

Topology-Transparent Broadcast Scheduling with Erasure Coding in Wireless Networks

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Abstract—Broadcasting is an important function in wireless networks. Ensuring broadcast efficiency and reliability with low communication overhead is challenging, especially with error-prone links. In this letter, we employ erasure coding together with topology-transparent scheduling as a coded transmission strategy in the MAC (Medium Access Control) layer rather than the physical layer to combat collisions and channel errors, implementing an efficient and reliable broadcast algorithm in wireless networks without introducing any additional communication overhead. We achieve the optimal frame structure that maximizes the average network throughput, and investigate the performance of our proposed algorithm in terms of the average network throughput and the packet failure probability. Simulation results show that our proposed algorithm with erasure coding outperforms other existing topology-transparent broadcast algorithms and the conventional TDMA dramatically.

Index Terms—Efficient and Reliable Broadcast, Erasure Coding, Topology-Transparent Scheduling.

I. INTRODUCTION

EFFICIENT and reliable broadcast is required in many scenarios such as command broadcast in battlefield communication networks and safety message broadcast in vehicular networks. Ensuring broadcast efficiency and reliability is a challenging task because of the error-prone characteristics of wireless links, node mobility, and limited wireless bandwidth. Contention-based scheduling approaches neither support reliable broadcast nor guarantee bounded delay in multi-hop networks. This is because the probability of all neighbours receiving a broadcast packet successfully is small when the network load is high, resulting in many retransmissions. It is shown that contention-based approaches suffer from serious instability and unfairness issues in multi-hop ad hoc networks [13]. That is why most throughput and delay guaranteeing networks, such as tactical networks EPLRS (Enhanced Position Location Reporting System) [7] and JTRS (Joint Tactical Radio System) [10], use time-division multiple-access (TDMA) as their MAC protocols. Topology-dependent scheduling algorithms require each node to maintain accurate network connectivity information, which is impractical in wireless networks with dynamic topologies. Topology-transparent algorithms [3, 6, 12] are proposed to guarantee at least one collision-free time slot in each frame. However, they are only applicable to unicast traffic. Cai et al. [2]

proposed a broadcast scheduling algorithm, transmitting the same message repeatedly to guarantee exactly one successful broadcast transmission per frame, but the throughput is low. A constant weight code based approach [5] was proposed to provide certain reliability for safety message broadcast in vehicular networks.

A probabilistic topology-transparent broadcast scheduling algorithm (P-TTS) is proposed in [8], which improves the average throughput at the expense of reduced reliability. The main differences of this work and [8] are as follows. First, we focus on solving the problem that acknowledgement mechanisms cannot be efficiently implemented in broadcast scenarios, which is the main reason leading to the inefficiency of previous topology-transparent broadcast algorithms. However, [8] spent no effort on this issue. Inspired by the similarity between erasure coding and topology-transparent scheduling, we employ them together as a coded transmission strategy in the MAC layer, without introducing any additional communication overhead, to combat the collisions and channel errors rather than only using erasure coding in the physical layer to control errors in the data communication. Moreover, [8] has traded reliability for a relatively higher average throughput compared to that in [2], while we achieve the optimal throughput, which is much higher than those of the previous papers including [8], at no expense of reduced reliability. Finally, unlike the previous work including [8], we consider the error-prone characteristics of wireless links, which is one of the most challenging issues in reliable broadcast.

In this letter, we employ erasure coding to combat collisions and channel errors in topology-transparent scheduling, implementing an efficient and reliable solution for broadcast in wireless networks. Moreover, the employed erasure coding scheme does not introduce any overhead. As far as we know, this is the first work improving both efficiency and reliability of a topology-transparent broadcast scheduling over error-prone channels. We study the performance of our proposed algorithm analytically and by simulation, in terms of the average network throughput and the packet failure probability.

II. SYSTEM MODEL

Consider a mobile ad hoc network with N nodes. If Node u is in the interference range of Node v , u is considered as an interfering neighbour of v . The degree of a node v , $d(v)$, defines the number of interfering neighbours of v . Due to the characteristics of mobile ad hoc networks, the maximum degree D is much smaller than N , and assumed to remain constant while the network topology changes [4]. Each node broadcasts its packets to all nodes in its communication range. The number of such nodes is upper-bounded by D , since the

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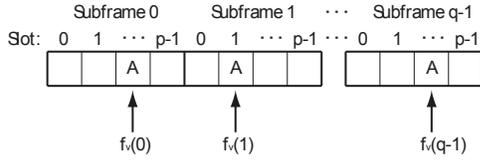


Fig. 1. The frame structure.

interference range is typically larger than the communication range.

Time is divided into frames, which consist of equal-sized synchronized transmission slots. Synchronization can be achieved by Global Positioning System (GPS). For broadcast traffic, the acknowledgement mechanism cannot be easily implemented [2]. We assume a protocol interference model. The transmission from Node u to one of the nodes in its communication range Node v succeeds when 1) Node v is not transmitting, 2) other interfering neighbours of v are not transmitting, and 3) there is no channel error over the link from Node u to Node v .

Let BER , L , and p_e be the bit error rate, the packet length in bytes, and the packet error rate, respectively. The packet error rate is determined by the packet length and bit error rate. Note that there is no physical coding scheme implemented. When the bit errors are assumed to be independent, we have:

$$p_e = 1 - (1 - BER)^{8L}. \quad (1)$$

An $[n, k]$ erasure code can be used to protect k packets with $n - k$ redundant packets. Let $G_{k \times n}$ be the generator matrix of an $[n, k]$ erasure code. A vector \mathbf{x}_k of k elements can be encoded to a vector \mathbf{y}_n of n elements according to the following:

$$\mathbf{y}_n = \mathbf{x}_k \mathbf{G}_{k \times n}. \quad (2)$$

For the decoder, any k out of n elements of the vector \mathbf{y}_n are assumed to be received successfully (we denote it by \mathbf{y}_k). We keep columns of the generator matrix $G_{k \times n}$ according to these k elements, and delete the other columns. Thus, we get a $k \times k$ matrix $G'_{k \times k}$. The inverse of the matrix $G'_{k \times k}$ exists [9]. The original vector can be decoded as follows:

$$\mathbf{x}_k = \mathbf{y}_k (\mathbf{G}'_{k \times k})^{-1}. \quad (3)$$

III. PROPOSED ALGORITHM

In the proposed algorithm, a frame is divided into q subframes, each of which consists of p fixed-length time slots. Each node v is assigned a unique polynomial of degree k over $GF(p)$ (p is a prime or prime power), $f_v(x) = \sum_{i=0}^k a_i(v)x^i \pmod{p}$. Node v selects the time slot $f_v(i)$ to transmit in Subframe i , where $i \in \{0, 1, 2, \dots, q-1\}$. $f_v(x)$ and $(f_v(0), f_v(1), \dots, f_v(q-1))$ are denoted by time slot allocation function (TSAF) and time slot location vector (TSLV) of Node v , respectively [6]. The frame structure is shown in Fig. 1.

Consider a network with N mobile nodes and maximum node degree D . In order to guarantee that every node has at

least one collision-free time slot to transmit, two constraints should be satisfied [3, 6]. Thus, we have:

$$p^{k+1} \geq N, \quad (4)$$

$$q \geq kD + 1. \quad (5)$$

This guarantee only depends on the global parameters (N and D) rather than detailed network connectivity information. Specific network realizations have no effect on the operation and performance of our algorithm. This is why it is called topology-transparent scheduling.

Assume that each node encodes M (where $M \leq q - kD$) queued packets to q packets using a $[q, M]$ erasure code and transmits the i -th encoded packet in Subframe $i-1$, where $i = 1, 2, \dots, q$. We assume a heavy traffic condition with all nodes backlogged. That is, M data packets are always available for encoding and transmission at each node in every frame.

We define the network throughput as the average number of packets successfully broadcasted per slot in the network. It has been found that $k = 1$ for most cases [6]. Thus, without loss of generality, we use $k = 1$ in the following analysis for simplicity.

First, consider a transmission from Node u to Node v . We assume the worst case here. There are up to D interfering nodes and all are transmitting. Given q , let N^l denote the number of ways for a given TSLV of Node u , $TSLV_u$, to select D other TSLVs, the union of which intersects $TSLV_u$ in exactly l specific positions. There are $\binom{q}{l}$ ways to choose l positions intersected from q transmission positions, and $\binom{p^2-1}{D}$ ways to select D TSLVs from all the remaining p^2-1 TSLVs. The probability that there are $q-l$ collision-free slots for the transmission from Node u to Node v is $\frac{\binom{q}{l} N^l}{\binom{p^2-1}{D}}$. Note that less than M encoded packets transmitted from Node u being received successfully leads to the decoding failure at the receiver v as discussed in Section II. Thus, the packet failure probability of the transmission from Node u to Node v , denoted by P_f^{uv} , can be expressed as follows:

$$P_f^{uv} \leq \sum_{l=0}^D \frac{\binom{q}{l} N^l}{\binom{p^2-1}{D}} \sum_{i=0}^{M-1} \binom{q-l}{i} (1-p_e)^i p_e^{q-l-i}. \quad (6)$$

Consider the TSAF of Node u , $TSAF_u$. We categorize the remaining $\beta = p^2 - 1$ TSAFs into $p+1$ different subsets F_i (where $i = 0, 1, \dots, p$) according to their coincidences with $TSAF_u$. We define the coincidence of any two polynomials as the root of the difference of these two polynomials. That is, if $f_u(j) - f_v(j) = 0$, j is the coincidence of $f_u(x)$ and $f_v(x)$. The TSAFs in F_i (where $i = 0, 1, \dots, p-1$) have the coincidence i with $TSAF_u$ and TSAFs in F_p have no coincidence with $TSAF_u$.

Consider an arbitrary polynomial $f(x) = ax + b \pmod{p}$ with degree one, where $a, b \in \{0, 1, \dots, p-1\}$. Keeping the slope of $f(x)$ invariant and varying b , we get a sequence of polynomials $g_i(x) = ax + b_i \pmod{p}$, where $b_i \in \{0, \dots, b-1, b+1, \dots, p-1\}$, that have no coincidence with $f(x)$. The number of the sequence of polynomials is $p-1$. That is, $|F_p| = p-1$. Similarly, consider $f(x) = ax + b \pmod{p}$ and an arbitrary integral number x_0 , where $a, b, x_0 \in \{0, 1, \dots, p-1\}$. Fixing the point $(x_0, f(x_0))$ and varying a_i , where $a_i \in$

$\{0, \dots, a-1, a+1, \dots, p-1\}$, we obtain a sequence of lines (polynomials) passing the point $(x_0, f(x_0))$ besides $f(x)$, the number of which is $p-1$. Thus, $|F_i| = p-1$, where $i = 0, 1, \dots, p-1$. Consider l specific integral numbers, namely, $p_1, p_2, \dots, p_l \in \{0, 1, \dots, q-1\}$, and we denote by S_{q-l} the set of the other $q-l$ integral numbers, which are smaller than q . We classify TSAFs other than $TSAF_u$ into three different groups:

- *Group 1*: The number of TSAFs, that have the coincidence p_i with $TSAF_u$, where $i = 1, 2, \dots, l$, is $l(p-1)$.
- *Group 2*: The number of TSAFs, that have the coincidence x with $TSAF_u$, where $x \in S_{q-l}$, is $(q-l)(p-1)$.
- *Group 3*: The number of TSAFs, that have the coincidence x , where $x = q, q+1, \dots, p-1$, or no coincidence with $TSAF_u$, is $(p-q+1)(p-1)$.

Let A_{p_i} (where $i = 1, 2, \dots, l$) be the set of events in which none of the chosen D TSAFs from Groups 1 and 3 has the coincidence p_i with $TSAF_u$. Note that the number of TSAFs which have the coincidence p_i (where $i = 1, 2, \dots, l$) with $TSAF_u$ is $l(p-1)$ and we choose D TSAFs from Groups 1 and 3. Thus, the cardinality of the intersection of any m sets from A_{p_i} , where $i = 1, 2, \dots, l$, is $\binom{p^2-1-(q-l+m)(p-1)}{D}$. N^l is equal to the cardinality of the complementary set of $\bigcup_{i=1}^l A_{p_i}$. Note that there are $\binom{p^2-1-(q-l)(p-1)}{D}$ ways to select D codewords from Groups 1 and 3. Thus,

$$N^l = \binom{\beta - (q-l)\alpha}{D} - \left| \bigcup_{i=1}^l A_{p_i} \right|, \quad (7)$$

where $l = 1, 2, \dots, D$, $\beta = p^2 - 1$, and $\alpha = p - 1$.

Applying the Inclusion-Exclusion Principle, we can obtain:

$$N^l = \binom{\beta - (q-l)\alpha}{D} - \sum_{m=1}^l (-1)^{m-1} \binom{l}{m} \binom{\beta - (q-l+m)\alpha}{D}, \quad (8)$$

and $N^0 = \binom{\beta - q\alpha}{D}$.

Recall that the number of nodes in the communication range of one node must be no larger than D . We assume that all (up to D) transmissions corresponding to a single broadcast communication are independent. Note that different receivers may have common neighbours, and correlation thus exists. The simulation results shown in Section IV demonstrate that this assumption does not affect the accuracy of our results. Note that if one of the (up to D) transmissions from one source to one of the nodes in its communication range fails, the broadcast transmission fails. Note that the frame length is pq time slots. Thus, the average network throughput of our proposed algorithm is:

$$T \geq NM \frac{(1 - P_f^{uv})^D}{pq}. \quad (9)$$

The value of M governs the tradeoff between the average network throughput and the packet failure probability. Given (N, D) , packet error rate p_e , and M , we can obtain the optimal number of subframes q_{opt} by maximizing the average network throughput and guaranteeing $P_f \leq \rho$ according

TABLE I
OPTIMAL NUMBER OF SUBFRAMES, NETWORK THROUGHPUT, AND PACKET FAILURE PROBABILITY FOR DIFFERENT NUMBERS OF PACKET ENCODED.

M	1	2	3	4	5
q_{opt}	8	9	10	11	13
T	1.23	2.18	2.92	3.45	3.71
$P_f (10^{-2})$	0.03	0.26	1.14	3.64	1.93

to (9). The optimal numbers of subframes corresponding to different values of M are shown in Table I. The average network throughput and packet failure probability are also shown accordingly. Obviously, it is better to choose a larger M in this scenario. However, it is better to choose a smaller M when the channel condition is very poor, as shown in the next section.

IV. PERFORMANCE EVALUATION

In this section, we quantitatively compare the average network throughput with the MGD algorithm [2], the P-TTS algorithm [8], the POC algorithm [5], and the conventional TDMA fixed assignment scheme.

We conduct simulations on two graph models, namely, the geometric model for the average performance and the D-regular graph model for the worst performance. In the geometric model, all nodes are distributed uniformly and randomly in a region A of $1000 \text{ m} \times 1000 \text{ m}$. Given D , we set the interference range of each node R_I such that the probability that the number of interfering neighbours of an arbitrary node exceeding D , which is $\sum_{i=D+1}^{N-1} \binom{N-1}{i} \left(\frac{\pi R_I^2}{A}\right)^i \left(1 - \frac{\pi R_I^2}{A}\right)^{N-1-i}$, is smaller than 0.05. For example, $R_I = 89 \text{ m}$ if $(N, D) = (200, 9)$. If there exist more than D nodes in the interference range of a node, the nodes other than D randomly selected interfering nodes are assumed to be non-interfering. This guarantees that the maximum node degree is D . In the D-regular graph model [11], the degree of each of the N nodes is set to D , i.e., each node has exactly D interfering neighbours. In order to validate our worst case analysis, we simply assume that the interference range equals the communication range. In reality, the performance is better, since the communication range is typically smaller than the interference range. We set $BER = 10^{-5}$ and the packet length $L = 512$ bytes. For each result, we run each simulation for 300 randomly generated topologies.

Given that $(N, D) = (200, 9)$, we have $k = 1$, $p = 17$, and thus $D + 1 = 10 \leq q \leq p = 17$. We investigate the optimal average network throughput of our algorithm with M varying from one to seven. We obtain the optimal number of subframes that maximizes the average network throughput and guarantees $P_f \leq 0.05$ and $M \leq q - kD$ according to the discussion in Section III. In Fig. 2(a), we can observe that the average network throughput of our algorithm increases with increasing number of encoded packets. This is because our algorithm can broadcast successfully multiple encoded packets rather than one packet within each frame time. Our proposed algorithm dramatically outperforms other algorithms when $M \geq 2$. When $M = 1$, the throughput of our proposed algorithm is reduced to that of MGD, and less than those

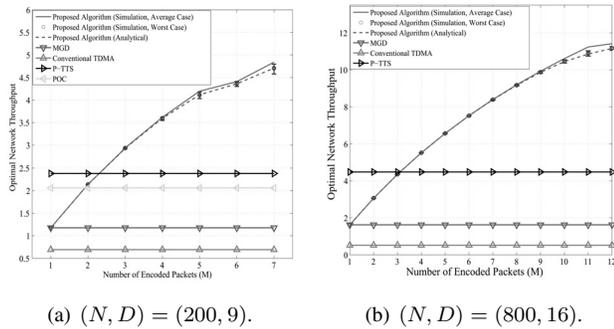


Fig. 2. The effect of M on the optimal network throughput.

of P-TTS and POC. However, the packet failure probability of P-TTS is over 0.19, which is much larger than that of our algorithm. The cardinality of the codewords of POC algorithm is too small to assign a unique codeword for each of the 200 nodes.¹ Besides, the performance is worse than that of P-TTS. Thus, we do not include the POC algorithm in the following simulations. Similar results for the case of $(N, D) = (800, 16)$ are shown in Fig. 2(b) as well.

Given that $(N, D) = (200, 9)$, we set $q = p = 17$, and M is set to achieve the optimal average network throughput such that $P_f \leq 0.05$ and $M \leq q - kD$. As observed in Fig. 3, our algorithm performs best under different BERs varying from 10^{-6} to 10^{-4} . When $BER = 10^{-6}$, we choose $M = 8$ to achieve the best throughput, and $M = 3$ to provide reliability when $BER = 10^{-4}$. This shows that M governs the tradeoff between the efficiency and the reliability.

It is shown in Figs. 2-3 that the simulation results of worst performance match our analytical results closely. The slight difference between simulations of average performance and analytical results is due to the number of neighbours for any node being not larger than D . As stated in (9), the average network throughput depends mainly on the packet failure probability. A small M results in a very small packet failure probability. When the packet failure probability is small, the average network throughput obtained by simulation would be close to that obtained analytically. The difference is indistinguishable in the figure. In order to make the figures clear, we only show the confidence interval of the worst performance.

V. CONCLUSION

In this paper, we propose an approach to employ erasure coding and topology-transparent scheduling in the MAC layer as an efficient and reliable solution for broadcast over error-prone channels in mobile ad hoc networks. We investigate analytically the performance of our proposed algorithm in terms of the average network throughput and the packet failure

¹We use an $(91, 18, 10)$ code of length 91, Hamming distance 18, and weight 10 for $(N, D) = (200, 9)$, which is the code with the largest known cardinality. However, the cardinality of $(91, 18, 10)$ is 91, which is much smaller than the number of network nodes. Interested readers are referred to [1, 9] and the references therein. The results in Fig. 3(a) can be achieved under the assumption that the reuse method of the codewords [5] introduces no overhead.

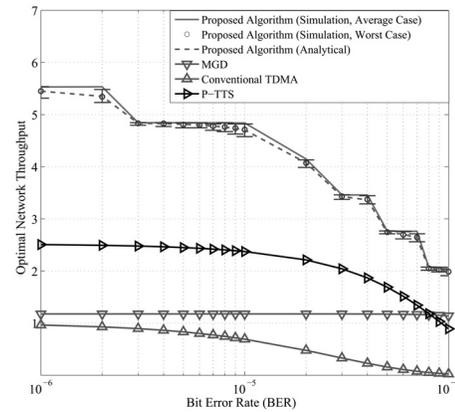


Fig. 3. The effect of bit error rate on the optimal network throughput.

probability, and compare it with existing algorithms. Our simulation results show that the proposed algorithm outperforms other existing algorithms under study.

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