Optimal Scheduling With Vehicle-to-Grid Regulation Service

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Abstract—In a vehicle-to-grid (V2G) system, aggregators coordinate the charging/discharging schedules of electric vehicle (EV) batteries so that they can collectively form a massive energy storage system to provide ancillary services, such as frequency regulation, to the power grid. In this paper, the optimal charging/discharging scheduling between one aggregator and its coordinated EVs for the provision of the regulation service is studied. We propose a scheduling method that assures adequate charging of EVs and the quality of the regulation service at the same time. First, the scheduling problem is formulated as a convex optimization problem relying on accurate forecasts of the regulation demand. By exploiting the zero-energy nature of the regulation service, the forecast-based scheduling in turn degenerates to an online scheduling problem to cope with the high uncertainty in the forecasts. Decentralized algorithms based on the gradient projection method are designed to solve the optimization problems, enabling each EV to solve its local problem and to obtain its own schedule. Our simulation study of 1000 EVs shows that the proposed online scheduling can perform nearly as well as the forecast-based scheduling, and it is able to smooth out the real-time power fluctuations of the grid, demonstrating the potential of V2G in providing the regulation service.

Index Terms—Charging/discharging scheduling, decentralized algorithm, electric vehicles (EVs), regulation service, vehicle-to-grid (V2G).

I. INTRODUCTION

VEHICLE-TO-GRID (V2G) refers to the utilization of grid-connected electric vehicles (EVs) to provide power and energy services to the power grid [1], [2]. It seeks to utilize the battery packs installed on EVs as distributed energy storage. Power flow can be unidirectional when EVs provide ancillary services by modulating their charging rates, and bidirectional when EVs are also allowed to discharge their batteries to deliver energy back to the grid. As the adoption of EVs grows, V2G can help increase the stability, reliability, and flexibility of the grid by providing ancillary services, such as frequency regulation, spinning reserves, and reactive power support [3].

Frequency regulation is the service for compensating the random and uncorrelated power fluctuations of the grid [4]. Due to the quick-start and fast-response characteristics of battery storage, EVs are suitable for providing the regulation service. However, because of the randomness of the regulation demand, the long-term energy consumption of frequency regulation is usually considered to be zero [4]–[6]. Such zero-energy property may be in conflict with the positive energy consumption of EVs and complicate the charging/discharging process of their batteries. Generally speaking, the V2G operation requires sophisticated control of the charging/discharging behaviors of EVs so as to tackle the following three major challenges:

1) the stochasticity and variability of the regulation demand;
2) the potential conflicts between charging needs of EVs and the provision of the regulation service; and
3) computational efforts and privacy issues incurred by the scheduling process of EVs.

In this paper, we propose an optimal charging/discharging scheduling method, which is able to resolve all of these three challenges at the same time for the real-time control of EVs participating in the V2G regulation service. Before introducing our method, we present a general review on the existing work in order to identify the research gap.

Existing research on the control for the V2G regulation service can be classified into two approaches: 1) the frequency deviation approach [6]–[8] and 2) the incentive approach [5], [9]–[12]. The frequency deviation approach aims to minimize the real-time frequency deviation of the grid based on the droop characteristics and the local frequency measurement. However, the schemes proposed in [6]–[8] cannot achieve the global optimum of allocating the power and energy for a set of EVs over a given time horizon, since the distributed control methods they apply do not involve the coordination of the charging/discharging schedules of EVs.

The incentive approach makes use of incentives to aggregators or EV users to encourage them to provide the regulation service through coordinated scheduling of EVs. The concept of power aggregation of EVs is introduced in [5]. Aggregators gather the relatively small power of individual EVs so that the EV fleet can collectively form a massive energy storage system. Han et al. [5] propose a charging policy to maximize the profit earned for an EV. In [10], the framework in [5] is extended to deal with the discharging problem with a decentralized control scheme. Accounting for the uncertainty of market prices and the regulation signal, Donadee and Ilic [9] formulates a Markov decision problem (MDP) to optimize the baseline charging rates and regulation capacities of EVs. Shi and Wong [11] and Wang et al. [12] study the real-time charging/discharging control problem. Shi and Wong [11] also apply the MDP method to tackle the price uncertainty, while model predictive control (MPC) is adopted in [12]. The disadvantage of the incentive approach [5], [9]–[12] is that it may result in
the inability of V2G in providing power for frequency regulation, since the service providers, i.e., the aggregator and/or EVs, only seek to maximize their own revenues rather than optimize the quality of the regulation service received by the power grid.

As far as we know, a coordinated control scheme of EVs that ensures global optimality of the quality of the regulation service is lacking in the existing research [5]–[12]. This work fills the gap by proposing an optimal scheduling method, which employs a power profile approach, different from the frequency deviation approach and the incentive approach. Our method focuses on the profiles of the V2G power and the actual regulation demand, i.e., the difference between the actual generation and load of the grid. The power profile approach has been applied in the coordinated charging of EVs [13], [14]. However, Gan et al. [13] and Sortomme et al. [14] only consider the charging control of EVs, whereas our work utilizes EVs to provide the regulation service when ensuring that the charging needs of EVs are satisfied. Moreover, accurate forecasts of nonEV demands are necessary for the scheduling process in [13] and [14]. In this paper, we exploit the zero-energy characteristics of the regulation service to derive an online scheduling method that does not require the forecasts of the regulation demand.

This paper is distinguished from the existing literature [5]–[12]. First, the proposed scheme has the advantage over the frequency deviation approach [6]–[8]. The latter approach just tries to offset the immediate fluctuations by local control, whereas we formulate a global optimization problem that aims to obtain an optimal charging/discharging schedules of EVs over a given time horizon and solve it in a coordinated manner. Second, unlike the incentive approach [5], [9]–[12], the proposed control objective, which will be introduced in Section III-A, seeks to optimize the quality of the regulation service, while the revenues of the service providers are not considered explicitly. Nonetheless, our method enables an EV to provide the regulation service during its plug-in period while guaranteeing adequate charging simultaneously. Although Han et al. [5], Donadee and Ilic [9], Han et al. [10], Shi and Wong [11], and Wang et al. [12] ensure every EV will be charged to a desired level of state of charge (SOC), their schemes divide the plug-in period of an EV into two modes: the charging mode when the EV is charging up its battery so as to meet its charging need, and the regulation mode when the battery can get charged/discharged in response to the regulation demand. A drawback of such schemes is that EVs have to occasionally switch between the two modes and, therefore, are not always available to provide the regulation service to the grid. In this sense, our method elongates the active period of EVs to provide the regulation service and therefore does not sacrifice the income of the service providers.

This paper addresses the charging/discharging scheduling problem for an aggregator coordinating EVs to provide frequency regulation in a V2G system. Our contributions are as follows.

1) We propose a control scheme that optimizes the quality of the regulation service and guarantees the adequate charging of EVs in a V2G context.

2) Based on a forecast-based scheduling which requires the forecasts of the regulation demand, an online scheduling for the V2G regulation service is derived to cope with the high uncertainty of the regulation demand.

3) Decentralized algorithms are designed to solve the scheduling problems optimally so as to achieve scalability in real-world V2G systems.

Compared to our prior work [15], this paper conducts a more comprehensive and rigorous study on the scheduling problem with substantial improvements as follows.

1) We propose new formulations for the forecast-based scheduling problem and the online scheduling problem.

2) We account for the effects of the charging and discharging efficiencies of a battery on the quality of the regulation service.

3) We prove the convexities of the new forecast-based and the new online scheduling problems, and proving the convergences of the decentralized algorithms for the two scheduling problems.

4) We discuss the complexities and convergence rates of the proposed decentralized algorithms.

This paper is organized as follows. In Section II, the multilevel V2G system architecture is introduced with a particular focus on the aggregator–EV protocol, which specifies the process for an aggregator to schedule its subordinate EVs. In Section III, according to the proposed control objective for the V2G regulation service, the charging/discharging scheduling problem is formulated as convex optimization problems with two schemes, namely, forecast-based scheduling and online scheduling. The decentralized algorithms to solve the two optimization problems are discussed in Section IV. Case studies are presented and the simulation results are analyzed in Section V. Finally, Section VI draws the conclusion.

II. SYSTEM FRAMEWORK

The multilevel V2G system architecture, shown in Fig. 1, as proposed in our prior work [15] consists of three key components: the grid operator, a set of aggregators, and a set of EVs. It has a hierarchical structure with multiple levels of nodes. The utility grid operator, the aggregators, and the EVs are the root node, the nonleaf nodes, and the leaf nodes, respectively. For convenience, the aggregators directly connected to EVs are called aggregators of EVs. Correspondingly, all the other aggregators are called aggregators of aggregators. Each aggregator node can be viewed as the “root node” of a subtree of aggregators and EVs. The size of a subtree is determined by the size of its subordinate EV fleets and other geographical, economic, and technical factors, such as the communication radius, delay, and cost between nodes at different levels. For instance, a parking lot or a certain area of a large parking lot can install an aggregator of EVs. A number of such parking areas can be controlled by an aggregator of aggregators.

As illustrated in Fig. 1, there are three types of operation protocols, namely, the grid operator–aggregator protocol, the aggregator–aggregator protocol, and the aggregator–EV protocol. Each protocol operates between a node and its immediate subordinate nodes. In this paper, the focus is on the design
III. PROBLEM FORMULATION

A. Control Objective for V2G Regulation Service

The existing control schemes [5]–[12] for the V2G regulation service seek to offset the regulation demand with the charging/discharging power of EVs so that the sum of the regulation demand and the aggregate power of EVs is zero. However, since frequency regulation is a zero-energy service [4], the charging needs of EVs, which consume energy, is unlikely to be met merely by offsetting the regulation demand.

The imbalance between generation and load originates from their respective uncertainties. Given that such imbalance cannot be fully compensated by EVs, we propose to schedule the charging/discharging power of EVs to absorb the uncertainties of generation and load. In other words, EVs are employed to smooth out the power imbalance fluctuations of the grid by minimizing the variance of the profile of the total power, which is the sum of the regulation demand and the aggregate power of EVs.

From an economic perspective, the proposed control objective is prone to minimize the costs for the system to meet the regulation demand and the charging needs of EVs. Without V2G or other storage technologies, conventional generators have to be used to supply frequency regulation at a great expense, including the high-ramping costs and the lost opportunity costs of the underutilized regulation capacity [4]. By implementing the proposed control objective, the random fluctuations of the regulation demand are absorbed by EVs. Therefore, the resultant total power of the regulation demand and the powers of EVs can be met by conventional generators with minimal ramping costs, and the required generation reserve for the regulation service is minimized as well. Our simulation results in Section V-D illustrate the effectiveness of the proposed optimal scheduling method. The profiles of the total power become flat and minimize the ramping costs of the balancing conventional generators.

B. Models and Constraints

The coordinated scheduling process, which determines the charging/discharging scheduling of EVs, is the core of the aggregator–EV protocol. Consider a scenario in which an aggregator of EVs coordinates its EVs to meet the regulation request. The regulation requests are the aggregators’ shares of the regulation demand according to their contracts for the regulation service.

The framework and information flow of the aggregator–EV protocol are presented in Fig. 2. The aggregator of EVs receives the request signal for the regulation service, i.e., the regulation request, from its upper level node, which may be the grid operator or an aggregator of aggregators. Then, the aggregator of EVs starts to coordinate the charging/discharging schedules of its subordinate EVs to meet the regulation request. The coordinated scheduling process can be an iterative process by implementing a decentralized algorithm so that the computation can be effectively distributed to EVs. In each round of the iteration, the aggregator calculates the control signal according to the charging/discharging schedules received from a set of its subordinate EVs and broadcasts the control signal to the EVs. Then, every EV adjusts its own schedule based on the control signal and reports its new decision to the aggregator. The iterative process ends when the predetermined stopping criteria are met. Finally, the aggregator reports the commitment for the regulation service to its upper level node.
EV will provide either unidirectional or bidirectional V2G regulation service according to its contract to the aggregator. Let \( \theta_{bi} \in [0, 1] \) denote the proportion of the EVs that participate in bidirectional V2G. Note that those EVs participating in unidirectional V2G will not discharge their batteries to provide regulation up.

Denote the plug-in time and plug-out time of EV \( n \) as \( T_{n,in} \) and \( T_{n,out} \), respectively. When EV \( n \) is plugged in, its charging/discharging power should follow:

\[
P_{n,\text{discharge}}(t) \leq P_{n,\text{charge}}, \quad t \in [T_{n,in}, T_{n,out}] 
\]

where \( P_{n,\text{discharge}} \leq 0 \) and \( P_{n,\text{charge}} > 0 \) denote the limits of discharging power and charging power of EV \( n \), respectively.

Denote the profile of the actual regulation requests as \( R(T) := (R(T_1), R(T_2), \ldots, R(T_{N_T}))^T \), where \((\cdot)^T\) denotes transposition. Denote the charging/discharging schedule of EV \( n \) as \( P_n(T) := (P_n(T_1), P_n(T_2), \ldots, P_n(T_{N_T}))^T \), and the schedules of all EVs as \( P_N(T) := (P_1(T), P_2(T), \ldots, P_{N_EV}(T)) \). Then, the profile of the total power, which is the sum of the regulation requests and the aggregate power of EVs, is defined as follows:

\[
P_{\text{total}}(T) := R(T) + P_A(T)
\]

where \( P_A(T) \) denotes the profile of the aggregate power of EVs defined as

\[
P_A(T) := \sum_{n \in N} P_n(T).
\]

Let \( \text{SOC}_{n,0} \), \( \text{SOC}_{n}(T_k) \), and \( C_n \) be the initial SOC, SOC at the end of \( T_k \), and capacity of the battery pack of EV \( n \), respectively. Considering the energy conversion efficiency between the power grid and the batteries of EVs, \( \text{SOC}_{n}(T_k) \) can be calculated as

\[
\text{SOC}_{n}(T_k) = \text{SOC}_{n,0} + \Delta t \sum_{i=1}^{k} \eta(P_n(T_i))P_n(T_i)
\]

where \( \eta(P_n(T)) \) calculates the energy conversion efficiency of a given charging/discharging power \( x \) of an EV. Assume that the charging and discharging efficiencies are, respectively, identical among the EVs. Then, \( \eta(P_n(T)) \) is defined as

\[
\eta(P_n(T)) := \begin{cases} 
\eta_{ch}, & \text{if } x \geq 0 \\
\frac{1}{\eta_{dch}}, & \text{if } x < 0
\end{cases}
\]

where \( \eta_{ch} \) and \( \eta_{dch} \) are the charging and discharging efficiencies of the EVs, respectively, and we have

\[
0 < \eta_{ch}, \eta_{dch} \leq 1.
\]

Two constraints for the SOC of the battery pack during the plug-in period of EV \( n \), where \( n \in N \) are proposed as

\[
\text{SOC}_{n}(T_{N_T}) \geq \text{SOC}_{n,\text{MinCh}}
\]

\[
\text{SOC}_{n,\text{min}} \leq \text{SOC}_{n}(T_k) \leq \text{SOC}_{n,\text{max}}, \quad T_k \in T.
\]

SOC\(_{n,\text{MinCh}}\) denotes the minimum value of SOC that EV \( n \) needs to reach before it is plugged out. The constraint represented by (7) ensures that EV \( n \) will have been charged up with enough energy for the next trip when it is plugged out. SOC\(_{n,\text{min}}\) and SOC\(_{n,\text{max}}\) denote the lower and upper SOC limits, respectively, of EV \( n \) for all \( T_k \in T \). The constraint in (8) prevents deep discharging or over-charging of the battery so as to prolong the battery life.

C. Formulation of Forecast-Based Scheduling

Assume that, before the participation period \( T \), the aggregator receives the forecasting profile \( R_f(T) := (R_f(T_1), R_f(T_2), \ldots, R_f(T_{N_T}))^T \) of its actual regulation requests \( R(T) \), and is able to communicate with \( N_{EV} \) EVs that are going to participate in the V2G regulation service during \( T \). Then, according to the control objective proposed in Section III-A, it should coordinate these \( N_{EV} \) EVs to determine their optimal charging/discharging schedules \( P_N(T) \) by the following optimization problem

\[
\min_{P_N(T)} U_f(P_A(T))
\]

such that \( \forall n \in N, (1), (7), \) and (8) hold

where \( U_f(P_A(T)) \) calculates the variance of the profile of the total power \( P_{\text{total}}(T) \). Therefore, we have

\[
U_f(P_A(T)) := \text{Var}(P_{\text{total}}(T))
\]

\[
= \frac{1}{N_T} \sum_{T_i \in T} \left( R_f(T_i) + P_A(T_i) \right)^2 - \frac{1}{N_T} \left( \sum_{T_j \in T} (R_f(T_j) + P_A(T_j)) \right)^2
\]

where \( \text{Var}(\cdot) \) denotes the function for calculating variance.

Theorem 1: The optimization problem in (9) is a convex optimization problem.

Proof: See the Appendix, Section A.

D. Formulation of Online Scheduling

In practice, the regulation demand is derived from the regulation signals measured in real time. Hence, it is more realistic to adopt online scheduling, which schedules the charging/discharging power of EVs in response to the real-time input of a regulation request. Consider a scenario of online scheduling where at each time slot \( T_k \in T \), the aggregator receives the real-time signal of a regulation request \( R(T_k) \) and then coordinates the EVs to update their charging/discharging
schedules from $T_k$ to $T_{N_T}$, i.e., $\{P_n(T_j)|n \in \mathcal{N}, k \leq j \leq N_T\}$, so that $\text{Var}(P_{\text{total}}(T))$ is minimized. We have

$$\text{Var}(P_{\text{total}}(T)) = \frac{1}{N_T} \sum_{i=1}^{k} \left( R(T_i) + \sum_{n \in \mathcal{N}} P_n(T_i) - A(P_{\text{total}}(T)) \right)^2$$

$$+ \frac{1}{N_T} \sum_{j=k+1}^{N_T} \left( R_f(T_j) + \sum_{n \in \mathcal{N}} P_n(T_j) - A(P_{\text{total}}(T)) \right)^2 \tag{11}$$

where $A(P_{\text{total}}(T))$ denotes the average of $P_{\text{total}}(T)$ as

$$A(P_{\text{total}}(T)) := \frac{1}{N_T} \left( \sum_{i=1}^{k} R(T_i) + \sum_{j=k+1}^{N_T} R_f(T_j) + \sum_{n \in \mathcal{N}} \sum_{T_i \in \mathcal{T}} P_n(T_i) \right). \tag{12}$$

From (11), the forecasts of the future regulation requests $R_f(T_j), j = k+1, k+2, \ldots, T_k$, can be approximated by

$$R_f(T_j) = \mathbb{E}(R(T_j)|\{R(T_i)|1 \leq i \leq k\}). \tag{13}$$

However, the calculation of the conditional expectation in (13) requires the distribution of the regulation demand which is not known a priori. Nonetheless, since frequency regulation is a zero-energy service, the expectation of the total energy that the regulation service requires is zero over a long period of time. Therefore, we can make the following assumption

$$\mathbb{E}(R_S(T)) = 0 \tag{14}$$

where

$$R_S(T) := \sum_{T_i \in \mathcal{T}} R(T_i). \tag{15}$$

From (13) and (14), we have

$$\sum_{j=k+1}^{N_T} R_f(T_j) = \mathbb{E} \left( \sum_{j=k+1}^{N_T} R(T_j) | \{R(T_i)|1 \leq i \leq k\} \right)$$

$$= - \sum_{i=1}^{k} R(T_i). \tag{16}$$

By applying the Cauchy–Schwarz Inequality, we can derive a lower bound of the second summation in (11) when $k \leq N_T - 1$ as follows:

$$\frac{1}{N_T} \sum_{j=k+1}^{N_T} \left( R_f(T_j) + \sum_{n \in \mathcal{N}} P_n(T_j) - A(P_{\text{total}}(T)) \right)^2 \geq \frac{1}{N_T(N_T-k)} \left( \sum_{j=k+1}^{N_T} R_f(T_j) + \sum_{n \in \mathcal{N}} \sum_{j=k+1}^{N_T} P_n(T_j) \right)^2$$

$$- (N_T-k)A(P_{\text{total}}(T))^2. \tag{17}$$

The equality of (17) holds if and only if the following condition is satisfied

$$\forall k+1 \leq j, l \leq N_T, R_f(T_j) + P_n(T_j) = R_f(T_l) + P_n(T_l). \tag{18}$$

Condition (18) also minimizes (11). Therefore, the lower bound derived in (17) can be used to approximate (11) since we seek to minimize the variance of $P_{\text{total}}(T)$.

Based on (16) and (17), an approximation of $\text{Var}(P_{\text{total}}(T))$ in (11) for $k \leq N_T - 1$ is derived and used as the objective function for the online scheduling problem as follows:

$$U_o(Q_A(T_k)) := \frac{1}{N_T} \sum_{i=1}^{k} \left( R(T_i) + \sum_{n \in \mathcal{N}} P_n(T_i) \right)^2$$

$$+ \frac{\alpha(k)}{N_T} \left( \sum_{i=1}^{k} R(T_i) + \sum_{n \in \mathcal{N}} FP_n(T_k) \right)^2$$

$$- \frac{1}{N_T} \left( \sum_{n \in \mathcal{N}} \left( \sum_{i=1}^{k} P_n(T_i) + FP(T_k) \right) \right)^2 \tag{19}$$

where

$$Q_A(T_k) = (P_A(T_k), FP_A(T_k))^T := \sum_{n \in \mathcal{N}} Q_n(T_k) \tag{20}$$

and

$$\alpha(k) := \begin{cases} \frac{1}{N_T-k} & \text{if } 1 \leq k \leq N_T - 1 \\ 0 & \text{if } k = N_T. \end{cases} \tag{21}$$

In (20), $Q_n(T_k) := (P_n(T_k), FP_n(T_k))^T$ denotes the control variables or schedule of EV $n, \forall n \in \mathcal{N}$, at $T_k$, where $FP_n(T_k)$ is the sum of the future charging/discharging powers of EV $n$ as follows:

$$FP_n(T_k) := \sum_{i=k+1}^{N_T} P_n(T_i). \tag{22}$$

Note that the charging/discharging powers of the EVs before $T_k$, i.e., $\{P_n(T_i)|n \in \mathcal{N}, 1 \leq i \leq N_T\}$, are not included in the control variables since they are already historical data.

Denote the schedules of all EVs at $T_k$ as $Q_{\mathcal{N}}(T_k) := (Q_{1}(T_k), Q_{2}(T_k), \ldots, Q_{N_{\text{EV}}}(T_k))$. The online scheduling problem for the V2G regulation service is formulated as follows:

At each $T_k \in \mathcal{T}$

$$\min_{Q_{\mathcal{N}}(T_k)} U_o(Q_A(T_k)) \tag{23a}$$

such that for all $n \in \mathcal{N}$

$$\eta(P_n(T_k))P_n(T_k) + \eta(FP_n(T_k))FP_n(T_k)$$

$$\geq \frac{C_n}{\Delta} (\text{SOC}_{n,\text{MinCh}} + \text{SOC}_{n,\text{MOS}(T_k)} - \text{SOC}_{n}(T_{k-1})), \tag{23b}$$

and (1) and (8) hold.
From (23b), \( \text{SOC}_{n, \text{MOS}}(T_k) \) denotes the SOC “margin of safety” of EV \( n \in \mathcal{N} \) defined as follows:

\[
\text{SOC}_{n, \text{MOS}}(T_k) := \begin{cases} 
\mu(\text{SOC}_{n, \text{max}}) - \text{SOC}_{n, \text{MinCh}}, & T_k \in [T_{n,\text{in}}, (1 - \tau)T_{n,\text{in}} + \tau T_{n,\text{out}}] \\
0, & \text{otherwise}
\end{cases}
\]

(24)

where \( \mu \in [0, 1] \) quantifies the relative amount of the safety margin, and \( \tau \in [0, 1] \) determines the ratio of the time that the safety margin is in effect. The parameters \( \mu \) and \( \tau \) are identical among the EVs.

Constraint (23b) is derived from (7) with \( \text{SOC}_{n, \text{MOS}}(T_k) \) added to the charging requirement of EV \( n \), where \( n = 1, 2, \ldots, N_{\text{EV}} \), in (7). The purpose of introducing the margin of safety for charging is to cope with the uncertainty of the regulation requests. Because the objective function (19) for the online scheduling problem in (23) approximates \( \text{Var}(P_{\text{total}}(T)) \) in (11) based on the zero-sum assumption in (14) of the regulation requests \( R(T) \), such approximation may be inaccurate when the assumption stated in (14) does not hold in some cases. By introducing the margin of safety for charging in (23b), the EVs would buffer some more energy on top of their minimum charging requirements to meet the extra energy needs for regulation up, i.e., \( R_S(T) > 0 \).

**Theorem 2:** For all \( T_k \in \mathcal{T} \), the optimization problem in (23) is a convex optimization problem.

**Proof:** See the Appendix, Section B.

Although the proposed online scheduling problem in (23) only optimizes an approximation of (11), it is more practical than the forecast-based scheduling problem in (9) because it does not require the forecasts of the regulation requests. In addition, (23) incurs much lower computational complexity than (9) since it reduces the number of control variables significantly.

IV. DECENTRALIZED SCHEDULING ALGORITHMS

In this section, two decentralized algorithms, namely, Algorithm 1 (including Algorithms 1A and 1B) and Algorithm 2 (including Algorithms 2A and 2B), are proposed to solve the forecast-based scheduling problem in (9) and the online scheduling problem in (23), respectively. They are inspired by the decentralized algorithm proposed for optimal EV charging control [13] and based on the gradient projection method [16].

Since Algorithms 1 and 2 distribute the computational efforts to EVs, the aggregator only needs to perform simple arithmetic for calculating the control signals with a computational complexity equal to \( O(N_{\text{EV}}) \). In each round of the iterations, every EV should update its own schedule by solving an optimization problem. The computational complexity of an EV is \( O(1) \) in terms of the scale of the set of EVs. Therefore, Algorithms 1 and 2 are highly scalable.

**Theorem 3:** In Algorithm 1 the schedules \( P_{n}(T) \) converge to one of the optimal solutions for the forecast-based scheduling problem in (9) as \( m \to \infty \).

**Proof:** See the Appendix, Section C.

**Algorithm 1A. Forecast-Based Scheduling for Aggregator**

**Input:** The participation period \( \mathcal{T} \). Before the participation period \( \mathcal{T} \) starts, the aggregator receives the forecasting profile of the regulation requests \( R_f(\mathcal{T}) \) and knows the number of EVs \( N_{\text{EV}} \).

**Output:** The charging/discharging schedules of EVs \( P_{n}(T) := (P_1(T), P_2(T), \ldots, P_{N_{\text{EV}}}(T)) \).

Choose a parameter \( \beta \) such that \( 0 < \beta < \frac{N_{\text{EV}}}{N_{\text{EV}} - 1} \).

Wait for the initial schedule \( P_0(n)(T) \) of every EV \( n \in \mathcal{N} \). Set the iteration number \( m \leftarrow 1 \), repeat Steps 1–3.

1. Calculate the control signal \( s^m_j(T) := (s^m_{f,j}(T_1), s^m_{f,j}(T_2), \ldots, s^m_{f,j}(T_{N_{\text{EV}}})) \) as follows:

\[
s^m_{f,j}(T) := \beta \nabla U_j(P_{\mathcal{A}}^{m-1}(T)).
\]

Therefore, \( \forall T_k \in \mathcal{T} \),

\[
s^m_{f,j}(T_k) := \frac{\partial U_j(P_{\mathcal{A}}^{m-1}(T))}{\partial P_{\mathcal{A}}(T_k)}
= \frac{2\beta}{N_{\text{EV}}} (R_f(T_k) + \sum_{n \in \mathcal{N}} P_{n}^{m-1}(T_k))
- \frac{2\beta}{N_{\text{EV}}^2} \left( \sum_{T_j \in \mathcal{T}} (R_f(T_j) + \sum_{n \in \mathcal{N}} P_{n}^{m-1}(T_j)) \right).
\]

Broadcast the control signal \( s^m_{f,j}(T) \) to all EVs.

2. Wait for the updated schedule \( P_{n}^{m}(T) \) reported by every EV \( n \in \mathcal{N} \).

3. If the stopping criteria of Algorithm 1B are not met, set \( m \leftarrow m + 1 \) and go to Step 1. Otherwise, broadcast the message that the iteration process ends to all EVs.

**Return** \( P_{n}(T) = P_{n}^{m}(T) \).

**Algorithm 2. Online Scheduling**

In Algorithm 2, at any time slot \( T_k \in \mathcal{T} \), the schedules \( Q_{n}(T_k) \) converge to one of the optimal solutions for the online scheduling problem in (23) as \( m \to \infty \).

**Proof:** See the Appendix, Section D.

In Algorithms 1 and 2, \( \langle \cdot, \cdot \rangle \) represents the dot product operation and \( ||\cdot|| \) denotes the Euclidean norm.

A. Forecast-Based Scheduling

For the forecast-based scheduling, it is assumed that all \( N_{\text{EV}} \) EVs are available to run Algorithm 1 under the coordination of the aggregator before the participation period \( \mathcal{T} \) starts. Since the forecasting inputs of the regulation requests over such a long time horizon \( \mathcal{T} \) (a span of hours in our context) are highly unreliable, forecast-based scheduling is not practical in the real world. Nonetheless, Algorithm 1 can still be useful to obtain the best possible scheduling results as the performance bound when we assume that \( R_f(T) \) is accurate in the simulation.

The stopping criteria of Algorithm 1 can be based on the number of iterations performed, i.e., \( m = M_f \), where \( M_f \) is the maximum number of iterations, and/or the convergence of
Algorithm 1B. Forecast-Based Scheduling for each EV $n \in \mathcal{N}$

**Input:** The participation period $\mathcal{T}$. EV $n \in \mathcal{N}$ knows its own constraints (1), (7), and (8).

**Output:** The charging/discharging schedule of EV $n$, $P_n(T)$.

Initialize the schedule $P_n^0(T)$ such that $P_n^0(T)$ lies in the boundary of the region defined by the constraint (7). Then report $P_n^0(T)$ to the aggregator.

Set the iteration number $m \leftarrow 1$, repeat Steps 1–3.

1) Wait for the updated control signal $s_f^m(T)$ broadcast by the aggregator.

2) Calculate a new schedule $P_n^m(T)$ as

$$P_n^m(T) := \arg \min_{P_n(T)} \left( \left\langle s_f^m(T), P_n(T) \right\rangle + \frac{1}{2} \left\| P_n(T) - P_n^{m-1}(T) \right\|^2 \right)$$

such that (1), (7) and (8) hold

Report $P_n^m(T)$ to the aggregator.

3) If the aggregator has not announced that the iteration process has ended, set $m \leftarrow m + 1$ and go to Step 1.

**Return** $P_n^m(T) = P_n^m(T)$.

the control signal $s_f^m(T)$ within the convergence tolerance, i.e.,

$$\left\| s_f^m(T) - s_f^{m-1}(T) \right\| \leq \epsilon_f, \text{ where } \epsilon_f > 0 \text{ is the convergence tolerance.}$$

The forecast-based scheduling problem in (9) minimizes the variance of $P_{\text{total}}(T)$. Therefore, without considering the constraints of (9), the optimal solutions of (9) should follow:

$$\forall T_i \in \mathcal{T}, P_{\text{total}}(T_i) = A(P_{\text{total}}(T)). \quad (28)$$

The value of $A(P_{\text{total}}(T))$ does not affect the optimality of (9) as long as (28) is satisfied. Since constraints (7) and (8) allow the final SOC of an EV $n \in \mathcal{N}$ to be within a given range, i.e., $\text{SOC}_n(T_k) \in [\text{SOC}_{n, \text{MinCh}}, \text{SOC}_{n, \text{max}}]$, the total energy consumption of EVs, $\sum_{T_k \in \mathcal{T}} P_A(T_k)$, is not fixed. The proposed Algorithm 1 does not determine the value of $A(P_{\text{total}}(T))$. Nevertheless, the value of $A(P_{\text{total}}(T))$ is fixed in the optimization problem. The iteration number $m$ is fixed in the initialization step, since the searching for the optimal schedules $P_{\mathcal{N}}(T)$ starts from $P_{\mathcal{N}}^0(T)$.

In each round of the iterations of Algorithm 1, the aggregator calculates and broadcasts the control signal $s_f^m(T) \in \mathbb{R}^{N_T \times 1}$ from the schedules $P_{\mathcal{N}}^{m-1}(T) \in \mathbb{R}^{N_T \times N_{\text{EV}}}$ received from the EVs. Every EV $n \in \mathcal{N}$ needs to solve the optimization problem (27) to obtain its updated schedule $P_n^m(T)$ in $\mathbb{R}^{N_T \times 1}$ and reports $P_n^m(T)$ to the aggregator. Therefore, the total communication overhead $C_{f}$ of Algorithm 1 is calculated as

$$C_{f} := D \cdot m_f \cdot N_T \cdot (N_{\text{EV}} + 1). \quad (29)$$

Algorithm 2A. Online Scheduling for Aggregator

**Input:** At any time slot $T_k \in \mathcal{T}$, the aggregator knows the total number of time slots $N_T$, and the number of EVs $N_{\text{EV}}$, and has received the regulation requests $\{R(T_i)\}_{1 \leq i \leq k}$.

**Output:** The charging/discharging schedules of EVs at $T_k$, $Q_{N}(T_k) := (Q_1(T_k), Q_2(T_k), \ldots, Q_{N_{\text{EV}}}(T_k))$.

Choose a parameter $\beta$ such that $0 < \beta < \frac{N_T}{N_{\text{EV}}}$. Wait for the initial schedule $Q_n^0(T_k)$ of every EV $n \in \mathcal{N}$. Set the iteration number $m \leftarrow 1$, repeat Steps 1–3.

1) Calculate the control signal $s_o^m(T_k) \in \mathbb{R}^{2 \times 1}$ as follows:

$$s_o^m(T_k) := \beta \nabla U_o(Q_A^{m-1}(T_k))$$

$$= \beta \left( \frac{\partial U_o(Q_A^{m-1}(T_k))}{\partial P_A(T_k)} \right) - \frac{\partial U_o(Q_A^{m-1}(T_k))}{\partial P_A(T_k)} \right) T$$

$$= 2\beta \left( \frac{\partial \alpha(k)}{\partial \beta} + \frac{P_A^{m-1}(T_k)}{N_T} \right)$$

$$+ \left( \frac{2\beta}{N_T} \sum_{i=1}^{k} P_A(T_i) + P_A^{m-1}(T_k) \right)$$

$$+ F_P^{m-1}(T_k). \quad (30)$$

Broadcast the control signal $s_o^m(T_k)$ to all EVs.

2) Wait for the updated schedule $Q_n^m(T_k)$ reported by every EV $n \in \mathcal{N}$

3) If the stopping criteria of Algorithm 1B are not met, set $m \leftarrow m + 1$ and go to Step 1. Otherwise, broadcast the message that the iteration process ends to all EVs.

**Return** $Q_{N}(T_k) = Q_{N}^m(T_k)$.

where $D$ and $m_f$ denote the size of a one-dimensional (1-D) control variable, e.g., $P_n(T_k)$, and the number of iterations performed, respectively.

B. Online Scheduling

For the online scheduling, Algorithm 2 is performed at every slot $T_k \in \mathcal{T}$ to update the schedules of EVs according to the newly received regulation request $R(T_k)$.

Similar to those of Algorithm 1, the stopping criteria of Algorithm 2B can be $m = M_o$ where $M_o$ is the maximum number of iterations, and/or $\left\| s_o^m(T_k) - s_o^{m-1}(T_k) \right\| \leq \epsilon_o$, where $\epsilon_o > 0$ is the convergence tolerance.

At every slot $T_k \in \mathcal{T}$, the optimization results $Q_{N}(T_k)$ are influenced by the profiles of the current and past regulation requests $\{R(T_i)\}_{1 \leq i \leq k}$ and the historical charging/discharging power of the EVs $\{P_n(T_k)\}_{n \in \mathcal{N}, 1 \leq i \leq k}$. In each round of the iterations of Algorithm 2, the aggregator calculates and broadcasts the control signal $s_o^m(T_k) \in \mathbb{R}^{2 \times 1}$ from the schedules $Q_{N}^{m-1}(T_k) \in \mathbb{R}^{2 \times N_{\text{EV}}}$ received from the EVs. Every EV $n \in \mathcal{N}$ needs to solve the optimization problem (27) to obtain its updated schedule $Q_n^m(T_k) \in \mathbb{R}^{2 \times 1}$.
Three scheduling algorithms, namely, Algorithm 1 for the forecast-based scheduling problem in (9), Algorithm 2 for the online scheduling problem in (23), and an extended version, which is introduced below, of the optimal decentralized charging (ODC) algorithm proposed in [13], will be investigated by computer simulation.

Since the ODC algorithm proposed in [13] does not consider discharging of EV batteries, we extend ODC by enabling discharging to fit in our context. Hence, the optimization problem of ODC with discharging (ODCD) is discussed as follows:

\[
\min_{P_N(T)} \sum_{T_i \in T} u \left( R_f(T_i) + \sum_{n \in \mathcal{N}} P_n(T_i) \right)
\]

such that for all \( n \in \mathcal{N}, (1) \) holds, and

\[
SOC_n(T_{N_T}) = SOC_{n,\text{MinCh}}
\]

where \( u : \mathbb{R} \to \mathbb{R} \) is strictly convex. According to Theorem 2 of [13], the optimal total power profile obtained by (34) is independent on the choice of \( u \).

According to Property 1 of [13], ODC is able to obtain a flat total power profile by scheduling the charging activities of EVs. However, when discharging is introduced and ODC is extended to ODCD, such valley-filling property of ODC may not be inherited by ODCD. It will be studied in the simulation.

### B. Performance Metric

According to the proposed control objective for the V2G regulation service, the variance of the profile of the total power \( \text{Var}(P_{\text{total}}(T)) \) is used as the performance metric.

A smaller \( \text{Var}(P_{\text{total}}(T)) \) implies a more flattened profile of the total power, indicating that the fluctuations of the regulation requests are better absorbed by the aggregated EV power, and, therefore, a better performance.

### C. Simulation Setup

The simulation scenario is an aggregator coordinating \( N_{EV} = 1000 \) EVs to decide their charging/discharging schedules from 19:00 in the evening to 7:00 on the next morning. This 12-h period of time is divided equally into \( N_T = 144 \) slots of length \( \Delta t = 5 \) min.

The hypothetic EV group consists of four models of EVs currently on the market: Chevrolet Volt with a 16.5-kWh battery pack [17], Ford C-MAX Energi with a 7.6-kWh battery pack [18], Nissan Leaf SV with a 24-kWh battery pack [19], and Tesla Model S with a 60-kWh battery pack [20]. Each of the four models accounts for 25% of the 1000 EVs. All EVs are assumed to have been contracted to provide the V2G regulation service, either unidirectionally or bidirectionally. According to the standard Level 2 charging in the USA [21], we assume that the charging power of Chevrolet Volt and C-MAX Energi can vary from 0 to 4.0 kW. Nissan Leaf SV and Tesla Model S have their dedicated 240-V chargers with charging powers of 6.6 kW [19] and 10 kW [20], respectively. Thus, we assume that the charging power of Leaf SV and Model S can vary from 0 to 6.6 kW and from 0 to 10 kW, respectively. Furthermore, the discharging power of all of the four models is assumed to vary from \(-4 \) kW to 0 for bidirectional V2G. According to Shao et al. [22], the distribution of plug-in time of EVs is close to a normal distribution. Hence, in the simulation, the plug-in times of EVs are generated based on a normal distribution with the mean at 19:00 and the standard deviation is equal to 1 h first, and then any plug-in time before 19:00 is set to be 19:00. Similarly, the plug-out times are also generated based on a normal distribution with the mean at 7:00 and the standard deviation is equal to 1 h first. Then, any plug-out time after 7:00 is set to be 7:00. The values or distributions of the parameters for EVs are summarized in Table I.
A total of 31 sets of the 12-h profiles of regulation signals are extracted from the 32-day period. According to Rebourset al. [24], the regulation signal is normalized to be within the range of \([-1, 1]\), and related to the regulation demand linearly. The ratio between the regulation demand and regulation signal is set by the balancing authority of a specific control area. Since, the most of the 1000 EVs in the hypothetic EV group have charging and discharging power limits of 4 kW and \(-4\) kW, respectively, and the regulation service is usually bid on an MW basis [2], it is reasonable to assume that the aggregator would receive the assigned regulation requests within the range of \([-2, 2]\) MW in this case. Hence, the ratio between the regulation requests and regulation signal would be 2 MW in the simulation. According to (14), the expectation of the sum of the regulation requests over the 12-h participation period is assumed to be zero. Fig. 3 shows the histogram of the sum of the 12-h regulation requests. The minimum and maximum values of the sum \(R_S(T)\) are \(-13.442\) and \(8.161\) MW. It can be observed that the total energy needs of frequency regulation over the specified 12-h period may be nonzero although in the most cases the total energy needs are close to zero.

Both Algorithm 1 for forecast-based scheduling and ODCD require the forecasting profile of the regulation requests. In the simulation, it is assumed that the forecasts of the regulation requests are accurate, i.e., \(R_f(T_t) = R(T_t), \forall T_t \in T\).

For the stopping criteria of Algorithm 1, \(\epsilon_f = 10^{-5}\) and \(M_f = 100\). As for Algorithm 2, \(\epsilon_o = 10^{-5}\) and \(M_o = 50\).

The simulation has been run in MATLAB, Release 2013b.

\section*{D. Simulation Results}

The scheduling results of the three algorithms with various values of the participation ratio of the bidirectional V2G \(\theta_{bi}\) and the sum of the regulation requests \(R_S(T)\) are studied. Table II lists the performances of the three algorithms with \(\theta_{bi}\) equal to 1, 0.5, and 0, respectively, in three special cases of \(R_S(T)\), namely, the minimum absolute, maximum, and minimum values of \(R_S(T)\), respectively, among the 31 sets of the profiles of the regulation requests.

First, we compare the scheduling results in the ideal case when the sum of the regulation requests is close to zero. Among the 31 sets of profiles of regulation requests, the one with its sum closest to zero has \(R_S(T) = -24.09\) kW. Fig. 4 presents the results of the three scheduling algorithms when \(\theta_{bi}\) is equal to 1, 0.5, and 0, respectively.

As shown in Fig. 4(a) with \(\theta_{bi} = 1\), i.e., all EVs participate in the bidirectional V2G, the total power profile of the proposed forecast-based scheduling (the dashed curve) is flat, indicating that the power fluctuations represented by the profile of the regulation requests are smoothed out under the assumption of accurate forecasts of the regulation requests. The total power profile of the proposed online scheduling (the dashed-dotted curve) is almost as flat as the dashed curve. Note that the dashed curve and the dashed-dotted curve are close to two constant positive loads of about 731 and 874 kW, respectively.

As indicated in Section III-A, the constant loads are approximately equal to the power consumptions for satisfying the charging needs of EVs since the sum of the regulation requests is close to zero. The dashed-dotted curve is higher than the dashed curve because the safety margin for charging stated in (24) of online scheduling entails a higher energy consumption of EVs.

As shown in Fig. 4(b) with \(\theta_{bi} = 0.5\), the power profiles of forecast-based scheduling and online scheduling are still flat although 50% of EVs participate in the unidirectional V2G for which discharging is not allowed. However, when all EVs participate in unidirectional V2G, as shown in Fig. 4(c) with \(\theta_{bi} = 0\), the performances of the two scheduling algorithms deteriorate since the EVs are unable to discharge their batteries.
to maintain a flat profile of the total power when the regulation requests are large. The results of online scheduling at different values of \( \theta_{bi} \) imply that there exists a minimum \( \theta_{bi} \) for a given upper bound of \( \text{Var}(P_{\text{total}}) \). In other words, it may not always be necessary to have all EVs enabled with the bidirectional V2G.

The reliability and resilience of the proposed online scheduling in some extreme cases when \( R_S(T) \) is far beyond zero are also investigated. Fig. 5 presents the simulation results of the three scheduling algorithms when the sum of the regulation requests \( R_S(T) \) is the maximum and the minimum, respectively, among the 31 sets of the profiles of the regulation requests. The total power profiles of online scheduling (the dashed-dotted curves) are still nearly as flat as those of forecast-based scheduling (the dashed curves) even when the zero-energy assumption specified in (14) is seriously violated and \( R_S(T) \) takes large absolute values. Although the safety margin of online scheduling results in a total load a bit higher than that of forecast-based scheduling, it makes online scheduling resilient to different regulation requests. In addition, since the total power profile is flattened by online scheduling, such total load can be economically met by load following or even generation dispatch. Therefore, online scheduling will not impair the stability of the grid.

The proposed forecast-based scheduling and online scheduling outperform ODCD. As shown in Figs. 4 and 5, ODCD fails to produce flat total power profiles even though the forecasts of the regulation requests have been assumed to be accurate. There are obvious spikes in the total power profiles, shown by the dots and solid curves, corresponding to ODCD when the power demands of the regulation-up requests are so high that the EV fleet should collectively provide discharging power to the grid. This is because ODCD does not account for the effect of the round-trip efficiency of a battery. It indicates that the method proposed in [13] is not suitable to deal with the discharging control of EVs. Moreover, ODCD is impractical for the scheduling control of the regulation service since it requires the forecasting profile of regulation requests.
To summarize, the simulation results further corroborate our statements in Sections III-C and III-D that the optimization result of forecast-based scheduling can serve as a performance bound since it is the best possible schedules when the forecasts of the regulation requests are accurate, but online scheduling is more suitable and practical for real-world implementation because it does not depend on the forecasts of the regulation requests, determines the schedules in real time, and has a comparable performance to forecast-based scheduling.

### E. Convergence Rates

The convergence rates of Algorithm 1 for forecast-based scheduling for various values of the ratio of the bidirectional V2G $\theta_{bi}$ and the convergence tolerance $\epsilon_f$ are presented in Table III. The numbers of iterations shown in Table III are averaged over the 31 sets of the profiles of the regulation requests. It can be observed that given $\epsilon_f$, the number of iterations increases as $\theta_{bi}$ decreases. This is because as $\theta_{bi}$ decreases, the number of EVs that are allowed to discharge their batteries decreases, and hence it becomes more difficult for EVs to suppress the peaks of the regulation requests and flatten the total power profile. According to (29) and Table III, the communication overheads of Algorithm 1 can be obtained. For instance, when $\theta_{bi} = 1$ and $\epsilon_f = 10^{-5}$, the average number of iterations at each time slot is 5.2. Thus, the average communication overhead per slot $C_{O,bi}$ in this case is about 81.331 kB according to (33). Since Algorithm 2 should be performed $N_T = 144$ times during the participation period $T$, the total overhead throughout $T$ is about 11.437 MB.

### VI. Conclusion

The optimal scheduling for an aggregator coordinating its EVs to provide the V2G regulation service is studied. Based on the zero-energy characteristics of frequency regulation, we propose an online scheduling method which does depend on the forecast of the regulation demand and allows each EV to determine its own schedule in real time. Our method jointly guarantees adequate charging of EVs and optimizes the quality of the regulation service. A simulation study of 1000 hypothetic EVs shows that the proposed online scheduling algorithm performs nearly as well as the forecast-based scheduling algorithm, demonstrating the practicability of online charging/discharging control for the provision of the V2G regulation service. Future work will extend the methods and models proposed in this paper to solve the inter-level control in a multilevel V2G system.

### APPENDIX

#### A. Proof of Theorem 1

**Lemma 1:** The objective function (10), $U_f : \mathbb{R}^{N_T} \rightarrow \mathbb{R}$ of the optimization problem (9) is convex.

**Proof:** $\forall 1 \leq i, j \leq N_T$, according to the first-order partial derivatives of $U_f(P_A(T))$, $\frac{\partial U_f(P_A(T))}{\partial P_A(T_i)}$ derived in (26), the second-order derivatives are as follows:

$$
\frac{\partial^2 U_f(P_A(T))}{\partial P_A(T_i)^2} = \frac{2}{N_T^2} (N_T - 1)
$$

$$
\frac{\partial^2 U_f(P_A(T))}{\partial P_A(T_i) \partial P_A(T_j)} = -\frac{2}{N_T^2}
$$

therefore

$$
\nabla^2 U_f(P_A(T)) = \frac{2}{N_T^2} \begin{bmatrix}
N_T - 1 & -1 & -1 & \cdots & -1 \\
-1 & N_T - 1 & -1 & \cdots & -1 \\
-1 & -1 & \ddots & \ddots & \vdots \\
: & : & \ddots & N_T - 1 & -1 \\
-1 & -1 & \cdots & -1 & N_T - 1
\end{bmatrix}
$$

Due to the symmetry of $\nabla^2 U_f(P_A(T))$, it is obvious that all the principal minors of order $i$ are equal to the leading principal minor of the same order $i, i = 1, 2, \ldots, N_T$. Further, by applying mathematical induction, it can be proved that all the leading
principal minors of $\nabla^2 U_f(P_A(T))$ are nonnegative. Hence, $\nabla^2 U_f(P_A(T))$ is positive semidefinite. In addition, since the domain of $U_f$, $\text{dom}U_f = R^{N_T}$, is convex, it follows from the second-order conditions for convex function [25] that the function $U_f : R^{N_T} \rightarrow R$ is convex.

**Lemma 2:** The feasible set of the optimization problem (9) is convex.

**Proof:** The feasible set of (9) is defined by the constraints (1), (7), and (8). It is obvious that the region defined by (1) is convex. The constraint (7) can be transformed into (37). The constraint (8) can be transformed into (38) and (39).

$$\forall n \in \mathcal{N}, k \in \{1, 2, \ldots, N_T\},$$

$$\sum_{i=1}^{N_T} \eta(P_n(T_i)) P_n(T_i) \geq \frac{C_n}{\Delta t} \left( \text{SOC}_{n, \text{MinCh}} - \text{SOC}_{n, 0} \right) \quad (37)$$

$$\sum_{i=1}^{k} \eta(P_n(T_i)) P_n(T_i) \geq \frac{C_n}{\Delta t} \left( \text{SOC}_{n, \text{min}} - \text{SOC}_{n, 0} \right) \quad (38)$$

$$\sum_{i=1}^{k} \eta(P_n(T_i)) P_n(T_i) \leq \frac{C_n}{\Delta t} \left( \text{SOC}_{n, \text{max}} - \text{SOC}_{n, 0} \right). \quad (39)$$

We prove that the region defined by (37), which equivalent to (7), is convex. Assume that $P_{n,1}(T), P_{n,2}(T)$ are any two points that satisfy (37). $\forall 0 \leq \lambda_1, \lambda_2 \leq 1, \lambda_1 + \lambda_2 = 1$, denote

$$P_{n,3}(T) := \lambda_1 P_{n,1}(T) + \lambda_2 P_{n,2}(T). \quad (40)$$

It can be shown that the following inequality holds $\forall T_i \in \mathcal{T}$

$$\eta(P_{n,3}(T_i)) P_{n,3}(T_i) \geq \lambda_1 \eta(P_{n,1}(T_i)) P_{n,1}(T_i)$$

$$+ \lambda_2 \eta(P_{n,2}(T_i)) P_{n,2}(T_i). \quad (41)$$

It follows from (41) that $P_{n,3}(T)$ also satisfies (37). Hence, the feasible set defined by (37) is convex. Similarly, it can be shown that, $\forall k \in \{1, 2, \ldots, N_T\}$, the feasible set defined by (38) is also convex. When $k = 1$, it is obvious that the region defined by (39) is convex. Then, by applying mathematical induction, it can be shown that $\forall k \in \{1, 2, \ldots, N_T\}$, the region defined by (39) is also convex.

To conclude, the region defined by each of the constraints (1), (7), (8) is convex. Therefore, the feasible set of (9) is convex.

**Theorem 1:** The optimization problem in (9) is a convex optimization problem.

**Proof:** According to Lemmas 1 and 2, it follows from the definition of convex optimization problem [25] that (9) is a convex optimization problem.

**B. Proof of Theorem 2**

**Lemma 3:** For all $T_k \in \mathcal{T}$, the objective function (19), $U_o : R^2 \rightarrow R$ of the optimization problem (23) is convex.

**Proof:** $\forall 1 \leq k \leq N_T - 1$, according to the gradient of $U_o(Q_A(T_k)), \nabla U_o(Q_A(T_k))$, derived in (30), the Hessian of $U_o(Q_A(T_k))$ is as follows:

$$\nabla^2 U_o(Q_A(T_k)) = \frac{2}{N_T^2} \left[ \begin{array}{cc} N_T - 1 & -1 \\ -1 & \frac{N_T - 1}{N_T - k} - 1 \end{array} \right]. \quad (42)$$

When $k = N_T$, the Hessian of $U_o(Q_A(T_{N_T}))$ is as follows:

$$\nabla^2 U_o(Q_A(T_{N_T})) = \frac{2}{N_T^2} \left[ \begin{array}{cc} N_T - 1 & 0 \\ 0 & 0 \end{array} \right]. \quad (43)$$

It can be checked that both (42) and (43) are positive semidefinite. In addition, since the domain of $U_o$, $\text{dom}U_o = R^2$, is convex, it follows from the second-order conditions for convex function [25] that the function $U_o : R^2 \rightarrow R$ is convex.

**Lemma 4:** For all $T_k \in \mathcal{T}$, the feasible set of the optimization problem (23) is convex.

**Proof:** The feasible set of (23) is defined by constraints (1), (8), and (23b). Similar to the proof of the convexity of the region defined by (37) in Lemma 2, it can be shown that, for all $T_k \in \mathcal{T}$, the region defined by (23b) is convex. In addition, it is obvious the the region defined by each of the constraints (1) and (8) is convex. Therefore, for all $T_k \in \mathcal{T}$, the feasible set of (23) is convex.

**Theorem 2:** The optimization problem in (23) is a convex optimization problem.

**Proof:** According to Lemmas 3 and 4, it follows from the definition of convex optimization problem [25] that (23) is a convex optimization problem.

**C. Proof of Theorem 3**

**Lemma 5:** $\forall m \geq 1$, the following inequality holds:

$$\langle \nabla U_f(P_{A}^{m-1}(T)) - \nabla U_f(P_{A}^{m}(T)), P_{A}^{m-1}(T) - P_{A}^{m}(T) \rangle \leq \frac{2}{N_T} \|P_{A}^{m-1}(T) - P_{A}^{m}(T)\|^2. \quad (44)$$

**Proof:** According to the first-order partial derivatives of $U_f(P_A(T))$ derived in (26), we have

$$\langle \nabla U_f(P_{A}^{m-1}(T)) - \nabla U_f(P_{A}^{m}(T)), P_{A}^{m-1}(T) - P_{A}^{m}(T) \rangle \leq \frac{2}{N_T} \|P_{A}^{m-1}(T) - P_{A}^{m}(T)\|^2. \quad (45)$$

**Lemma 6:** $\forall n \in \mathcal{N}, m \geq 1$, the following inequality holds:

$$\langle s_n^{m}(T), P_{n}^{m}(T) - P_{n}^{m-1}(T) \rangle \geq - \|P_{n}^{m}(T) - P_{n}^{m-1}(T)\|^2. \quad (46)$$

**Proof:** See the proof of Lemma 1 of [13].

**Theorem 3:** In Algorithm 1, the schedules $P_{A}^{m}(T)$ converge to one of the optimal solutions for the forecast-based scheduling problem in (9) as $m \rightarrow \infty$. 
\[
U_f(P_m^m(T)) \leq U_f(P_m^{m-1}(T)) - \langle \nabla U_f(P_m^m(T)), P_m^{m-1}(T) - P_m^m(T) \rangle \\
\leq U_f(P_m^{m-1}(T)) - \langle \nabla U_f(P_m^{m-1}(T)), P_m^{m-1}(T) - P_m^m(T) \rangle \\
+ \frac{2}{N_T} \|P_m^{m-1}(T) - P_m^m(T)\|^2 \\
= U_f(P_m^{m-1}(T)) - \frac{1}{\beta} \sum_{n \in \mathcal{N}} \langle s_m^m(T), P_n^{m-1}(T) - P_n^m(T) \rangle \\
+ \frac{2}{N_T} \|P_m^{m-1}(T) - P_m^m(T)\|^2 \\
\leq U_f(P_m^{m-1}(T)) - \frac{1}{\beta} \sum_{n \in \mathcal{N}} \|P_n^{m-1}(T) - P_n^m(T)\|^2 \\
+ \frac{2}{N_T} \|P_m^{m-1}(T) - P_m^m(T)\|^2 \\
\leq U_f(P_m^{m-1}(T)) + \left( \frac{2}{N_T} - \frac{1}{\beta N_{EV}} \right) \|P_m^{m-1}(T) - P_m^m(T)\|^2 \\
\leq U_f(P_m^{m-1}(T)). \tag{47}
\]

The first inequality holds due to the first-order condition [25] of the convex function \(U_f\). The second inequality is due to Lemma 5, the third inequality is due to Lemma 6, the fourth inequality is due to the Cauchy–Schwarz inequality, and the fifth inequality is due to \(0 < \beta < \frac{2 N_{EV}}{N_T}\).

According to (47), \(U_f(P_m^m(T))\) is nonincreasing as \(m\) increases. Further, it is easy to check that \(U_f(P_m^m(T)) = U_f(P_m^{m-1}(T))\) if and only if \(P_N^m(T) = P_N^{m-1}(T)\). If \(P_N^m(T) = P_N^{m-1}(T)\), it follows from the proof of Theorem 3 of [13] that \(P_N^m(T)\) minimizes \(U_f\). To conclude, \(P_N^m(T)\) minimizes \(U_f\) as \(m \to \infty\). \(\blacksquare\)

### D. Proof of Theorem 4

**Lemma 7:** \(\forall m \geq 1, T_k \in \mathcal{T}\), the following inequality holds:

\[
\langle \nabla U_o(Q_m^{m-1}(T_k)) - \nabla U_o(Q_n^m(T_k)), Q_n^{m-1}(T_k) - Q_m^m(T_k) \rangle \\
\leq \frac{2}{N_T} \|Q_n^{m-1}(T_k) - Q_m^m(T_k)\|^2. \tag{48}
\]

**Proof:** Similar to the proof of Lemma 5, inequality (48) can be proved by simple derivation.

\(\forall m \geq 1, T_k \in \mathcal{T}\), according to the gradient of \(U_o(Q_n(T_k))\), \(\nabla U_o(Q_n(T_k))\), derived in (30), we have

\[
\langle \nabla U_o(Q_m^{m-1}(T_k)) - \nabla U_o(Q_n^m(T_k)), Q_n^{m-1}(T_k) - Q_m^m(T_k) \rangle \\
= \frac{2}{N_T} \left( (P_m^{m-1}(T_k) - P_m^m(T_k))^2 + \alpha(k) \frac{2}{N_T} (FP_m^{m-1}(T_k)) \\
- FP_m^m(T_k))^2 - \frac{2}{N_T} (P_m^{m-1}(T_k) - P_m^m(T_k) \right) \\
+ FP_m^{m-1}(T_k) - FP_m^m(T_k))^2 \\
\leq \frac{2}{N_T} \left( (P_m^{m-1}(T_k) - P_m^m(T_k))^2 + (FP_m^m(T_k) - (1 + \alpha(k)) \frac{2}{N_T} (FP_m^{m-1}(T_k) - FP_m^m(T_k))^2 \\
- \frac{2}{N_T} (Q_n^{m-1}(T_k) - Q_m^m(T_k))^2. \tag{49}
\]

The second equality is due to \(\alpha(k) \leq 1\).

**Lemma 8:** \(\forall n \in \mathcal{N}, m \geq 1, T_k \in \mathcal{T}\), the following inequality holds:

\[
\langle s_n^m(T_k), Q_n^m(T_k) - Q_n^{m-1}(T_k) \rangle \\
\geq - \|Q_n^m(T_k) - Q_n^{m-1}(T_k)\|^2. \tag{50}
\]

**Proof:** See the proof of Lemma 1 of [13].

### Theorem 4: In Algorithm 2, at any time slot \(T_k \in \mathcal{T}\), the schedules \(Q_n^m(T_k)\) converge to one of the optimal solutions for the online scheduling problem in (23) as \(m \to \infty\).

**Proof:** Similar to the proof of Theorem 3, by applying the first-order condition of the convex function \(U_o\), Lemmas 7 and 8, and the Cauchy–Schwarz inequality, successively, it can be shown that, \(\forall T_k \in \mathcal{T}\), \(U_o(Q_n^m(T_k))\) is nonincreasing as \(m\) increases. It can be checked that \(U_o(Q_n^m(T_k)) = U_o(Q_n^{m-1}(T_k))\) if and only if \(Q_n^m(T_k) = Q_n^{m-1}(T_k)\), and such \(Q_n^m(T_k)\) minimizes \(U_o\). To conclude, \(Q_n^m(T_k)\) minimizes \(U_o\) as \(m \to \infty\). \(\blacksquare\)

### REFERENCES

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