1. The general problem

2. The wave vector

3. Solving the Fresnel Coefficients for Perpendicular & Parallel Polarizations using Boundary Condition

4. Some special cases

1 The general problem

In time domain

\[
\begin{align*}
E^I(r, t) &= E^I_0 \cos(\omega t - k^I \cdot r) \hat{k}_E \\
E^R(r, t) &= E^R_0 \cos(\omega t - k^R \cdot r) \hat{k}_R \\
E^T(r, t) &= E^T_0 \cos(\omega t - k^T \cdot r) \hat{k}_E \\
H^I(r, t) &= H^I_0 \cos(\omega t - k^I \cdot r) \hat{k}_H \\
H^R(r, t) &= H^R_0 \cos(\omega t - k^R \cdot r) \hat{k}_R \\
H^T(r, t) &= H^T_0 \cos(\omega t - k^T \cdot r) \hat{k}_H
\end{align*}
\]

Simplified in phasor domain

\[
\begin{align*}
E^I(r) &= E^I_0 \exp(-j k^I \cdot r) \hat{k}_E \\
E^R(r) &= E^R_0 \exp(-j k^R \cdot r) \hat{k}_R \\
E^T(r) &= E^T_0 \exp(-j k^T \cdot r) \hat{k}_E \\
H^I(r) &= H^I_0 \exp(-j k^I \cdot r) \hat{k}_H \\
H^R(r) &= H^R_0 \exp(-j k^R \cdot r) \hat{k}_R \\
H^T(r) &= H^T_0 \exp(-j k^T \cdot r) \hat{k}_H
\end{align*}
\]
The problem is generalized as follows

Given the following information

\[
\begin{align*}
\text{Incident wave information:} & \quad \begin{cases} 
E_i^I : \text{magnitude of incident wave} \\
\hat{k} : \text{direction of incident wave} \\
\omega : \text{frequency of the incident wave} \\
\end{cases} \\
\text{Material medium information:} & \quad \begin{cases} 
\varepsilon : \text{permeativity} \\
\mu : \text{permeability} \\
\sigma : \text{conductivity} \\
\end{cases}
\end{align*}
\]

To find out the following unknowns

\[
\begin{cases} 
\text{Magnitude of reflected, transmitted wave} \\
\text{Direction of reflected, transmitted wave} \\
\end{cases}
\]

2 The Wave vector \( \hat{k} \)

\[
\begin{align*}
\hat{k}_I & : \text{the incident wave vector} \\
\hat{k}_R & : \text{the reflected wave vector} \\
\hat{k}_T & : \text{the transmitted wave vector}
\end{align*}
\]

Recall, any vector \( \hat{V} = \hat{V} |V| = \hat{V} V \), thus

\[
\begin{align*}
k^I & : \text{the magnitude of } \hat{k}^I \\
k^R & : \text{the magnitude of } \hat{k}^R \\
k^T & : \text{the magnitude of } \hat{k}^T
\end{align*}
\]

Furthermore

\[
\begin{align*}
k_1 & : \text{the magnitude of wave vector in media 1} \\
k_2 & : \text{the magnitude of wave vector in media 2}
\end{align*}
\]

where \( k_i = \omega \sqrt{\mu_i \varepsilon_i} \)

Thus

\[
k^I = k^R = k_1 = \omega \sqrt{\mu_1 \varepsilon_1} \quad k^T = k_2 = \omega \sqrt{\mu_2 \varepsilon_2}
\]

And recall, vector can be decompose into it’s basis vector

\[
\begin{align*}
\hat{k}_I = k_{I_x} \hat{x} + k_{I_z} \hat{z} = k^I \sin \theta_i \hat{x} + k^I \cos \theta_i \hat{z} \\
\hat{k}_R = k_{R_x} \hat{x} - k_{R_z} \hat{z} = k^R \sin \theta_r \hat{x} - k^R \cos \theta_r \hat{z} \\
\hat{k}_T = k_{T_x} \hat{x} + k_{T_z} \hat{z} = k^T \sin \theta_T \hat{x} + k^T \cos \theta_T \hat{z}
\end{align*}
\]
Thus

\[
\begin{align*}
\hat{k}^I &= \frac{k^I}{|k^I|} = \frac{k_{Ix}\hat{x} + k_{Iz}\hat{z}}{k^I} = \sin \theta_i \hat{x} + \cos \theta_i \hat{z} \\
\hat{k}^R &= \frac{k^R}{|k^R|} = \frac{k_{Rx}\hat{x} - k_{Rz}\hat{z}}{k^R} = \sin \theta_r \hat{x} - \cos \theta_r \hat{z} \\
\hat{k}^T &= \frac{k^T}{|k^T|} = \frac{k_{Tx}\hat{x} + k_{Tz}\hat{z}}{k^T} = \sin \theta_T \hat{x} + \cos \theta_T \hat{z}
\end{align*}
\]

And

\[
\begin{align*}
\hat{k}^I \cdot \mathbf{r} &= k_{Ix}x + k_{Iz}z = k^I (\sin \theta_i x + \cos \theta_i z) \\
\hat{k}^R \cdot \mathbf{r} &= k_{Rx}x - k_{Rz}z = k^R (\sin \theta_r x - \cos \theta_r z) \\
\hat{k}^T \cdot \mathbf{r} &= k_{Tx}x + k_{Tz}z = k^T (\sin \theta_T x + \cos \theta_T z)
\end{align*}
\]

3 Perpendicular Polarizations

- Expression of \(E^{I,R,T}\) and \(H^{I,R,T}\) and their direction term, propagation term
- Fresnel Relationship and Fresnel Coefficient
- Solving Fresnel Equations
- Special case: Normal incident and only one media

By looking at the diagram

\[
\begin{align*}
\begin{cases}
\text{Perpendicular case: all direction of } E \text{ are } +\hat{y}, \text{ so } \hat{k}^{I,R,T}_E = +\hat{y} \\
\text{Parallel case: direction of } H \text{ are } \pm \hat{y}, \text{ so } \hat{k}^{I,T}_H = +\hat{y}, \hat{k}^R_H = -\hat{y}
\end{cases}
\end{align*}
\]

Thus

\[
\begin{align*}
| \begin{cases}
E^I (\mathbf{r}) = E^I_0 \exp j (-k_{Ix}x - k_{Iz}z) \hat{y} & H^I (\mathbf{r}) = H^I_0 \exp j (-k_{Ix}\hat{x} - k_{Iz}\hat{z}) (\hat{k}^I_H) \\
E^R (\mathbf{r}) = E^R_0 \exp j (-k_{Rx}x + k_{Rz}z) \hat{y} & H^R (\mathbf{r}) = H^R_0 \exp j (-k_{Rx}\hat{x} + k_{Rz}\hat{z}) (\hat{k}^R_H) \\
E^T (\mathbf{r}) = E^T_0 \exp j (-k_{Tx}x - k_{Tz}z) \hat{y} & H^T (\mathbf{r}) = H^T_0 \exp j (-k_{Tx}\hat{x} - k_{Tz}\hat{z}) (\hat{k}^T_H)
\end{cases} | \\
\begin{cases}
E^I (\mathbf{r}) = E^I_0 \exp j (-k_{Ix}x - k_{Iz}z) (\hat{k}^I_E) & H^I (\mathbf{r}) = H^I_0 \exp j (-k_{Ix}\hat{x} - k_{Iz}\hat{z}) (\hat{y}) \\
E^R (\mathbf{r}) = E^R_0 \exp j (-k_{Rx}x + k_{Rz}z) (\hat{k}^R_E) & H^R (\mathbf{r}) = H^R_0 \exp j (-k_{Rx}\hat{x} + k_{Rz}\hat{z}) (-\hat{y}) \\
E^T (\mathbf{r}) = E^T_0 \exp j (-k_{Tx}x - k_{Tz}z) (\hat{k}^T_E) & H^T (\mathbf{r}) = H^T_0 \exp j (-k_{Tx}\hat{x} - k_{Tz}\hat{z}) (\hat{y})
\end{cases}
\end{align*}
\]
Apply Fresnel Relationship and Relationship between $H_0$ and $E_0$

$$\begin{cases} E_0^R = R E_0^I \\ E_0^T = T E_0^I \end{cases}$$ and $H_0 = \frac{E_0}{\eta} \implies \begin{cases} E_0^I \\ H_0^I = \frac{E_0^I}{\eta} \\ E_0^R = R E_0^I \\ H_0^R = \frac{R E_0^I}{\eta} \\ E_0^T = T E_0^I \\ H_0^T = \frac{R E_0^I}{\eta_2} \end{cases}$

Furthermore, consider the following diagram to find direction of $H$ ($\perp$ case) and $E$ ($\parallel$ case)

$$\begin{cases} \hat{k}_H^I = -\cos \theta_i \hat{x} + \sin \theta_i \hat{z} \\ \hat{k}_R^H = \cos \theta_r \hat{x} + \sin \theta_r \hat{z} \\ \hat{k}_T^H = -\sin \theta_r \hat{x} + \cos \theta_r \hat{z} \end{cases}$$

$$\begin{cases} \hat{k}_E^I = \cos \theta_i \hat{x} - \sin \theta_i \hat{z} \\ \hat{k}_E^R = \cos \theta_r \hat{x} + \sin \theta_r \hat{z} \\ \hat{k}_E^T = \cos \theta_T \hat{x} - \sin \theta_T \hat{z} \end{cases}$$
Therefore

\[
\begin{align*}
E^I (r) &= E^I_0 \exp j (-k_{Ix}x - k_{Iz}z) \hat{y} \quad H^I (r) = \frac{E^I_0}{\eta_1} \exp j (-k_{Ix} \hat{x} - k_{Iz} \hat{z}) (-\cos \theta_i \hat{x} + \sin \theta_i \hat{z}) \\
E^R (r) &= R_{\perp E} E^I_0 \exp j (-k_{Rx}x + k_{Rz}z) \hat{y} \quad H^R (r) = \frac{R_{\perp E} E^I_0}{\eta_1} \exp j (-k_{Rx}x + k_{Rz}z) (\cos \theta_r \hat{x} + \sin \theta_r \hat{z}) \\
E^T (r) &= T_{\perp E} E^I_0 \exp j (-k_{Tx}x - k_{Tz}z) \hat{y} \quad H^T (r) = \frac{T_{\perp E} E^I_0}{\eta_2} \exp j (-k_{Tx}x - k_{Tz}z) (-\sin \theta_T \hat{x} + \cos \theta_T \hat{z})
\end{align*}
\]

Therefore

\[
\begin{align*}
E^I (r) &= E^I_0 \exp j (-k_{Ix}x - k_{Iz}z) (\cos \theta_i \hat{x} - \sin \theta_i \hat{y}) \\
H^I (r) &= \frac{E^I_0}{\eta_1} \exp j (-k_{Ix} \hat{x} - k_{Iz} \hat{z}) (\cos \theta_i \hat{x} + \sin \theta_i \hat{y}) \\
E^R (r) &= R_{\parallel E} E^I_0 \exp j (-k_{Rx}x + k_{Rz}z) (\cos \theta_r \hat{x} + \sin \theta_r \hat{y}) \\
H^R (r) &= \frac{R_{\parallel E} E^I_0}{\eta_1} \exp j (-k_{Rx}x + k_{Rz}z) (-\sin \theta_r \hat{x} + \cos \theta_r \hat{y}) \\
E^T (r) &= T_{\parallel E} E^I_0 \exp j (-k_{Tx}x - k_{Tz}z) (\cos \theta_T \hat{x} - \sin \theta_T \hat{y}) \\
H^T (r) &= \frac{T_{\parallel E} E^I_0}{\eta_2} \exp j (-k_{Tx}x - k_{Tz}z) (-\sin \theta_T \hat{x} + \cos \theta_T \hat{y})
\end{align*}
\]

Now the remaining works to do is to find those Fresnel Coefficient : $R$ and $T$

Those value can be found by using boundary condition : basically, the term $e^{-jkz}$ disappear when putting $z = 0$

Therefore

\[
\begin{align*}
E^I (x) &= E^I_0 \exp j (-k_{Ix}x) \hat{y} \quad H^I (x) = \frac{E^I_0}{\eta_1} \exp j (-k_{Ix} \hat{x}) (-\cos \theta_i \hat{x} + \sin \theta_i \hat{y}) \\
E^R (x) &= R_{\perp E} E^I_0 \exp j (-k_{Rx}x) \hat{y} \quad H^R (x) = \frac{R_{\perp E} E^I_0}{\eta_1} \exp j (-k_{Rx}x) (\cos \theta_r \hat{x} + \sin \theta_r \hat{y}) \\
E^T (x) &= T_{\perp E} E^I_0 \exp j (-k_{Tx}x) \hat{y} \quad H^T (x) = \frac{T_{\perp E} E^I_0}{\eta_2} \exp j (-k_{Tx}x) (-\sin \theta_T \hat{x} + \cos \theta_T \hat{y})
\end{align*}
\]

Apply boundary condition

"Tangential component of $E$ fields are equal"

"Tangential component of $H$ fields are equal"

Note , Generally $H_{t1} - H_{t2} = J_s$ , but this is the case in equilibrium

For the boundary at $z$ , the tangential directions are $x$ and $y$

That is

\[
\begin{align*}
\text{\textbf{\perp}} \quad \text{y-component of } (E^I + E^R)_{z=0} &= \text{y-component of } (E^T)_{z=0} \\
\text{\textbf{\perp}} \quad \text{x-component of } (H^I + H^R)_{z=0} &= \text{x-component of } (H^T)_{z=0}
\end{align*}
\]
\[
\begin{align*}
&\{ \text{x-component of } (E^I + E^R) \text{ at } z=0 = \text{x-component of } (E^T) \text{ at } z=0 \\
&\{ \text{y-component of } (H^I + H^R) \text{ at } z=0 = \text{xy-component of } (H^T) \text{ at } z=0 \\
\end{align*}
\]

Which is

\[
\begin{align*}
&\left\{ \begin{array}{l}
E^I(x) = E^I_0 \exp j\left(-k_{Ix}x\right) \\
\quad + \\
E^R(x) = R_{\perp E}E^I_0 \exp j\left(-k_{Rx}x\right) \\
\quad + \\
E^T(x) = T_{\perp E}E^I_0 \exp j\left(-k_{Tx}x\right)
\end{array} \right. \\
&\left\{ \begin{array}{l}
H^I(x) = \frac{E^I_0}{\eta_1} \exp j\left(-k_{Ix}\hat{x}\right) (-\cos \theta_i) \\
\quad + \\
H^R(x) = \frac{R_{\perp E}E^I_0}{\eta_1} \exp j\left(-k_{Rx}\hat{x}\right) (\cos \theta_r) \\
\quad + \\
H^T(x) = \frac{T_{\perp E}E^I_0}{\eta_2} \exp j\left(-k_{Tx}\hat{x}\right) (-\cos \theta_T)
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
&\left\{ \begin{array}{l}
E^I(x) = E^I_0 \exp j\left(-k_{Ix}x\right) (\cos \theta_i) \\
\quad + \\
R_{\perp E}E^I_0 \exp j\left(-k_{Rx}x\right) (\cos \theta_r) \\
\quad + \\
T_{\perp E}E^I_0 \exp j\left(-k_{Tx}x\right) (\cos \theta_T)
\end{array} \right. \\
&\left\{ \begin{array}{l}
H^I(x) = \frac{E^I_0}{\eta_1} \exp j\left(-k_{Ix}\hat{x}\right) \\
\quad + \\
R_{\perp E}E^I_0 \exp j\left(-k_{Rx}\hat{x}\right) (-1) \\
\quad + \\
T_{\perp E}E^I_0 \exp j\left(-k_{Tx}\hat{x}\right)
\end{array} \right.
\end{align*}
\]

i.e.

\[
\begin{align*}
&\left\{ \begin{array}{l}
E^I_0 \exp j\left(-k_{Ix}x\right) \\
\quad + \\
R_{\perp E}E^I_0 \exp j\left(-k_{Rx}x\right) (\cos \theta_r) \\
\quad + \\
T_{\perp E}E^I_0 \exp j\left(-k_{Tx}x\right) (\cos \theta_T)
\end{array} \right. \\
&\left\{ \begin{array}{l}
E^I_0 \exp j\left(-k_{Ix}x\right) (\cos \theta_i) \\
\quad + \\
R_{\perp E}E^I_0 \exp j\left(-k_{Rx}x\right) (-1) \\
\quad + \\
T_{\perp E}E^I_0 \exp j\left(-k_{Tx}x\right)
\end{array} \right.
\end{align*}
\]
Rearrange
\[
\begin{align*}
\exp j (-k_{Ix}x) + R_{\perp E} \exp j (-k_{Rx}x) &= T_{\perp E} \exp j (-k_{Tx}x) \\
\left\{ \begin{array}{c}
\frac{1}{\eta_1} \left[ \exp j (-k_{Ix}x) - \cos \theta_i \right] + R_{\perp E} \exp j (-k_{Rx}x) \cos \theta_r = T_{\perp E} \exp j (-k_{Tx}x) \cos \theta_T \\
\frac{1}{\eta_1} \left[ \exp j (-k_{Ix}x) - R_{\parallel E} \exp j (-k_{Rx}x) \right] = T_{\parallel E} \exp j (-k_{Tx}x)
\end{array} \right.
\end{align*}
\]

Since the boundary condition should hold for all \(x\), then
\[k_{Ix} = k_{Rx} = k_{Tx} \triangleq k_x \quad (1)\]
i.e. They are Law of reflection and refraction
\[\begin{align*}
k_{Ix} = k_{Rx} &\iff k_1 \sin \theta_i = k_1 \sin \theta_r \iff \theta_i = \theta_r \\
k_{Ix} = k_{Tx} &\iff k_1 \sin \theta_i = k_2 \sin \theta_2
\end{align*}\]
Then since
\[\begin{align*}
(k^I)^2 &= k_{Ix}^2 + k_{Iz}^2 = k_1^2 = \omega^2 \mu_1 \varepsilon_1 \\
(k^R)^2 &= k_{Rx}^2 + k_{Rz}^2 = k_1^2 = \omega^2 \mu_1 \varepsilon_1 \\
(k^T)^2 &= k_{Tx}^2 + k_{Tz}^2 = k_2^2 = \omega^2 \mu_2 \varepsilon_2
\end{align*}\]
Therefore
\[\begin{align*}
k_{Iz} &= \sqrt{k_{Iz}^2 - k_{Ix}^2} = \sqrt{k_1^2 - k_2^2} = \sqrt{k_1^2 - k_{Rx}^2} = k_{Rz} \\
k_{Tz} &= \sqrt{k_{Tz}^2 - k_{Tx}^2} = \sqrt{k_2^2 - k_{xz}^2}
\end{align*}\]
Then
\[k_{Iz} = k_{Rz} \triangleq k_z \quad (2)\]
Therefore the Boundary Condition Equation can be simplified (factor out common term \(e^{-jk_zx}\))
\[\begin{align*}
\left\{ \begin{array}{c}
1 + R_{\perp E} = T_{\perp E} \\
\frac{1}{\eta_1} \left[ - \cos \theta_i + R_{\perp E} \cos \theta_r \right] = \frac{T_{\perp E}}{\eta_2} (- \cos \theta_T)
\end{array} \right. \\
\left\{ \begin{array}{c}
\cos \theta_i + R_{\parallel E} \cos \theta_r = T_{\parallel E} \cos \theta_T \\
\frac{1}{\eta_1} \left[ 1 - R_{\parallel E} \right] = \frac{T_{\parallel E}}{\eta_2}
\end{array} \right.
\end{align*}\]
And since \(\theta_i = \theta_r\) (Law of reflection), rearrange same term together
\[\left\{ \begin{array}{c}
1 + R_{\perp E} = T_{\perp E} \\
\frac{1 - R_{\perp E}}{\eta_1 \cos \theta_i} = T_{\perp E} \frac{\eta_1}{\eta_2} \cos \theta_i
\end{array} \right. \\
\left\{ \begin{array}{c}
1 + R_{\parallel E} = \frac{\cos \theta_i}{\cos \theta_T} T_{\parallel E} \\
1 - R_{\parallel E} = \frac{\eta_1}{\eta_2} T_{\parallel E}
\end{array} \right.\]
Solving, the Fresnel Coefficients are
\[
\begin{align*}
T_{\perp E} &= \frac{2\eta_2 \cos \theta_i}{\eta_1 \cos \theta_T + \eta_2 \cos \theta_i} \\
R_{\perp E} &= \frac{\eta_2 - \eta_1}{\eta_1 \cos \theta_T + \eta_2 \cos \theta_i} \\
T_{\parallel E} &= \frac{2\eta_2 \cos \theta_i}{\eta_1 \cos \theta_T + \eta_2 \cos \theta_i} \\
R_{\parallel E} &= \frac{\eta_2 \cos \theta_T - \eta_1 \cos \theta_i}{\eta_1 \cos \theta_T + \eta_2 \cos \theta_i}
\end{align*}
\]

4 Special cases

4.1 Normal incidence

Normal incident: When \( i = 0 = T \)
\[
T_{\perp E} = T_{\parallel E} = \frac{2\eta_2}{\eta_1 + \eta_2} \\
R_{\perp E} = R_{\parallel E} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}
\]

There are the same

Further, when \( \eta_2 = \eta_1 \) (only one media)
\[
T_{\perp E} = T_{\parallel E} = 1 \text{ (all transmitted)} \\
R_{\perp E} = R_{\parallel E} = 0 \text{ (No reflection)}
\]

4.2 Berewster Angle

\[
\begin{align*}
T_{\perp E} &= \frac{2\eta_2 \cos \theta_i}{\eta_1 \cos \theta_T + \eta_2 \cos \theta_i} \\
R_{\perp E} &= \frac{\eta_2 - \eta_1}{\eta_1 \cos \theta_T + \eta_2 \cos \theta_i} \\
T_{\parallel E} &= \frac{2\eta_2 \cos \theta_i}{\eta_1 \cos \theta_T + \eta_2 \cos \theta_i} \\
R_{\parallel E} &= \frac{\eta_2 \cos \theta_T - \eta_1 \cos \theta_i}{\eta_1 \cos \theta_T + \eta_2 \cos \theta_i}
\end{align*}
\]

Notice that it is possible for \( R_{\perp} = 0 \) and \( R_{\parallel} = 0 \) when
\[
\eta_2 \cos \theta_i = \eta_1 \cos \theta_T \text{ for } R_{\perp} \\
\eta_2 \cos \theta_T = \eta_1 \cos \theta_i \text{ for } R_{\parallel}
\]

Do the following (1. turn cos into sin term, 2. Apply Law of refraction to \( \sin \theta_T = \frac{k_1}{k_2} \sin \theta_i \))
\[
\eta_2^2 \left( 1 - \sin^2 \theta_i \right) = \eta_1^2 \left( 1 - \frac{k_1^2}{k_2^2} \sin^2 \theta_i \right) \text{ for } R_{\perp}
\]
\[
\eta_2^2 \left( 1 - \frac{k_1^2}{k_2^2} \sin^2 \theta_i \right) = \eta_1^2 \left( 1 - \sin^2 \theta \right) \text{ for } R_{\parallel}
\]

i.e.
\[
\sin \theta_i = \frac{\eta_1^2 - \eta_2^2}{\eta_1 \sqrt{\frac{k_1^2}{k_2^2} - \eta_2^2}} \text{ for } R_{\perp}
\]
\[
\sin \theta_i = \frac{\eta_1^2 - \eta_2^2}{\eta_1 \sqrt{\frac{k_1^2}{k_2^2} - \eta_2^2}} \text{ for } R_{\parallel}
\]
Expand $k, \eta$

For $R_\perp$

$$\sin \theta_i = \sqrt{\frac{\mu_1 - \mu_2}{\varepsilon_1 \mu_1 \varepsilon_1 - \mu_2} \frac{\varepsilon_2}{\mu_2 \varepsilon_2}} = \sqrt{\frac{\mu_1 - \mu_2}{\varepsilon_1 \mu_1 \varepsilon_1 - \mu_2} \frac{\varepsilon_2}{\mu_2 \varepsilon_2} \times \frac{\varepsilon_2 - \mu_2}{\mu_1}}$$

$$\sin \theta_i = \sqrt{\frac{\varepsilon_2 - \mu_2}{\mu_1}}$$

For $R_{||}$

$$\sin \theta_i = \sqrt{\frac{\mu_1 - \mu_2}{\varepsilon_1 \mu_1 \varepsilon_1 - \mu_2} \frac{\varepsilon_2}{\mu_2 \varepsilon_2}} = \sqrt{\frac{\mu_1 - \mu_2}{\varepsilon_1 \mu_1 \varepsilon_1 - \mu_2} \frac{\varepsilon_2}{\mu_2 \varepsilon_2} \times \frac{\varepsilon_2 - \mu_2}{\mu_1}}$$

$$\sin \theta_i = \sqrt{\frac{\varepsilon_2 - \mu_2}{\mu_1}}$$

Therefore, denote this angle as Brewster Angle $\theta_B$

$$\sin \theta_{B,\perp} = \sqrt{\frac{\varepsilon_2 - \mu_2}{\mu_1}} \quad \sin \theta_{B,||} = \sqrt{\frac{\varepsilon_2 - \mu_2}{\mu_1}}$$

A special case, when $\mu_1 = \mu_2 = \mu_0$ (non-magnetic material)

$$\sin \theta_{B,\perp} = \infty (impossible) \quad \sin \theta_{B,||} = \sqrt{\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 \varepsilon_2}} = \sqrt{\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 \varepsilon_2}}$$

By $1 + \tan^2 \theta = \sec^2 \theta$

$$\tan \theta = \sqrt{\sec^2 \theta - 1} = \sqrt{\frac{1}{\cos^2 \theta} - 1} = \sqrt{\frac{1}{1 - \sin^2 \theta} - 1}$$

Plug in $\sin \theta_{B,||}$

$$\tan \theta_B = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

i.e. It is impossible for $R_\perp = 0$ for non-magnetic material, but possible for $R_{||}$