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Abstract—The growing penetration of renewable energy sources in electricity generation will bring challenges to the power grid operations due to the intermittency and fluctuation of renewables. In this article, we employ an energy storage system (ESS) in a grid-connected renewable energy system (RES) to serve the electrical load from the power grid. We study the control algorithms of the ESS to smooth the renewable generation and analyze the tradeoff between the ESS size and the system performance, i.e., renewable utilization and operation cost. We provide performance guarantees of the proposed algorithms and propose an optimization framework to configure the ESS. To analyze the reliability of the RES, an analytical framework using the Markov modulated fluid queue is devised. It is shown in our simulation studies that the devised power output smoothing algorithm outperforms other existing algorithms in terms of operation cost under different sizes of ESSs. We also find that, to satisfy the renewable utilization requirement, the required ESS size blows up with both the amplitude of fluctuation and the intermittency level of the profile gap between the renewable generation and the load.

Index Terms—Energy storage system (ESS), Internet of Energy (IoE), power output smoothing (POS), renewable energy system (RES), smart grid.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$t$</td>
<td>A particular time slot.</td>
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<tr>
<td>$T$</td>
<td>Total operation time in $\Delta t$.</td>
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<tr>
<td>$\Delta t$</td>
<td>Length of a time slot in minutes.</td>
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<tr>
<td>$D(t)$</td>
<td>Network load at Time Slot $t$ in kW.</td>
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<tr>
<td>$R(t)$</td>
<td>Renewable generated power at time slot $t$ in kW.</td>
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<tr>
<td>$B$</td>
<td>Size of the ESS in kWh.</td>
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<tr>
<td>$T_L$</td>
<td>Total lifetime of the ESS in $\Delta t$.</td>
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<tr>
<td>$P_c(t)$</td>
<td>Charging power of the ESS at time slot $t$ in kW.</td>
</tr>
<tr>
<td>$P_d(t)$</td>
<td>Discharging power of the ESS at time slot $t$ in kW.</td>
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<tr>
<td>$P_{max}$</td>
<td>Maximum charging power of the ESS in kW.</td>
</tr>
<tr>
<td>$P_{max}$</td>
<td>Maximum discharging power of the ESS in kW.</td>
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<tr>
<td>$\eta$</td>
<td>Charging efficiency of the ESS.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discharging efficiency of the ESS.</td>
</tr>
<tr>
<td>$S(t)$</td>
<td>State of charge of the ESS at time slot $t$ in kWh.</td>
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<tr>
<td>$P_{out}(t)$</td>
<td>Actual power output from the RES at time slot $t$ in kW.</td>
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<tr>
<td>$P_e(t)$</td>
<td>Power injection from other generation units at time slot $t$ in kW.</td>
</tr>
<tr>
<td>$P_s(t)$</td>
<td>Spilled power from the RES at time slot $t$ in kW.</td>
</tr>
<tr>
<td>$p(t)$</td>
<td>Unit cost of buying electricity from the controllable generation unit at time slot $t$ in dollars/kWh.</td>
</tr>
<tr>
<td>$s(t)$</td>
<td>Unit spillage cost at time slot $t$ in dollars/kWh.</td>
</tr>
<tr>
<td>$C_g$</td>
<td>Electricity cost stemmed from the controllable generation unit in dollars.</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Renewable spillage cost in dollars.</td>
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<tr>
<td>$C_c$</td>
<td>Construction cost for ESS in dollars.</td>
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<tr>
<td>$R_C$</td>
<td>Contract revenue of the RES in dollars.</td>
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<tr>
<td>$C_{avg}$</td>
<td>Daily average cost in dollars.</td>
</tr>
<tr>
<td>$U_{RG}$</td>
<td>Renewable energy utilization.</td>
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I. INTRODUCTION

Due to concerns on global warming, many countries have established green policies to restrain the greenhouse gas emissions, such as employing more distributed renewable energy sources in place of thermal power plants. With the integration of modern communication and Internet of things technologies, distributed renewable energy sources with distributed and embedded intelligence are interfaced to the smart grid, which forms the Internet of Energy (IoE) [1]. A major goal of the IoE is to provide reliable and cost-efficient electricity services to the demand-side of the power grid [2]. To support the efficient and reliable operation of IoE, techniques to integrate renewable energy sources are, thus, very important in the smart grid.
In fact, how to effectively integrate distributed renewable energy resources in the smart grid remains challenging. The renewables cannot provide reliable power output to the power grid due to its fluctuation [3] and intermittency [4] properties. Moreover, due to the stochastic natures of the renewables, the daily profile of the renewable generated power is hard to be accurately predicted [5]. The uncertainty induced by the renewables can cause power imbalance in the power grid, which may result in frequency deviations and jeopardize the grid stability.

With the aforementioned challenges, research on the power output smoothing (POS) techniques for renewable generation gains popularity in recent years. Basically, POS aims to flatten the renewable generation profile so as to assist in the integration of the renewable generation in the operation of IoE. How to implement POS for a renewable energy system (RES) rests in the way to level off the fluctuations of renewable generation based on the uncertainty of renewables. Energy storage system (ESS), which can reserve excessive renewable energy through charging and compensate for energy shortage in the power grid through discharging, is considered to be a good candidate to help cope with the fluctuations of renewable generation. Several previous studies investigated smoothing the power fluctuations of renewable generation with the help of ESS [6]–[9], [11], [13], [14]. To stabilize the photovoltaic (PV) power output, a model predictive control (MPC) strategy was proposed in [7] to smooth the power and voltage profiles. The proposed strategy requires no communication between the ESS and the PV system, since control commands can be sent along with local measurement data. In [8], a heuristic smoothing control method was proposed for an ESS station to help reduce the fluctuations of the renewable generated power. The proposed method determined power output levels based on a dynamic filtering controller and, then, regulated the state-of-charge (SOC) of each battery based on the proposed power allocation method. In [14], an SOC feedback strategy for ESS control was designed to smooth out the fluctuations of wind power while keeping the SOC of the ESS within its proper range. The POS schemes based on filter design were devised in [6] and [9]. In [9], Li-ion capacitors were deployed for filtering power variations and control techniques were devised based on low-pass filter (LPF). In [6], a general filter was designed to improve the performance of wind power smoothing using LPF. A metaheuristic method was, then, proposed to calculate the capacity and the power rating of the ESS. The aforementioned approaches [6]–[9], [14] fail to provide theoretical performance guarantee on POS. Moreover, they only aim at smoothing the real-time fluctuations of renewable generation and do not react to grid loads as well as electricity prices. Hence, the smoothed power output still cannot be injected to the power grid nor utilized for the operation of IoE.

To satisfy the demand of grid load and respond to electricity prices, some offline POS schemes have been devised to dispatch the renewable generation based on the MPC framework [10]–[13]. MPC repetitively takes control actions from the solution of an optimization problem with forward-looking objective and predicts the future system state variables for the optimization. The combination of optimization process and prediction process aims to handle the uncertainty of renewables. In [11], a control approach based on stochastic MPC was proposed for an ESS so as to minimize the daily operation cost of an RES. In [10], stochastic MPC was applied to dynamic and adaptive reconfiguration of electrical distribution system with renewable penetration so as to handle the uncertainty and variability of renewable energy resources. In [12], a control approach for the joint charging management of plug-in electric vehicles and reconfiguration of electrical distribution system was proposed based on stochastic MPC. Wang et al. [13] developed an optimization and control method for a PV storage system based on MPC. The proposed method includes an optimization process to provide energy arbitrage and smooth the PV output, and a prediction process based on the least square method. The MPC framework applied in [10]–[13] needs to perform estimations for the future system states, e.g., wind speed for wind turbine [10]–[12], solar irradiance for PV panels [10]–[13], SOC of EVs [12], and electricity prices [13]. Therefore, effective and accurate forecast methods are still required to enable the dynamic MPC-based scheduling. Moreover, except [6], the above mentioned techniques [7]–[9], [11], [13], [14] do not take the effect of ESS on the performance of POS into account. Note that the configuration of ESS can impact on POS to a large extent. Hence, ESS should be optimally configured and the method for the configuration of ESS should be devised.

Some existing techniques considered the storage configuration problem in the RES [15], [16]. In [15], Anwar et al. proposed a two-stage strategy to configure ESS, in which the optimal power output at each time instant was determined in the first stage and the minimum size of ESS was then determined in the second stage. Saez-de-Ibarra et al. [16] proposed a two-stage optimization framework to determine the size of ESS in a renewable generation plant. The optimal operation profile of the battery for each day of the year was determined in the first stage and the optimal size of ESS was determined in the second stage based on the results obtained in the first. The approaches proposed in [15] and [16] suffer the defects that the determination of the size of ESS in the constructional level does not fully embed the power scheduling in the operational level. Therefore, the optimality of the approaches cannot be guaranteed. Moreover, the above works did not analyze the tradeoff between the ESS size and the improvement in the system performance, say, system reliability, when considering the generation and the load profiles. It should be noted that the profiles of generation and load can have impacts on this tradeoff, specifically: 1) How do the fluctuations of renewable generation and load impact on this tradeoff? 2) How shall we configure the ESS and power output based on the profiles of the generation and the load?

To answer the above questions, in this article, we propose POS algorithms for an RES and analyze the tradeoff between the ESS size and the improvement in system performance, i.e., the reliability and utilization enhancements, as well as cost reduction, on POS through theoretical analysis and simulation. We consider a grid-connected RES, with an ESS, which provides reliable power output to IoE. To utilize renewable generation to serve loads in distribution power network, a leaky bucket (LB)-inspired POS scheme is proposed to control the ESS. In the real operation, the configurations of ESS, i.e., the size of the ESS as well as the maximum charging and discharging rates, can impact the overall system performance and should be jointly determined. To do this, we propose an optimization framework for solving this joint storage configuration problem (JSCP) by taking the generation profile into consideration. To minimize the operation cost of the system, a price-adaptive power smoothing algorithm is then, proposed based on the Lyapunov optimization to deal with time-varying electricity prices. We conduct simulations to study the effect of ESS size on system performance under different operation algorithms. Through theoretical study and simulation, we also give some insights into how the ESS...
should be configured to satisfy the performance requirement based on different renewable generation profiles.

The preliminary conference version of this article can be found in [17]. In [17], we developed some control algorithms of an ESS so as to smooth the power output of an RES. In this article, we provide theorems to analyze the performance of the POS scheme proposed in [17]. We then extend the results in [17] by conducting a theoretical analysis to analyze the tradeoff between the ESS size and the system performance. We also perform simulations to verify such tradeoff. Moreover, a Lyapunov-based load-adaptive power smoothing algorithm is proposed in this article, which outperforms the heuristic-based load-adaptive power smoothing (HB-LAPS) algorithm in [17] on operation cost. We also provide the performance guarantee for the proposed algorithm theoretically. The specific contributions of this article are as follows.

1) To utilize renewable generation to serve network loads in distribution power network, a POS scheme is proposed to control the ESS. We prove that the proposed scheme can achieve the optimal renewable energy utilization.

2) We propose an optimization framework to configure the ESS. The proposed framework fully embeds the scheduling scheme in the operational level, which differs from the existing approaches [15], [16].

3) A price-adaptive power smoothing algorithm is devised to minimize the operation cost of the RES based on the Lyapunov optimization. Compared to [11] and [13], it requires no forecast information on generation and network load, thereby is more practical to be implemented. We also provide the deterministic performance guarantee of the algorithm on operation cost.

4) In this article, the wind power generation process is modeled as a Markov process. We conduct a theoretical analysis to analyze the reliability of an RES, measured by the expected loss of power supply, based on the Markov modulated fluid queue (MMFQ), which provides steady-state analysis for a system with Markovian arrivals. To the best of our knowledge, we are the first to conduct the theoretical analysis on the expected loss of power supply of an RES by using MMFQ.

The rest of this article is organized as follows. Section II introduces the system model. Section III describes the POS scheme for RES. An storage configuration problem is, then, formulated and solved. In Section IV, a theoretical framework to analyze the tradeoff between the ESS size and reliability is devised. To adapt to the electricity price, a Lyapunov-based price-adaptive power smoothing (LB-PAPS) algorithm is proposed in Section V. The performance evaluation results are discussed in Section VI and the article concludes in Section VII.

II. SYSTEM MODEL

As shown in Fig. 1, an RES is considered, which is connected to a power network with the power and information flows indicated. Consider that the RES operates over a scheduling horizon divided equally into $T$ slots. Each time slot has a duration of $\Delta t$ minutes. Each $t \in \{1, 2, \ldots, T\} = T$ denotes a time slot in the operation period. All system states only change at the beginning of a time slot. The total load of the power network is modeled as the network load $D(t)$. Note that $D(t)$ is not a single load but the equivalent aggregated load in the power network interface. The RES aims to provide stable power output to serve the network load. The RES consists of an ESS, a storage controller, and a wind power plant. The renewable generated power at time slot $t$ is denoted as $R(t)$ kW. The ESS has a lifetime of $T_L$ minutes, and its size is $B$ kWh where $B > 0$. It gets charged or discharged at the rate of $P_c(t) \leq \bar{P}_c$ or $P_d(t) \leq \bar{P}_d$ at time slot $t$, respectively, where $\bar{P}_c$ ($\bar{P}_d$) is the maximum charging (discharging) rate in kW, respectively. Note that the charging and discharging processes of the ESS cannot happen at the same time. The construction cost of ESS includes two parts, i.e., the cost in terms of the size of the ESS $c_B$ dollars/kWh and the cost in terms of the maximum charging rate as well as the maximum discharging rate, $c_c$ dollars/kW, and $c_d$ dollars/kW, respectively. The SOC of the ESS at time $t$ is denoted as $S(t)$ kWh. $0 < \eta < 1$ and $0 < \beta < 1$ represent the charging and discharging efficiencies of the ESS, respectively. The storage controller controls the ESS so as to smooth out the power output from the RES. The power output from the RES at time slot $t$, $P_{out}(t)$ kW, is given by

$$P_{out}(t) = R(t) - P_c(t) + P_d(t).$$

(1)

In traditional power grid without the integration of renewables, controllable generation units are dispatched to serve the network load. In our proposed power grid model with the integration of renewables, the RES is employed to provide stable power output to the network load $D(t)$, with the help of the controllable generation units. When $P_{out} < D(t)$, the power output from the RES is insufficient to satisfy the network load. The controllable generation units, then, compensate for the power shortage through injecting power of $P_{c}(t)$ kW such that

$$P_{c}(t) = D(t) - P_{out}(t).$$

(2)

The time-varying price of purchasing electrical energy from the controllable generation units is given by $p(t)$ dollars/kWh.

When $P_{out}(t) > D(t)$, which generally happens when the ESS is full and the renewable generated energy is excessive, namely, $R(t) > D(t)$, a portion of power output from the RES equal to $P_{d}(t) = P_{out}(t) - D(t)$ is spilled and the unit spillage cost is given by $s(t)$ dollars/kWh. We assume that $s(t) \leq p(t)$ since the electricity generation cost is much more expensive than the spillage cost in general.

III. POS CONTROL FOR RES

In this section, a POS scheme is devised for the RES to smooth out the power fluctuations of the renewable generation to serve the network load. The proposed framework includes an operation algorithm to control the charging/discharging scheme of the ESS and a planning problem to optimize the configurations of the ESS. The principle of the POS scheme is first introduced, which defines the control scheme of the ESS in the operational level. A joint storage configuration problem (JSCP) is, then, discussed, in which it optimally configures the size and the charging/discharging rates of the ESS as a planning problem.

Fig. 1. Architecture of the RES.
Algorithm 1: POS Scheme.

Input: Real-time renewable generation $R(t)$, SOC of the ESS $S(t)$, size of the ESS $B$, power output reference $P_r(t)$, $\bar{P}_c$, $\bar{P}_d$.

Output: $P_c(t)$, $P_d(t)$.

1: for each time slot $\Delta t$ do
2: Calculate charging demand $G_c(t) = \max\{0, R(t) - P_r(t)\}$.
3: Calculate discharging demand $G_d(t) = \max\{0, P_r(t) - R(t)\}$.
4: Calculate maximum charging bound before the ESS is full by current time slot $M_c(t) = \frac{B - S(t)}{\eta \Delta t}$.
5: Calculate maximum discharging bound before the ESS is empty by current time slot $M_d(t) = \frac{S(t) \cdot \beta}{\Delta t}$.
6: Calculate $P_c(t) = \min\{G_c(t), M_c(t), \bar{P}_c\}$.
7: Calculate $P_d(t) = \min\{G_d(t), M_d(t), \bar{P}_d\}$.
8: Update $S(t+1) = \eta P_c(t) \Delta t - \frac{P_a(t)}{\beta} \Delta t + S(t)$.
9: Return $P_c(t)$, $P_d(t)$.
10: end for

A. Design Principle

To implement POS, it is required to regulate the renewable energy sources with uncertainty. By employing ESS, the RES has additional capabilities to manage its power output. A control scheme of the ESS is, thus, needed to smooth the power output for power grid operation so as to keep power balance in the grid. When devising the control scheme, we found that the stochastic nature of renewable energy sources shares similarities with those of network traffic sources. Actually, in network domain, a well-known algorithm, named the LB mechanism [22], has been devised to regulate the network traffic sources (traffic shaping). Basically, the LB mechanism can smoothen the stochastic traffic sources by restricting the output traffic rate at a preset constant level and holding the awaiting packets for transmission in the network buffer. Inspired by the LB mechanism, a control scheme is devised and applied to POS for the RES. In this scheme, the way how the ESS manages the energy can be analogized as the way in which the LB handles the fluid. The RES serves the network load with a power output reference that equals the network load, i.e., $P_c(t) = D(t)$ kW. The ESS is controlled to match $P_{out}(t)$ to $P_c(t)$ as close as possible.

An algorithm is devised to implement the POS scheme as Algorithm 1. In Steps 2–3, the charging/discharging demands are calculated so as to match $P_{out}(t)$ to $P_c(t)$. In Steps 4–5, the charging/discharging bounds are calculated based on the SOC constraints. In Steps 6–7, $P_c(t)$ and $P_d(t)$ are calculated by jointly considering the SOC constraints, the charging/discharging demands, and the maximum charging/discharging rates. In Algorithm 1, the power output of RES is controlled to match the network load as close as possible. Moreover, the maximum charging/discharging rates, the SOC constraints, and charging/discharging efficiencies are required to be considered in the POS scheme. Since POS scheme only requires simple arithmetics and max/min calculations of current state variables, the time complexity of Algorithm 1 is $O(1)$.

Define the renewable energy utilization $U_{RG}$ in the RES as follows:

$$U_{RG} = \sum_{t=1}^{T} \frac{\min\{D(t), P_{out}(t)\} \Delta t}{\sum_{t=1}^{T} R(t) \Delta t}.$$  

**Theorem 1:** The devised POS scheme can achieve the optimal $U_{RG}$.

**Proof:** See Appendix A.

In the proposed scheme, the maximum charging and discharging rates in kW ($\bar{P}_c$ and $\bar{P}_d$), and the size of the ESS $B$ kWh, altogether have a great impact on the system performance. Therefore, these values are required to be configured appropriately. To optimally configure these values, a JSCP is formulated.

B. Problem Formulation

The control objective of the JSCP is to provide the stable power supply to meet the network load. Meanwhile, power shortage and power spillage should be avoided with minimum ESS cost. Therefore, the objective function should capture the operation costs due to power shortage and power spillage, and the construction cost of storage.

The cost of purchasing electrical energy from the controllable generation unit $C_g$ in dollars is given by

$$C_g = \sum_{t=1}^{T} G(t) = \sum_{t=1}^{T} \Delta t \cdot [P_r(t) - P_{out}(t)]^{+} p(t)$$

where $[x]^+$ means $\max\{x, 0\}$ and $G(t)$ is the electricity cost in time slot $t$. The renewable spillage cost $C_s$ is given by

$$C_s = \sum_{t=1}^{T} H(t) = \sum_{t=1}^{T} \Delta t \cdot [P_{out}(t) - P_r(t)]^{+} s(t)$$

where $H(t)$ is the renewable spillage cost in time slot $t$.

The construction cost of the ESS $C_c$ in dollars is given by

$$C_c = (c_b B + c_c \bar{P}_c + c_d \bar{P}_d) \cdot \frac{T \Delta t}{T_L}$$

which gives the weighted construction cost of the ESS for the scheduling horizon of $T$ time slots. $c_b B$ is the construction cost in terms of the size of the ESS and $c_c \bar{P}_c + c_d \bar{P}_d$ is the construction cost in terms of the maximum charging and discharging rates. For simplicity, the capacity decay of the ESS is not considered in this model since during the ESS lifetime, the capacity decay of ESS is not significant [24].

The objective function $f$ is, then, expressed as

$$f = C_g + C_s + C_c. \quad (7)$$

The charging/discharging power of the ESS follows Algorithm 1. The storage controller aims to equalize $P_{out}(t)$ and $P_r(t)$ when there exists power discrepancy between these two. Meanwhile, the maximum charging/discharging rates and the SOC constraints are considered.

The charging power of the ESS, $P_c(t)$ kW, is, thus, given by

$$P_c(t) = \min \left\{ \bar{P}_c, \frac{B - S(t)}{\eta \Delta t}, \max\{0, R(t) - P_r(t)\} \right\} \quad (8)$$

in which $\max\{0, R(t) - P_r(t)\}$ gives the amount of charging demand in kW in case of power excess, $\bar{P}_c$ is the maximum charging rate, and $\frac{B - S(t)}{\eta \Delta t}$ is the maximum charging bound to ensure that the ESS will not be charged over its upper bound $B$ during Slot $t$.

Similarly, the discharging power of the ESS, $P_d(t)$ kW, is

$$P_d(t) = \min \left\{ \bar{P}_d, \frac{S(t) \beta}{\Delta t}, \max\{0, P_r(t) - R(t)\} \right\} \quad (9)$$

in which $\max\{0, P_r(t) - R(t)\}$ gives the amount of discharging demand in kW in case of power shortage, $\bar{P}_d$ is maximum
discharging rate, and \( B(t) \) is the maximum discharging bound to ensure that the ESS will not be discharged under its lower bound 0 during slot \( t \).

Since the charging and discharging processes of the ESS cannot happen at the same time, we have
\[
P_c(t) \cdot P_d(t) = 0. \tag{10}
\]
The SOC of the ESS is updated as
\[
S(t+1) = \eta P_c(t) \Delta t - \frac{P_d(t)}{B} \Delta t + S(t) \tag{11}
\]
and \( S(1) \) denotes its initial SOC. Moreover, the SOC of the ESS is restricted by the size of the ESS, which follows:
\[
B \geq S(t) \geq 0. \tag{12}
\]

Based on the aforementioned control objective and constraints, the optimization for JSCP is formulated as
\[
\min_{B, T_c, T_d} \quad (7) \\
\text{subject to} \quad (1), (8)–(12). \tag{13}
\]

By solving (13), the optimal values of \( T_c, T_d \), and \( B \) can be found. Due to constraints (8)–(10), the problem is a non-linear equality constrained optimization (nonconvex) and, thus, intractable. We need to relax the problem so as to solve it more efficiently, which will be discussed in Section III-C.

C. Problem Relaxation

Here, JSCP (13) is relaxed as a mixed-integer linear program (MILP) problem. First, we convert constraint (8) into
\[
0 \leq P_c(t) \leq \frac{B - S(t)}{\eta \Delta t} \tag{14}
\]
\[
0 \leq P_c(t) \leq a_0 P_c \tag{15}
\]
\[
0 \leq P_c(t) \leq R(t). \tag{16}
\]

Similarly, constraint (9) can be converted into
\[
0 \leq P_d(t) \leq \frac{S(t) B}{\Delta t} \tag{17}
\]
\[
0 \leq P_d(t) \leq a_1 P_d \tag{18}
\]
\[
0 \leq P_d(t) \leq P_d(t). \tag{19}
\]

Note that \( a_0 \) and \( a_1 \) need to satisfy
\[
a_0 + a_1 = 1, \text{ where } a_0, a_1 \in \mathbb{N}. \tag{20}
\]

Equations (14)–(20) are linear constraints with mixed integers and sufficient conditions for (8)–(10). Then, a relaxed MILP problem is obtained as
\[
\min_{B, T_c, T_d, P_c(t), P_d(t), a_0, a_1} \quad (7) \\
\text{subject to} \quad (1), (11), (12), \text{ and (14)–(16)}. \tag{21}
\]

**Theorem 2:** Suppose that the electricity price \( p(t) \) and the unit spillage cost \( s(t) \) are constant. The optimal solutions for \( T_c, T_d, B \) in (21) are these of (13).

**Proof:** See Appendix B.

By Theorem 2, the solution of the original JSCP (13) can be obtained from (21), which is an MILP.

IV. RELIABILITY ANALYSIS OF RES

In Section III, we have devised POS scheme for the RES. For such an RES, it is important to theoretically analyze the performance of the system, like system reliability, under the proposed POS scheme. System reliability is an important metric for the power grid operation, which reflects to what extent the power supply can meet the network load. In our model, the system reliability is measured by the expected loss of power supply (LOPSE) \([25]\), which is the expected amount of shortage of the power supply compared to \( D(t) \)
\[
\text{LOPSE} = \frac{1}{T} \sum_{t=1}^{T} \max\{D(t) - P_{\text{out}}(t), 0\} \tag{22}
\]
where \( P_{\text{out}}(t) \) is the power output for RES at time \( t \).

In this section, an analytical framework is proposed to analyze the reliability of the RES with the proposed POS scheme. Based on the proposed framework, the tradeoff between the size of the ESS and the system performance, i.e., system reliability, can be evaluated.

The development of this section is as follows. First, the wind power generation process is modeled as a Markov process inspired by \([26]\) and \([27]\). Second, the mathematical model for the network load is provided. Then, we analyze the evolution of the SOC of the ESS using MMFQ \([28]\). Finally, the LOPSE is computed theoretically through the framework.

A. Wind Generated Power Modeling

To analyze an RES with stochastic renewable sources, we need to model the renewable generated power mathematically. In fact, renewable generated power shows stochastic fluctuations, day periodic patterns, and seasonal patterns. Thus, it can be described as a stochastic process. The Markov process for renewable generated power modeling is a simplified model that can capture the stochastic properties of the renewable generation and make it feasible to be analyzed \([27]\). In this article, we model the wind generation process as a continuous time Markov chain (CTMC) \([26]\), \([27]\). The phase-based Markov process divides the time-varying wind generated power into \( M \) phases or power levels. The transitions among different phases are, then, represented by an generator matrix \( A = [A_{ij}] \), \( i, j = 0, \ldots, M - 1 \). The generator matrix can be calculated using the method proposed in \([27]\).

To validate the effectiveness of the Markov chain model for renewable generated power, we conduct a simulation study to compare the Markov chain model with the real-world data. The real data of renewable generated power is obtained from \([29]\) and the Markov chain model is trained using the method proposed in \([27]\). According to \([27]\), time series model for wind generation is required to imitate the major statistical properties of the real data, i.e., the probability density distribution and the autocorrelation function. Based on our simulation results illustrated in Fig. 2, the probability density distribution of the generated power from Markov chain model can match well with the real-world data. The autocorrelation function of the generated power modeled by Markov chain can mimic the trend of the
autocorrelation function of the real-world data. In Fig. 2(b), the periodic fluctuations in the real-world data arise from the daily periodicity of the wind profile. Note that, due to the intractability of the analytical model, the periodicity of the wind generated power has not been incorporated in our model. This will not result in much inaccuracy in wind generation modeling, since the periodicity of wind generation is not significant. In Section VI-D2, we further show that although the periodicity is not incorporated in the wind generation model, the CTMC model still shows its accuracy in wind generation modeling since the analytical results using the CTMC model match well with the simulation results using the real-world data.

B. Network Load Model

Different from the wind generation process that is highly fluctuated and intermittent, the network load profile in a residential area reflects a periodic pattern due to the regular electricity consumption behaviors of residents [30]. To model the daily periodicity of the network load profile, the mathematical model of network load is formulated as

\[ D(t) = L(t - nR) \quad if \quad 0 \leq t - nR < R, \quad n \in \mathbb{N}. \]  \hspace{1cm} (23)

According to (23), the network load profile \( D(t) \) is a periodic function with period \( R = 1440 \) min. It is mapped to a daily load curve \( L(t) \) with domain \([0, R]\).

C. Theoretical Results for System Reliability Using MMFQ

After the wind generation process has been modeled as a CTMC, the RES with the Markovian energy arrival process and the energy departure process following Algorithm 1 can be analyzed using MMFQ [28]. MMFQ can describe the evolution of a system with the Markov arrival process and the constant-rate departure process. It can provide a framework for the steady-state analysis of such a system. The state of wind generation process at time \( t \) is represented by \( S_w(t) \) and the energy rate of ESS at drift state \( S_w(t) = s \) is denoted by \( d(s, t) = s - D(t) \), where \( s = 0, 1, \ldots, M - 1 \). (Here, \( d(s, t) > 0 \) means charging and \( d(s, t) < 0 \) means discharging.) We assume that the charging/discharging bounds for ESS are sufficiently large, i.e., \( d(0) \geq P_e \) and \( d(M - 1) \leq P_d \), so that the energy rate can always be satisfied. Note that even if this assumption does not hold, we can still remove the states in CTMC in which the energy rate cannot be satisfied and modify the generator matrix \( \mathbf{A} \) accordingly so that the CTMC can model the energy rate of the ESS. Transitions among \( d(s, t) \) are controlled by the generator matrix \( \mathbf{A} = [A_{ij}] \) as shown in Fig. 3. We define the drift generator matrix \( \mathbf{E}_r \) as \( \mathbf{E}_r = \text{diag}(d(0, r), d(1, r), \ldots, d(M - 1, r)) = \text{diag}(0, -L(r), 1 - L(r), \ldots, M - 1 - L(r)), \) where \( r \in [0, R] \) and \( R \) is the period of function \( D(t) \). To simplify the discussion, it is assumed that \( \beta = \eta = 1 \). Note that it is easy to extend the model when \( \beta \neq 1 \) and \( \eta \neq 1 \) by manipulating the generator matrix \( \mathbf{A} \). When the duration of each time slot \( \Delta t \) is small enough, the discrete processes discussed in Sections II and III can be considered as continuous processes and the evolution of the SOC of the ESS \( S(t) \), under the proposed POS scheme, is shown as

\[
\frac{dS(t)}{dt} = \begin{cases} 
\max\{0, d(s, t)\} & \text{if } S(t) = 0 \text{ and } S_w(t) = s \\
\min\{0, d(s, t)\} & \text{if } S(t) = B \text{ and } S_w(t) = s \\
\min\{0, d(s, t)\} & \text{if } 0 < S(t) < B \text{ and } S_w(t) = s 
\end{cases}
\]  \hspace{1cm} (24)

For an RES, we decide to analyze the system reliability given a certain renewable supply process denoted by the generator matrix \( \mathbf{A} \), the size of the ESS \( B \), and the load profile \( L(r) \). We first obtain the limiting probability vector of the SOC of the ESS using the MMFQ model [31] in order to calculate the system reliability. Based on [31], Theorems 3 and 4 are introduced to help obtain the limiting probability vector of the SOC of the ESS.

**Theorem 3:** We define \( \mathbf{F}(x, r) = \{F_1(1, x, r), F_2(2, x, r), \ldots, F_{M}(M, x, r)\} \) as the limiting probability vector of the SOC of the ESS under load state \( r \), where \( 0 \leq s \leq M \), is the drift state and \( s \) is the SOC. In the finite-storage case with ESS size \( B \), \( \mathbf{F}(x, r) \), which can be modeled as MMFQ, satisfies

\[
\mathbf{F}(x, r) = \frac{d\mathbf{F}(x, r)}{dx} \mathbf{E}_r
\]  \hspace{1cm} (25)

with the boundary conditions \( F(s, B, r) = \pi_s \) when \( d(s, r) < 0 \) and \( F(s, 0, r) = 0 \) when \( d(s, r) > 0 \), where \( \pi^+ = \{\pi_1, \pi_2, \ldots, \pi_M\} \) is the limiting probability vector for CTMC.

The above differential equation can be solved using the eigenvalue method [32]. The solution of \( \mathbf{F}(x, r) \) is

\[
\mathbf{F}(x, r) = \sum_{i=0}^{M-1} \alpha_i e^{\lambda_i x} \phi_i^r
\]  \hspace{1cm} (26)

where the pair (\( \lambda_i, \phi_i^r \)) is the eigenvalue and eigenvector for matrix pair \((\mathbf{E}_r, \mathbf{A})\) that satisfies \( \phi_i^r(\lambda_i^r \mathbf{E}_r - \mathbf{A}) = 0 \). The coefficients \( \alpha_i \) satisfies

\[
\sum_{i=0}^{M-1} \alpha_i \phi_i^r = 0, \quad j \in S_+
\]  \hspace{1cm} (27)

\[
\sum_{i=0}^{M-1} \alpha_i \phi_i^r = \pi_j^r, \quad j \in S_-
\]  \hspace{1cm} (28)

where \( S_+ \) and \( S_- \) denote the state in which \( d(s, r) > 0 \) and \( d(s, r) < 0 \), respectively.

**Theorem 4:** The limiting probability vector \( \mathbf{F}(x) = \{F(1, x), \ldots, F(M, x)\} \) under the load profile \( D(t) \) can be calculated as

\[
\mathbf{F}(x) = \int_0^R \mathbf{F}(x, r).
\]  \hspace{1cm} (29)

After the limiting probability vector of the SOC \( \mathbf{F}(x) \) is obtained, we can derive the system reliability measured by LOPSE, which is the expectation for the amount of power shortage given the size of the ESS \( B \)

\[
\text{LOPSE} = \sum_{i=0}^{M-1} \left[ -\min\left\{0, \int_{r=0}^R d(i, r) \right\} \right] F(i, 0).
\]  \hspace{1cm} (30)
V. PRICE-ADAPTIVE POWER SMOOTHING ALGORITHM

The proposed POS scheme in Section III aims to serve the network load with the optimal renewable energy utilization. Nevertheless, it fails to consider the time-varying electricity price and may yield high operation costs. With electricity price information transmitted through IoE in real time, a price-adaptive power smoothing algorithm is devised based on Lyapunov optimization theory. It aims to minimize the operation cost of the RES without forecast information on generation, load, and electricity price. In Section V-C, we provide the deterministic performance guarantee of the algorithm on operation cost compared to the global optimum.

The control objective of the RES is to determine \( P_c(t) \) and \( P_d(t) \) so as to minimize the time-averaged operation cost, which includes electricity cost \( G(t) \) and spillage cost \( H(t) \). Considering the ESS operation constraints, the power scheduling optimization problem \( \textbf{P1} \) is formulated as

\[
\begin{align*}
\mathbf{P1} : \quad & \mathcal{J}_{\text{min}} = \min_{\{P_c(t), P_d(t)\}} \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[G(t) + H(t)] \quad (31a) \\
& \text{subject to} \quad 0 \leq P_c(t) \leq \mathcal{P}_c \quad (31b) \\
& \quad 0 \leq P_d(t) \leq \mathcal{P}_d \\
& \quad (1), (10), (11), \text{and} (12). \quad (31c)
\end{align*}
\]

One challenge of solving the above optimization problem is that the future renewable generation, electricity price, and network load in \( \textbf{P1} \) are unknown system parameters. Moreover, the constraints on \( S(t) \), namely, (11) and (12), make \( \textbf{P1} \) become time-coupling, which means that the current control decision can influence the future control decisions. To solve \( \textbf{P1} \), an approach based on Lyapunov optimization is proposed, that requires no future information about the system parameters.

A. Problem Reformulation

In order to decouple the time-coupling constraints on ESS, we consider a relaxed version of the optimization problem \( \textbf{P1} \), namely, \( \textbf{P2} \), in which all the constraints associated with \( S(t) \), i.e., (11) and (12), are relaxed

\[
\begin{align*}
\mathbf{P2} : \quad & \mathcal{J}_{\text{min}} = \min_{\{P_c(t), P_d(t)\}} \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[G(t) + H(t)] \quad (32a) \\
& \text{subject to} \quad 0 \leq P_c(t) \leq \mathcal{P}_c \quad (32b) \\
& \quad 0 \leq P_d(t) \leq \mathcal{P}_d \\
& \quad (1), (27b), (27c), \text{and} (10). \quad (32b)
\end{align*}
\]

In this problem, the ESS with finite capacity is relaxed to have infinite storage. Hence, we have \( \mathcal{J}_{\text{min}} \leq \mathcal{J}_{\text{min}} \). The constraint of (32b) is introduced to guarantee the stability of SOC. Later, it is shown that the Lyapunov optimization approach for solving \( \textbf{P2} \) can provide tight bounds on \( S(t) \) to satisfy (11) and (12). Therefore, the relaxation from \( \textbf{P1} \) to \( \textbf{P2} \) is exact.

B. Lyapunov Optimization Approach

To track the SOC of ESS, we first adopt the concept of virtual queue in the formulation

\[
X(t) = S(t) - S_{th} \tag{33}
\]

where \( S(t) \) is the SOC of ESS and \( S_{th} \) is introduced so as to satisfy ESS constraints in Lyapunov optimization. The setting of \( S_{th} \) will be discussed in Theorem 5. We define the Lyapunov function with respect to the virtual queue as

\[
L(t) = \frac{1}{2}(X(t))^2 = \frac{1}{2}(S(t) - S_{th})^2. \tag{34}
\]

Here, the conditional Lyapunov drift with respect to \( L(t) \), which describes the one-slot expected change of the Lyapunov function, is given by

\[
\Delta(t) = \mathbb{E}[L(t + 1) - L(t) | X(t)]. \tag{35}
\]

**Lemma 1:** The Lyapunov drift is bounded as follows:

\[
\Delta(t) \leq \mathbb{E}[X(t)[\eta P_c(t) - \frac{1}{\beta} P_d(t)] \Delta t | X(t)] + \bar{T} \tag{36}
\]

where \( \bar{T} < \infty \) is a constant.

**Proof:** See Appendix C.

Based on the Lyapunov optimization framework, (35) is introduced to stabilize the virtual queue. To reflect the control objective which minimizes the generation cost, we add a function of the expected generation cost over one slot to (35) and obtain the following drift-plus-penalty function:

\[
\Delta_V(t) = \Delta(t) + V E[H(t) + G(t) | X(t)] \\
\leq \mathbb{E}[X(t)[\eta P_c(t) - \frac{1}{\beta} P_d(t)] \Delta t + V[H(t) + G(t) | X(t)] + \bar{T]. \tag{37}
\]

According to Lyapunov optimization theory [33], the decision is determined to greedily minimize the bound on the drift-plus-penalty function, say, the right-hand side of the inequality (37), for each time slot \( t \). The drift-plus-penalty function is composed of two parts, namely, the Lyapunov drift (35) and the weighted cost function \( V[G(t) + H(t)] \). Intuitively, to minimize, the Lyapunov drift can stabilize the virtual queue length. The second part \( V[G(t) + H(t)] \) is regarded as the penalty term, which describes the weighted operation cost. The weight \( V \) determines the tradeoff between minimizing the Lyapunov drift and minimizing the weighted cost function.

The Lyapunov optimization problem \( \textbf{P3} \) is formulated to solve \( \textbf{P2} \), that, in turn, solves the original problem \( \textbf{P1} \)

\[
\begin{align*}
\text{minimize} \quad & X(t)[\eta P_c(t) - \frac{1}{\beta} P_d(t)] \Delta t + V[G(t) + H(t)] \\
\text{subject to} \quad & (1), (27b), (27c), \text{and} (10). \quad (38)
\end{align*}
\]

Problem \( \textbf{P3} \) gives the control decisions \( P_c(t) \) and \( P_d(t) \) in time slot \( t \), where \( t \in T \). The SOC of the ESS \( S(t) \) is then updated based on (11).

**Theorem 5:** By choosing the parameters \( V \) and \( S_{th} \) as

\[
0 < V \leq \frac{(B - \mathcal{P}_d \Delta t - \mathcal{P}_c \Delta t) \eta}{p_{max} \mathcal{P}_d + s_{max}} \tag{39}
\]

\[
S_{th} = V p_{max} \mathcal{P}_d + \mathcal{P}_d \Delta t \tag{40}
\]

where \( p_{max} \) and \( s_{max} \) are the maximum values of \( p(t) \) and \( s(t) \), respectively, the decisions made by the optimization problem (38) are feasible for problem (31). More specifically, the sequence of SOC \( S(t) \) resulting from (38) satisfies

\[
0 \leq S(t) \leq B \tag{41}
\]

which satisfies (12).

**Proof:** See Appendix D.
Algorithm 2: LB-PAPS Algorithm.

Input: Real-time data for $R(t)$, network load $D(t)$, SOC of the ESS $S(t)$, electricity price $p(t)$, spillage cost $s(t)$.
Output $P_c(t), P_d(t)$.

1: for each Time $t$ do
2: Compute $P_c(t)$ and $P_d(t)$ based on the Lyapunov optimization problem P3.
3: Update $S(t)$ as (11).
4: end for

C. LB-PAPS Algorithm

Based on the above Lyapunov optimization framework, we now present the LB-PAPS algorithm shown in Algorithm 2 to determine the charging/discharging power of the ESS.

Theorem 6: The time average operation cost achieved by LB-PAPS algorithm satisfies

$$
\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} E[\hat{G}(t) + \hat{H}(t)] \leq \bar{T}_{\text{min}} + \frac{T}{T_{\text{avg}}}.
$$

Proof: See Appendix E.

Theorem 6 gives the performance guarantee of the LB-PAPS algorithm compared to the global optimum in P1. From (39) and (42), it can be induced that when the size of the ESS $B$ increases, the upper bound of $V$ increases and the optimality gap between the result achieved by the LB-PAPS algorithm and the global optimal, namely, $\bar{T}_{\text{min}}$, decreases. When $B$ goes to infinity, we can conclude that the time-averaged operation cost achieved by the LB-PAPS algorithm converges to the global optimal $\bar{T}_{\text{min}}$.

VI. PERFORMANCE EVALUATION

A. Simulation Setup

We conduct a case study of a small-scale wind power generator, which is located in the north of Los Angeles, CA, USA. A set of real data recorded for the whole 2012 is used in the case study, with 5-min metering interval [29]. The historical time-varying electricity price is obtained from [34]. The size of ESS varies to test its effect on the performance of RES. The maximum charging and discharging rates of the ESS are set as 8 and $-8$ kW, respectively. The whole operation period of the RES in the simulation is set to one month (30 days), with the duration of each time slot $\Delta t = 5$ min. The network load is assumed to be constant and equals 8 kW in Tests 1 and 2. In Tests 3 and 4, real-world metered load data of the PJM market from 1 January 2016 to 30 August 2017 [35] is adopted in the simulation. The metered load data are normalized with mean of 8 kW to match with the wind generation capacity. In Test 5, the network loads of $D(t)$ with mean of 8 kW and variance of different values are adopted to test the load adaptability of algorithms. Simulations are conducted for 20 times with different generation and load profiles. The results are given with the mean estimates for Tests 1 and 2, and the mean estimates and the 95% confidence intervals for Tests 3 and 5.

B. Scenarios for Comparison

To evaluate our approaches, we compare six scenarios:
(S1) with no ESS (base scenario);
(S2) with ESS, using the smoothing control method in [8];
(S3) with ESS, using the POS scheme devised in Section III-A;
(S4) with ESS, using the HB-LAPS algorithm in [17];
(S5) with ESS, using the LB-PAPS algorithm in Section V;
(S6) the global optimum on operation cost with ESS.

Note that S3 and S5 are the algorithms proposed in our article and are evaluated through the comparisons with S1, S2, S4, and S6.

In S1, no energy management approach is implemented. It is regarded as the base case for performance evaluation. For S2, the charging or discharging power of the ESS is calculated based on the dynamic rate limiter proposed in [8]. Basically, S2 keeps the renewable output unchanged if the current output is within the predefined range of the power fluctuation. Otherwise, it modifies and smoothens the renewable output by managing the charging and discharging behaviors of the ESS. In S3, $P_c(t)$ and $P_d(t)$ are computed according to Algorithm 1. It has been proved that S3 achieves the optimal utilization of renewable energy. Thus, S3 is taken as the benchmark to evaluate the renewable energy utilization. In S5, $S_{LB}$ is set based on (40) and $V$ is set as the right-hand side of (39). In S6, it is assumed that all the operation parameters, including the network load, renewable generation profile, and electricity prices, are known in advance. Therefore, the global optimum on operation cost can be obtained through minimizing (43). Note that S6 requires accurate predictions on operation parameters, that is not practical in real grid operations. S2–S5 are real-time algorithms and do not require predictive information.

C. Performance Metrics

Two performance metrics are considered in the simulation tests: $C_{avg}$ and $U_{RG}$. $C_{avg}$ denotes the daily average cost

$$
C_{avg} = \frac{T_d}{T} \sum_{t=1}^{T} [G(t) + H(t)]
$$

where $T_d$ denotes the time horizon of one day in slots. $U_{RG}$ denotes the renewable energy utilization. It measures the proportion of the renewable energy in serving the network load, and is given as

$$
U_{RG} = \frac{\sum_{t=1}^{T} \min\{P_{out}(t), D(t)\} \Delta t}{\sum_{t=1}^{T} R(t) \Delta t}.
$$

$U_{RG}$ can also be represented by LOPSE, which is given by (22) and (30)

$$
U_{RG} = \left(1 - \frac{\text{LOPSE}}{\sum_{t=1}^{T} P(t)} \right) \frac{\sum_{t=1}^{T} D(t)}{\sum_{t=1}^{T} R(t)}.
$$

An algorithm which results in a lower $C_{avg}$ and a higher $U_{RG}$ (lower LOPSE) indicates a better performance.

In order to evaluate the effect of renewable generation profile on the ESS configuration, we need to introduce some effective measurements to describe the fluctuation of the renewable generation profile. We adopt two metrics, i.e., standard deviation $\text{STD}_{RG}$ and standard deviation of moving average $\text{STD}_{MA}$, to describe, respectively, the amplitude of fluctuation and the intermittency of renewable generations

$$
\text{STD}_{RG} = \sqrt{\frac{1}{T} \sum_{t=1}^{T} \left( R(t) - \frac{1}{T} \sum_{t=1}^{T} R(t) \right)^2}
$$

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In this test, we focus on how the ESS affects the performance of the POS scheme. In the second test, we analyze how the ESS size and the renewable generation profile affect the renewable utilization. We also give some insights in how the ESS should be configured under different amplitudes of fluctuations and fluctuation frequencies of the renewable generation, so as to satisfy the required renewable utilization. In the third and fourth tests, Scenarios S1, S2, S3, S4, S5, and S6 are compared in terms of operation cost and renewable utilization, respectively. In the fifth test, we study the load adaptability of S5.

### D. Simulation Results

Five tests are performed to evaluate the performance of our methods and the effect of ESS size on the system performance. In the first test, we examine the effectiveness of our proposed POS scheme, i.e., S3, for serving the network load and show how the ESS affects the performance of the POS scheme. In the second test, we analyze how the ESS size and the renewable generation profile affect the renewable utilization. We also give some insights in how the ESS should be configured under different amplitudes of fluctuations and fluctuation frequencies of the renewable generation, so as to satisfy the required renewable utilization. In the third and fourth tests, Scenarios S1, S2, S3, S4, S5, and S6 are compared in terms of operation cost and renewable utilization, respectively. In the fifth test, we study the load adaptability of S5.

1) **Test 1**: We focus on smoothing power output of the RES to serve network load and compare S1 and S3 on renewable energy utilization and reliability under different sizes of ESS. This investigates the capability of the POS scheme to mitigate the power fluctuations. Fig. 4(a) illustrates the renewable energy utilization with different configurations on the size of the ESS $B$. It indicates that, as the size of the ESS increases, the renewable energy utilization achieved by the POS scheme improves but its marginal improvement decreases. It is reasonable because a larger ESS can buffer more renewable energy and, thus, increase the renewable energy utilization. Fig. 4(b) illustrates the system reliability, which is reflected by LOPSE calculated in Section IV, under S1 and S3. It reveals that S3 can improve the system reliability as the ESS size increases but its marginal improvement on reliability decreases according to the analytical results. The analytical results shown in Fig. 4(b) match well with the simulation results shown in Fig. 4(a) based on the relationship between $U_{RG}$ and LOPSE given by (45).

The analytical results shown in Fig. 4(b) match well with the simulation results shown in Fig. 4(a) based on the relationship between $U_{RG}$ and LOPSE given by (45).

2) **Test 2**: In this test, we focus on how the ESS affects the renewable utilization under different amplitudes of fluctuations of renewable generation and intermittency levels. We fix the renewable utilization of the RES to 90% and find what is the required ESS size to satisfy this utilization level under different amplitudes of fluctuations and intermittency levels. In the first simulation, we keep the mean of the renewable generated power unchanged but increase the amplitude of fluctuation through enlarging the gap between the renewable generated power and the mean of the renewable generated power. The relationship between STD$_{RG}$ and the size of the ESS $B$ is shown in Fig. 5(a). It reveals that when STD$_{RG}$ is larger than a threshold, the required ESS size blows up with STD$_{RG}$ to satisfy the required renewable utilization. In the second simulation, we fix STD$_{RG}$ and the mean of renewable generated power while changing the intermittency level through modifying STD$_{MA}$, where $W$ is set to 100. The relationship between STD$_{MA}$ and the size of the ESS $B$ is shown in Fig. 5(b). It reveals that, given that the mean and the variance of the renewable generated power unchanged, the size of the ESS blows up with STD$_{MA}$ to satisfy the required renewable utilization. It is because when the supply of renewable generated power becomes more intermittent, the ESS needs to reserve more energy when the renewable generated power is in excess to compensate for the power shortage with a longer period. Therefore, a larger size of the ESS is needed to satisfy the required renewable utilization level.

3) **Test 3**: We compare scenarios, S1–S6, with time-varying load to evaluate the achieved daily average cost while the size of the ESS varies. The results (both the mean estimate and the 95% confidence interval for each configuration) are shown in Fig. 6(a). We can see that the cost is the highest in S2 since S2 only aims to smooth out the high-frequency fluctuations of the power output, which does not consider the economical aspect of the RES. The cost in S1 is the second highest comparing to other scenarios with ESS, which indicates that the operation cost

\[
\text{STD}_{MA} = \sqrt{\frac{1}{T-2W} \sum_{t=1+W}^{T-W} \left( \frac{1}{2W+1} \sum_{i=t-W}^{t+W} R(i) \right) - \frac{1}{T} \sum_{t=1+W}^{T-W} R(t)^2}
\]

(47)

where $2W+1$ is the window size of the moving average and $\sum_{t=W+1}^{2W+1} R(i)$ is the moving average of the renewable generated power at time slot $t$. The renewable generated power with a larger STD$_{RG}$ has a larger amplitude of fluctuation. A larger STD$_{MA}$ suffers severer intermittency of the renewable generated power, where the occurrences of the supply shortages become more frequent.

Through Test 1, it is shown that S3 can yield both high renewable energy utilization and reliability while smoothing the renewable generated power. The gain achieved by S3 increases with the ESS size.
can be reduced by employing ESS. Among those scenarios with ESS, S5 outperforms S2, S3, and S4. The cost achieved by S5 is a bit higher than the global optimum in S6. According to (42), it can be proved theoretically that as the operation period and the size of the ESS go to infinity, the daily average cost achieved by S5 converges to the global optimum in S6.

4) Test 4: Similar to Test 3, we do the comparisons among S1–S5 in terms of renewable energy utilization. Fig. 6(b) illustrates the results achieved by these five scenarios, where both the mean estimate and the 95% confidence interval for each configuration are exhibited. Among these five scenarios, S3 yields the optimal utilization. It is because S3 always provides power output to serve the load whenever the renewable generation is available. The renewable energy utilization achieved by S4 is a little lower than S3. The reason why S3 and S4 outperform others is that both S3 and S4 take renewable energy utilization as their control objectives (S3 takes renewable energy utilization as its only control objective and S4 takes renewable energy utilization as one of its heuristic information). Among those scenarios, which do not take renewable energy utilization as their control objectives directly, S5 performs much better than S1 and S2. This shows that S5 can achieve good performance on daily average cost while maintaining relatively high utilization. S2 yields the lowest utilization since it does not aim to satisfy the load demand while filtering the high-frequency fluctuations of the renewable generated power.

5) Test 5: We focus on the effect of the load variation on our proposed LB-PAPS algorithm. We evaluate S5 under different variances of the network load and the size of the ESS is set to 100 kWh. The performance (both the mean estimate and the 95% confidence interval for each configuration) are exhibited in Fig. 7. From Fig. 7, we see that as the variance of the load increases, S5 can maintain the mean of the daily average cost and the renewable energy utilization at a constant level. It is because S5 can adaptively change the power output to counter the uncertainty of the load. Fig. 7 also reveals that the confidence intervals on the daily average cost and the renewable energy utilization becomes larger as the variance of the load increases. This indicates that the fluctuation of the load can degrade the degree of confidence on these two metrics.

VII. CONCLUSION

This article investigates the POS techniques for renewable generation in IoE and analyzes the tradeoff between ESS size and the performance, i.e., reliability, utilization, or cost, on POS, respectively. By jointly considering the constructional cost of the ESS and the operational cost of the RES, an optimization problem for joint storage configuration is formulated. The problem can be relaxed to an MILP under certain assumptions. An analytical framework is proposed to investigate the tradeoff between ESS size and the system reliability. The simulation results exhibit that the proposed LB-PAPS algorithm outperforms other existing algorithms in terms of operation cost and can maintain relatively high renewable energy utilization under different ESS capacities. We also find that, to satisfy the renewable utilization requirement, the required ESS size blows up with both the amplitude of fluctuation and the intermittency level of the renewable generation. It is noted that the technique proposed in this article applies to POS for the RES, as well as the potential applications on other research areas. For instance, the methods proposed and results obtained in this article can be applicable to the energy management and distribution in the vehicular energy network [36].

APPENDIX

A. Proof of Theorem 1

Due to the maximum charging/discharging rates of the ESS and the SOC constraints, the feasible domain of $P_c$ and $P_d$ is bounded by $0 \leq P_c \leq \min\{M_c(t), \overline{P}_c\} = \overline{A}_c$ and $0 \leq P_d \leq \min\{M_d(t), \overline{P}_d\} = \overline{A}_d$, respectively. Note that the charging and discharging processes of the ESS cannot happen at the same time, compared to the POS scheme, other charging schemes must fall into the following four cases: 1) $0 \leq P_c \leq G_c(t)$ and $P_d = 0$, 2) $G_c(t) \leq P_c \leq \overline{A}_c$ and $P_d = 0$, 3) $0 \leq P_d \leq G_d(t)$ and $P_c = 0$, and 4) $G_d(t) \leq P_c \leq \overline{A}_d$ and $P_c = 0$, where $G_c(t)$ and $G_d(t)$ are defined in Algorithm 1. Define the resulted $U_{RG}$ by using POS algorithm as $U_{RG}$. We now proof that in all the above cases, $U_{RG}$ cannot be higher than $U_{RG}^*$.

For cases 1), if $G_c(t) \leq \overline{A}_c$, satisfying 1) will decrease $S(t)$ by $[G_c(t) - P_c(t)]\eta \Delta t$ and will not affect $U_{RG}$. Since reducing $S(t)$ can only result in the reduction of $U_{RG}$, the obtained $U_{RG}$ cannot be higher than $U_{RG}^*$ in this way. If $G_c(t) \geq \overline{A}_c$, satisfying 1) will decrease $S(t)$ by $[\overline{A}_c - P_c(t)]\eta \Delta t$ and will not affect $U_{RG}$. Since reducing $S(t)$ can only result in the reduction of $U_{RG}$, the obtained $U_{RG}$ cannot be higher than $U_{RG}^*$ in this way. For case 2), if $G_c(t) \leq P_c \leq \overline{A}_c$, satisfying 1) will increase $S(t)$ by $[P_c(t) - G_c(t)]\eta \Delta t$ and decrease $U_{RG}$ by $\frac{|P_c(t) - G_c(t)|\eta \Delta t}{\sum_{i=1}^{L} (R_i(t) \Delta t)}$. In this way, even in the optimal case, the increase of $\{P_c(t) - G_c(t)\} \eta \Delta t$ in $S(t)$ can only increase $U_{RG}$ by $\frac{|P_c(t) - G_c(t)|\eta \Delta t}{\sum_{i=1}^{L} (R_i(t) \Delta t)}$, which is smaller than $\frac{|P_d(t) - G_d(t)|\eta \Delta t}{\sum_{i=1}^{L} (R_i(t) \Delta t)}$. Therefore, the obtained $U_{RG}$ cannot be higher than $U_{RG}^*$ by satisfying 2).

Similar to cases 1) and 2), it can be concluded that the obtained $U_{RG}$ cannot be higher than $U_{RG}^*$ by satisfying 3) and 4). Hence, we can draw the conclusion that the proposed POS scheme can achieve the optimal value of $U_{RG}$.

B. Proof of Theorem 2

Proof: Note that the feasible region of Problem (13) is a subset of the feasible region of Problem (21). Comparing Constraints (8), (9), and (10) in (13) with Constraints (14)–(20) in (21), we categorize the charging/discharging constraints of the ESS (14)–(20) in Problem (21) into four cases: 1) $P_d(t) = 0$ and $0 \leq P_c(t) < \max\{0, R(t) - P_c(t)\}$, 2) $P_d(t) = 0$ and $\max\{0, R(t) - P_c(t)\} < P_c(t) \leq R(t)$, 3) $P_c(t) = 0$ and $0 \leq P_d(t) < \max\{0, P_c(t) - R(t)\}$, and 4) $P_c(t) = 0$ and $\max\{0, P_c(t) - R(t)\} < P_d(t) \leq P_c(t)$.
According to (8), (9), and (10), these cases in (14)–(20) can be mapped to four states of the ESS 1 being undercharged, 2) being overcharged, 3) being underdischarged, and 4) being overdischarged, respectively. Given that \( p(t) \) is fixed, it can be concluded that \( f \) will not decrease by satisfying 2) and 3) since \( C_g \) cannot decrease by reducing the current energy provided and storing it in the ESS for future use.

To explain, we first consider case 2). Through satisfying 2) instead of letting \( P_d(t) = \max\{0, R(t) - P_r(t)\} \), \( S(t) \) is increased by \( \{P_r(t) - \max\{0, R(t) - P_r(t)\}\} \eta t \Delta t \) and the electricity cost at time \( t \), \( G(t) \), is increased by \( \{P_r(t) - \max\{0, R(t) - P_r(t)\}\} \|P_r(t)\|\Delta t \). In this way, the resulting electricity cost \( C_g \) is larger than the electricity cost \( C_g^* \) when satisfying \( P_d(t) = \max\{0, R(t) - P_r(t)\} \). It is because even in the optimal case, the increase of \( \{P_r(t) - \max\{0, R(t) - P_r(t)\}\} \eta \Delta t \) in \( S(t) \) results in the reduction of \( C_g \) by \( \max\{0, R(t) - P_r(t)\} \|P_r(t)\|\beta \Delta t \) in grid operation, which is smaller than than and cannot compensate for the electricity cost \( \{P_r(t) - \max\{0, R(t) - P_r(t)\}\} \|P_r(t)\|\Delta t \) arising by satisfying 2) at time \( t \). Therefore, satisfying 2) cannot reduce \( f \). In case 3), comparing to letting \( P_d(t) = \max\{0, P_r(t) - R(t)\} \), \( S(t) \) is increased by \( \min\{0, P_r(t) - R(t)\} \|P_r(t)\|\Delta t \) in \( S(t) \) results in the reduction of \( C_g \) by \( \max\{0, P_r(t) - R(t)\} \|P_r(t)\|\beta \Delta t \) in grid operation, which is equal to the electricity cost arising by satisfying 3) at time \( t \). Therefore, satisfying 3) cannot reduce \( f \).

Similar to the discussion in case 2) and 3), it can be concluded that \( f \) will not be reduced by satisfying 1) and 4).

Based on the above discussions, it is proved that Problem (13) always achieves a value of \( f \) not larger than that of Problem (21) for all the four situations, given that \( P_c, P_d \), and \( B \) are fixed. Coupled with the fact that the feasible region of Problem (13) is a subset of the feasible region of Problem (21), it can be concluded that solving Problem (13) for optimal values of \( P_c, P_d \), and \( B \) is equivalent to solving Problem (21) for optimal values of \( P_c, P_d \), and \( B \). Hence, the optimal solutions for \( P_c, P_d \), and \( B \) in Problem (21) are the same as those of Problem (13).

\[ C. \text{Proof of Lemma 1} \]

\[
\Delta(t) = \frac{1}{2} \mathbb{E}\left\{ \left(X(t + 1)^2 - X(t)^2\right) \right\} = \frac{1}{2} \left\{ \begin{array}{ll}
\mathbb{E}\left\{ (X(t) + \eta \bar{P}_c(t) \Delta t)^2 - X(t)^2 \right\} & \text{if } P_c(t) = 0 \\
\mathbb{E}\left\{ (X(t) - \frac{1}{\beta} P_d(t) \Delta t)^2 - X(t)^2 \right\} & \text{if } P_d(t) = 0 \\
\end{array} \right.
\leq \frac{1}{2} \mathbb{E}\left\{ (\eta \bar{P}_c(t) \Delta t)^2 + \left(\frac{1}{\beta} \bar{P}_d(t) \Delta t\right)^2 \right\} X(t)
\]

where \( \bar{T} = \frac{1}{2} \max\{ \eta \bar{P}_c(t)^2, \left(\frac{1}{\beta} \bar{P}_d(t)\right)^2 \} \).

\[ D. \text{Proof of Theorem 4} \]

To proof the left-hand side of (41), we first prove the following lemma.

**Lemma 2:** When \( S(t) < \frac{\bar{T}}{\beta} \Delta t \), it holds that \( P_d(t) = 0 \) according to (38).

**Proof:** We prove this lemma using the proof by contradiction. First, we assume that \( P_d(t) > 0 \) when \( S(t) < \frac{\bar{T}}{\beta} \Delta t \). Since the control objective is to minimize (38), it holds that if \( P_d(t) > 0 \)

\[
X(t) \frac{1}{\beta} \Delta t \geq -V_{\text{max}} \frac{1}{\beta} \Delta t \leq V_{\text{max}} (\text{using} (40))
\]

\[
S(t) \geq \frac{\bar{T}}{\beta} \Delta t
\]

(49) violates \( S(t) < \frac{\bar{T}}{\beta} \Delta t \). Therefore, it must be that when \( S(t) < \frac{\bar{T}}{\beta} \Delta t \), \( P_d(t) = 0 \).

According to Lemma 2, it holds that \( P_d(t) = 0 \) when \( S(t) < \frac{\bar{T}}{\beta} \Delta t \). When \( S(t) \geq \frac{\bar{T}}{\beta} \Delta t \), it holds that \( 0 \leq P_d(t) \leq \bar{P}_d \) based on (31c). Therefore, it can be concluded that the left-hand side of (41) \( 0 \leq S(t) \) holds.

To proof the right-hand side of (41), we introduce the following lemma.

**Lemma 3:** When \( S(t) > B - \bar{P}_c \eta \Delta t \), it holds that \( P_c(t) = 0 \) according to (38).

**Proof:** We prove this lemma using the proof by contradiction. First, we assume that \( P_c(t) > 0 \) when \( S(t) > B - \bar{P}_c \eta \Delta t \). Since the control objective is to minimize (38), it holds that if \( P_c(t) > 0 \)

\[
X(t) \eta \leq V_{\text{max}} \left[ S(t) - V_{\text{max}} \beta \Delta t \right] \eta \leq V_{\text{max}} \left( s_{\text{max}} + \max\{\eta \bar{P}_c \Delta t, \left(\frac{1}{\beta} \bar{P}_d \Delta t\right)^2 \} \right)
\]

\[
S(t) \leq B - \bar{P}_c \eta \Delta t
\]

(50) violates \( S(t) > B - \bar{P}_c \eta \Delta t \). Therefore, it must be that when \( S(t) > B - \bar{P}_c \eta \Delta t \), \( P_c(t) = 0 \).

According to Lemma 3, it holds that \( P_c(t) = 0 \) when \( S(t) > B - \bar{P}_c \eta \Delta t \). When \( S(t) \leq B - \bar{P}_c \eta \Delta t \), it holds that \( 0 \leq P_c(t) \leq \bar{P}_c \) based on (31b). Therefore, it can be concluded that the left-hand side of (41) \( S(t) \leq B \) holds.

Combining Lemmas 2 and 3 as well as the above discussion, we can conclude it holds that \( 0 \leq S(t) \leq B \).

\[ E. \text{Proof of Theorem 5} \]

Note that the solution of (38) minimize the drift-plus-penalty function (37) over its feasible region. We have

\[
\Delta v(t) \leq \mathbb{E}\left\{ \left[ \eta \bar{P}_c(t) - \frac{1}{\beta} \bar{P}_d(t) \right] \Delta t + V(\bar{G}(t) + \bar{H}(t)) \right\}
\]
\[
\begin{align*}
\leq \mathbb{E} \left[ X(t) | \eta P_d(t) - \frac{1}{\beta} P_d(t) \right] \Delta t + \mathbb{E} \left[ G(t) + H(t) \right] | X(t) + T
\end{align*}
\]

where the control actions by solving (38) is denoted with the superscript \( \tilde{\cdot} \) and the control actions by solving (32) is denoted with the superscript \( \tilde{\cdot} \). The second inequality is derived due to the stationary and the randomized policy in the problem (32). The equality holds based on \( E[\eta P_d(t) - \frac{1}{\beta} P_d(t)] X(t) = 0 \).

Summing from \( t = 0 \) to \( t = T - 1 \) on both sides of (51)

\[
\sum_{t=0}^{T-1} \Delta V(t) \leq \sum_{t=0}^{T-1} \left\{ \mathbb{E} \left[ V[\tilde{G}(t) + \tilde{H}(t)] \right] + \tilde{T} \right\}.
\]

Then, we expand (52) by using (35) and (37) and obtain

\[
\mathbb{E} \left\{ L(T) - L(0) \right\} + \sum_{t=0}^{T-1} \mathbb{E} \left[ V[\tilde{G}(t) + \tilde{H}(t)] \right]
\]

Divide both sides by \( V \cdot T \) and make \( T \rightarrow \infty \), we have

\[
\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \left[ \tilde{G}(t) + \tilde{H}(t) \right]
\]

\[
\leq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \tilde{T} + \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \left[ \tilde{G}(t) + \tilde{H}(t) \right]
\]

\[
= \frac{\tilde{T}}{V} + \bar{f}_{\min} \leq \frac{\tilde{T}}{V} + \bar{f}_{\min}.
\]

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