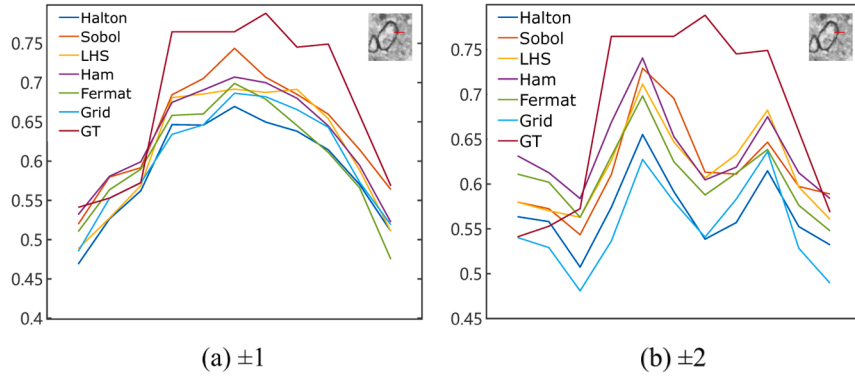


Fig. 11. Zoom-in views of the cell boundary with ± 1 -pixel and ± 2 -pixel random drifts.

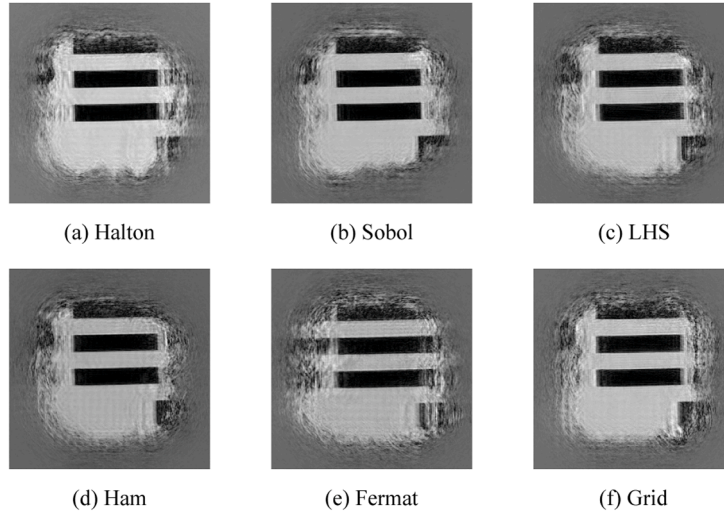


Fig. 12. Optical imaging results with different kinds of scanning patterns. The image from Sobol sequence (b) exhibits the fewest artifacts.

compared to other methods. This superior performance can be attributed to the intrinsic properties of its construction. The Sobol sequence is a digital (t, s) -sequence in base 2, with a quality factor t that grows only logarithmically with the dimension s . Its star discrepancy satisfies the asymptotic bound $O((\log N)^2/N)$ in the $s = 2$ case used here, which is provably optimal among all deterministic sequences, and the dyadic stratification property ensures that any initial segment of the sequence already constitutes a well-distributed set. Besides, ptychographic scanning only requires two-dimensional sampling, and the Sobol sequence maintains high uniformity in low-dimensional projections [39]. Specifically, consecutive two-dimensional projections of the Sobol sequence form $(1, 2)$ -sequences with quality factor $t = 1$ when constructed with appropriately chosen primitive polynomials. This guarantees that the projected points cover the unit square evenly without the striping artifacts or empty patches, which is suitable for 2D scanning in ptychography.

3.3. Optical experiment

We also validate the efficiency of the designed scanning patterns in optical ptychographic imaging system, which contains both noise and drift cases. The optical part is similar to the setup in [40]. A USAF resolution target is illuminated by a coherent light source with a wavelength of 543 nm. A charge coupled device (CCD) is placed 26 cm from the sample, and the cropped diffraction patterns with a resolution of 696×696 are captured. The sample is mounted on a 2D motorized stage and the object can be moved to the exact location in different scanning patterns with this device. First, a 13×13 grid pattern with a step size of $50 \mu\text{m}$ is

used to collect a dense dataset [41]. The reconstruction result from this dense grid serves as a proxy ground truth image for comparison. The radius of the reconstructed probe is 60 pixels, and the step size of the grid pattern is 10 pixels, leading to an average dwell time for all pixels of approximately 33. This proxy is not an independent and absolute reference, but it represents the best reconstruction achievable with a conventional dense grid scan under ideal conditions. This approximation is commonly adopted in ptychographic comparisons but has inherent limitations, the dense grid reconstruction may contain residual artifacts or missing fine details itself that are not present in a true ground truth. Therefore, while the proxy enables relative ranking of different scanning patterns, the reported R-factors and visual comparisons should be interpreted as relative performance indicators rather than absolute error metrics.

For fair comparison, we generate different kinds of scanning patterns with 81 scanning points as subsets of the original dataset using low-discrepancy sequences, Fermat spiral, and grid pattern. All of them have the same region of interest as the complete dataset. We use the same reconstruction algorithm with the same parameters to obtain the reconstruction results, and the imaging results are given in Fig. 12, where zoom-in views of the edges are shown in Fig. 13. We also calculate the R-factor of different results as

$$R = \frac{1}{N} \sum_j \frac{\|i_j - i_{0j}\|_2}{\|i_{0j}\|_2}, \quad (12)$$

where N is the total number of scanning points, i_j is the estimated diffraction pattern, and i_{0j} is the captured diffraction pattern. R-factor can indicate the data fidelity, and the results are summarized in Table 3.

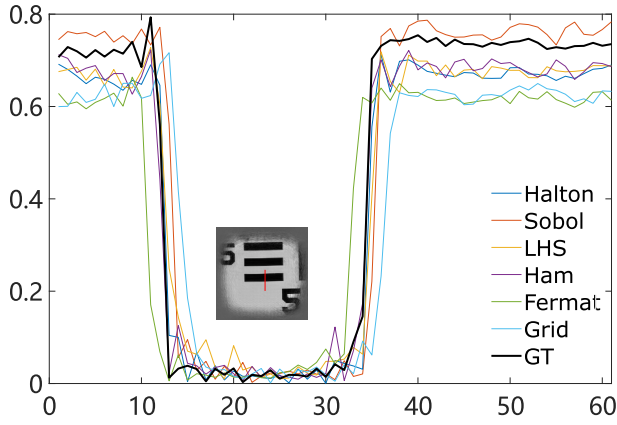


Fig. 13. Zoom-in views of the optical imaging results with different scanning patterns. Reconstruction results from low-discrepancy sequences show higher similarity with ground truth (GT) than commonly-used patterns.

We can find that imaging results from low-discrepancy sequences have lower R-factor, higher similarity with the ground truth image, indicating that they can achieve better imaging quality in real situations.

Table 3

R-factor of different scanning patterns in optical experiments, lower is better.

	Halton	Sobol	LHS	Ham	Fermat	Grid
R-factor (%)	9.53	9.28	9.33	9.73	11.33	9.99

In this section, we give both simulation and optical experiments to test the performances of different scanning patterns. In simulation experiment, the sample is a biological tissue, which is non-symmetric. The target object in optical experiment is a USAF resolution target, which is highly symmetric. The main influence of symmetry is the distribution of the Fourier spectrum. The high symmetry of the USAF target means its Fourier spectrum is concentrated along specific directions. Regular grid pattern and Fermat spiral have higher symmetry, which might therefore be expected to sample the most energetic Fourier components more efficiently than a truly non-symmetric pattern. Conversely, low-discrepancy sequences are designed to sample the Fourier domain uniformly, which may not show their full advantage on such a structured target. In summary, the combination of the simulation and optical experiments can demonstrate the superior performances of low-discrepancy sequences from different aspects.

4. Conclusion

In this paper, we propose a set of novel sampling patterns generated by low-discrepancy sequences for robust ptychographic imaging. In summary, low-discrepancy sequences offer three distinct advantages. First, the pattern can be progressively refined without redesign. Second, they can achieve provably lower discrepancy, leading to more uniform spatial coverage than random sampling. Finally, they have flatter power spectral density, which better preserves high-frequency texture information. These properties make the imaging process more robust to noise or scanning drift, and capture high-frequency texture information more efficiently, thereby improving reconstruction quality. Comparisons with conventional patterns such as grid and Fermat spiral demonstrate that our method achieves higher-quality ptychographic reconstructions. This work paves the way for designing advanced ptychographic imaging methods that leverage low-discrepancy patterns to improve scanning efficiency and robustness. Future work may extend these sequences to higher dimensions or adaptive sampling in both conventional and Fourier ptychography.

CRedit authorship contribution statement

Li Song: Writing – review & editing, Writing – original draft, Methodology, Data curation, Conceptualization; **Xu Liu:** Writing – original draft, Methodology, Conceptualization; **Zhenbo Ren:** Methodology, Investigation; **Kaiqiang Wang:** Methodology, Conceptualization; **Zhu Chen:** Writing – original draft, Supervision, Methodology, Conceptualization; **Jianglei Di:** Funding acquisition, Conceptualization; **Wenyi Ren:** Supervision, Project administration; **Edmund Y. Lam:** Supervision, Project administration, Conceptualization.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix A. Mathematical definitions of the sampling sequences

In this section, we give definitions of the five scanning patterns discussed in the paper.

Halton sequence:

The Halton sequence [31] is generated by choosing $r \geq 2$ as a prime number, then expanding the sequence of integers $1, 2, \dots, g, \dots, G$ in terms of the base r as

$$g = \sum_{l=0}^L b_l(g)r^l, \quad (A.1)$$

where $0 \leq b_l(g) \leq r-1$, $r^L \leq g < r^{L+1}$, and G is the number of elements in the sequence. Finally, the Halton sequence is obtained by taking the radical inverse of g by reflecting through the radical point

$$\phi_r(g) = 0.b_0(g)b_1(g)\dots b_L(g) \quad (A.2)$$

in base r . The standard Halton sequence in s dimensions is obtained by pairing s one-dimensional sequences based on s pairwise relatively prime integers.

Sobol sequence:

To generate one-dimensional Sobol sequence [32], an arbitrary sequence of binary-rational numbers $V_1, V_2, \dots, V_m$ is selected, where $0 < V_i < 1$. A new sequence of binary-rational numbers $r(i)$ called Sobol sequence can be obtained in the binary system by first expanding i as

$$i = e_1 + 2e_2 + \dots + 2^{m-1}e_m, \quad (A.3)$$

then

$$r(i) = e_1V_1 * e_2V_2 * \dots * e_mV_m, \quad (A.4)$$

where $*$ denotes digit-by-digit mod 2 binary addition. Other dimensions can also be generated similarly.

Latin hypercube sequence:

Latin hypercube sequence [33] can be generated by first dividing the range of each dimension into N_L strata of equal marginal probability, and then sampling once from each stratum randomly. Let this sample be X_{kj} , then for all j with a certain k , these elements form the sequences

in the k^{th} dimension. Finally, the components of the various dimensions are matched at random to generate the Latin hypercube sequence.

Hammersley sequence:

The Hammersley sequence [34] is a variant of the Halton sequence that is optimised for a fixed number of points N , which can be obtained as

$$x_h = \left(\frac{h}{N}, \phi_{r_1}(h), \phi_{r_2}(h), \dots, \phi_{r_{s-1}}(h) \right), \quad (\text{A.5})$$

where $\phi_r(h)$ is the radical point generated with the same method in Halton sequence, and $r_1, \dots, r_{s-1}$ are the first $s-1$ prime numbers.

Fermat Spiral:

Fermat Spiral [8] is generated in polar coordinates (r, θ) as

$$r = c\sqrt{n}, \theta = n\phi_0, \quad (\text{A.6})$$

where c is a scaling factor, n is the index number of the seed, and ϕ_0 is the divergence angle. This can be expressed in a compact form

$$r^2 = \frac{c^2}{\phi_0} \theta, \quad (\text{A.7})$$

which is an expression for parabolic spiral discussed by Fermat.

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