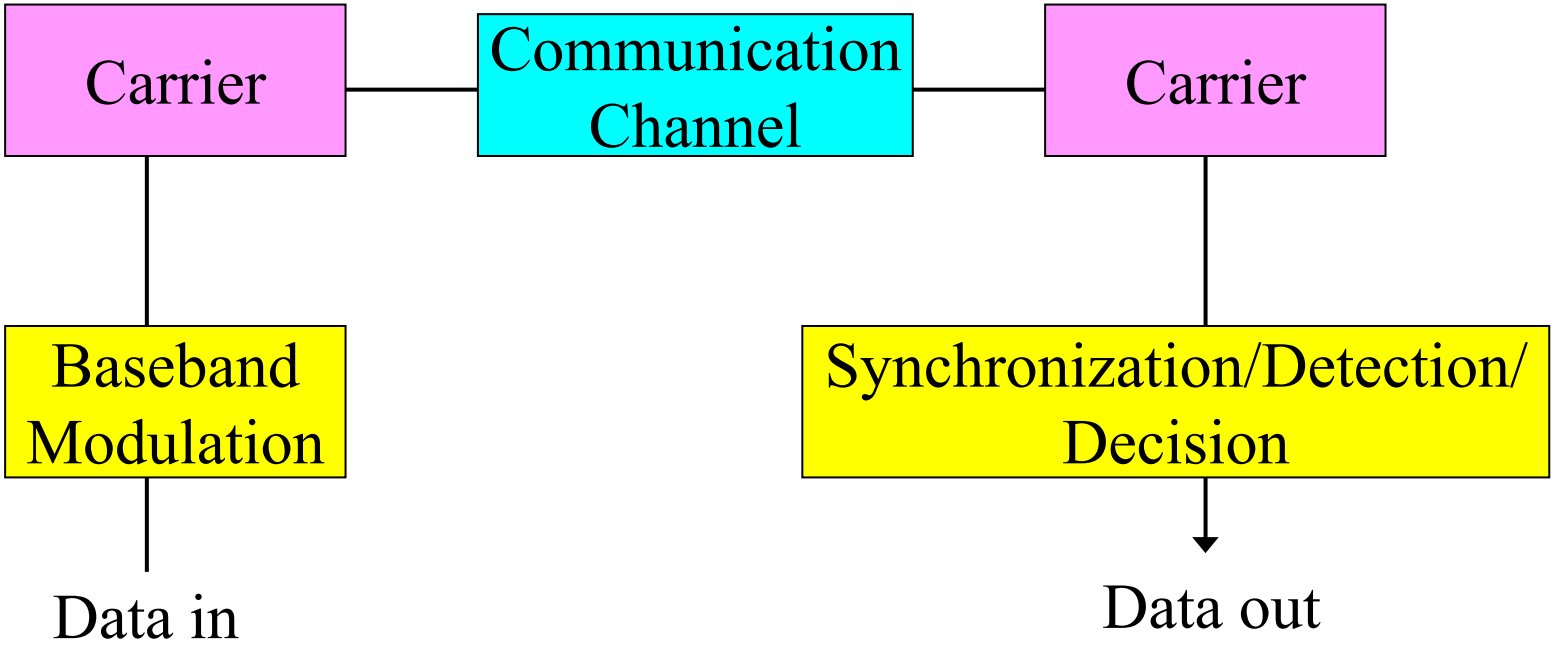
# Part 3. Digital Modulation

# What's Modulation & Demodulation?

## ➢ Digital modulation and demodulation:

- Modulation (demodulation) maps (retrieves) the digital information into (from) an analog waveform *appropriate for transmission over the channel*.
- Generally involve translating (recovering) the baseband digital information to (from) a bandpass analog signal at a *carrier frequency* that is very high compared to the baseband frequency.
- Examples: ASK, FSK, QPSK, 16QAM

# Modulation & Demodulation



# Why Carrier?

- *Effective radiation of electromagnetic waves* requires antenna dimensions comparable with the signal's wavelength:
  - Antenna for 3 kHz carrier would be ~100 km long
  - Antenna for 3 GHz carrier is 10 cm long
- *Frequency division multiplexing*
  - Shifting the baseband signals to different carrier frequencies
  - Sharing the communication channel resources

# Geometric Representation (1)

- *Digital modulation involves choosing a particular analog signal waveform $s_i(t)$ from a finite set S of possible signal waveforms based on the information bits applied to the modulator.*
- For *binary modulation* schemes, a binary information bit is mapped directly to a signal and S contains only 2 signals, representing 0 and 1.
- For *M-ary modulations*, S contains more than 2 signals and each represents more than a single bit of information. With a signal set of size M, it is possible to transmit up to $log_2 M$ bits per signal.

# Geometric Representation (2)

- Any element of set S, $S=\{s_1(t), s_2(t), \ldots, s_M(t)\}$, can be represented as *a point in a vector space* whose coordinates are basis signals $\phi_j(t)$, $j=1,2,\ldots,N$, such that

$$\int_{-\infty}^{\infty} \phi_i(t)\phi_j(t)dt = 0, i \neq j; \ (\rightarrow \text{orthogonal})$$

$$E = \int_{-\infty}^{\infty} \left[\phi_i(t)\right]^2 dt = 1; (\rightarrow \text{normalization})$$

$s_i(t)$ can be represented as a linear combination of the basis signals.

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \ i = 1,2,\cdots,M$$

# Example: BPSK Geometric Representation

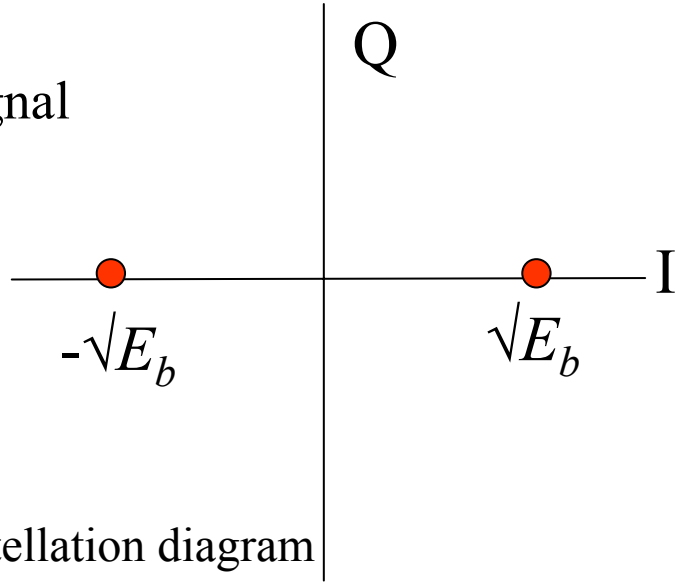
$$S_{BPSK} = \left\{ \begin{array}{l} \left[ s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + \theta_c\right) \right], \\ \left[ s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + \theta_c\right); \right] \end{array} \right\}; \quad 0 \le t \le T_b$$

$E_b =$ energy per bit; $T_b =$ bit period

For this signal set, there is a single basis signal

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos\left(2\pi f_c t + \theta_c\right); \quad 0 \le t \le T_b$$

$$S_{BPSK} = \left\{ \left[ \sqrt{E_b}\phi_1(t) \right], \left[ -\sqrt{E_b}\phi_1(t) \right] \right\}$$




Constellation diagram

# Constellation Diagram

- *A graphical representation of the complex envelope of each possible signal*
- The x-axis represents the in-phase component and the y-axis represents the quadrature component of the complex envelope
- The distance between signals on a constellation diagram relates to *how different the modulation waveforms are* and *how well a receiver can differentiate between them* when random noise is present.

# Performance Measures (1)

- *Two key performance measures of a modulation scheme are power efficiency and bandwidth efficiency*
- ***Power efficiency*** is a measure of how favorably the tradeoff between fidelity and signal power is made, and is expressed as *the ratio of the signal energy per bit ($E_b$) to the noise PSD ($N_0$) required to achieve a given probability of error (say $10^{-5}$):*

$$\eta_p = \frac{E_b}{N_0}$$

Small $\eta_p$ is preferred

# Performance Measures (2)

- ***Bandwidth efficiency*** describes the ability of a modulation scheme to accommodate data within a limited bandwidth, In general, it is defined as *the ratio of the data bit rate R to the required RF bandwidth B*:

$$\eta_B = \frac{R}{B}(\text{bps/Hz}) \qquad \text{Large } \eta_B \text{ is preferred}$$

- ***Channel capacity*** gives an upper bound of achievable bandwidth efficiency:

$$\eta_{B\max} = \frac{C}{B} = \log_2(1+\frac{S}{N})$$

# Modulation Schemes Classification

- ***Linear modulation:*** the amplitude of the transmitted signal, *s(t),* varies linearly with the modulating digital signal, *m(t)*.
  - *Bandwidth efficient but power inefficient*
  - Example: ASK, QPSK
- ***Nonlinear modulation:*** the amplitude of the transmitted signal, *s(t),* does not vary linearly with the modulating digital signal
  - *Power efficient but bandwidth inefficient*
  - Example: FSK, constant envelope modulation

# Demodulation (1)

- ***Coherent demodulation:*** requires a replica carrier wave of the same frequency and phase at the receiver
  - ***The received signal and replica carrier are cross-correlated***
  - Also known as *synchronous demodulation*
  - Carrier recovery methods
    - Using PLL to recover the carrier phase and frequency from the transmitted pilot carrier signal,
    - Recovering the carrier from the received signals using costas loop
  - Applicable to: PSK, FSK, ASK, etc.

# Demodulation (2)

➢ ***Example of BPSK coherent demodulator:***

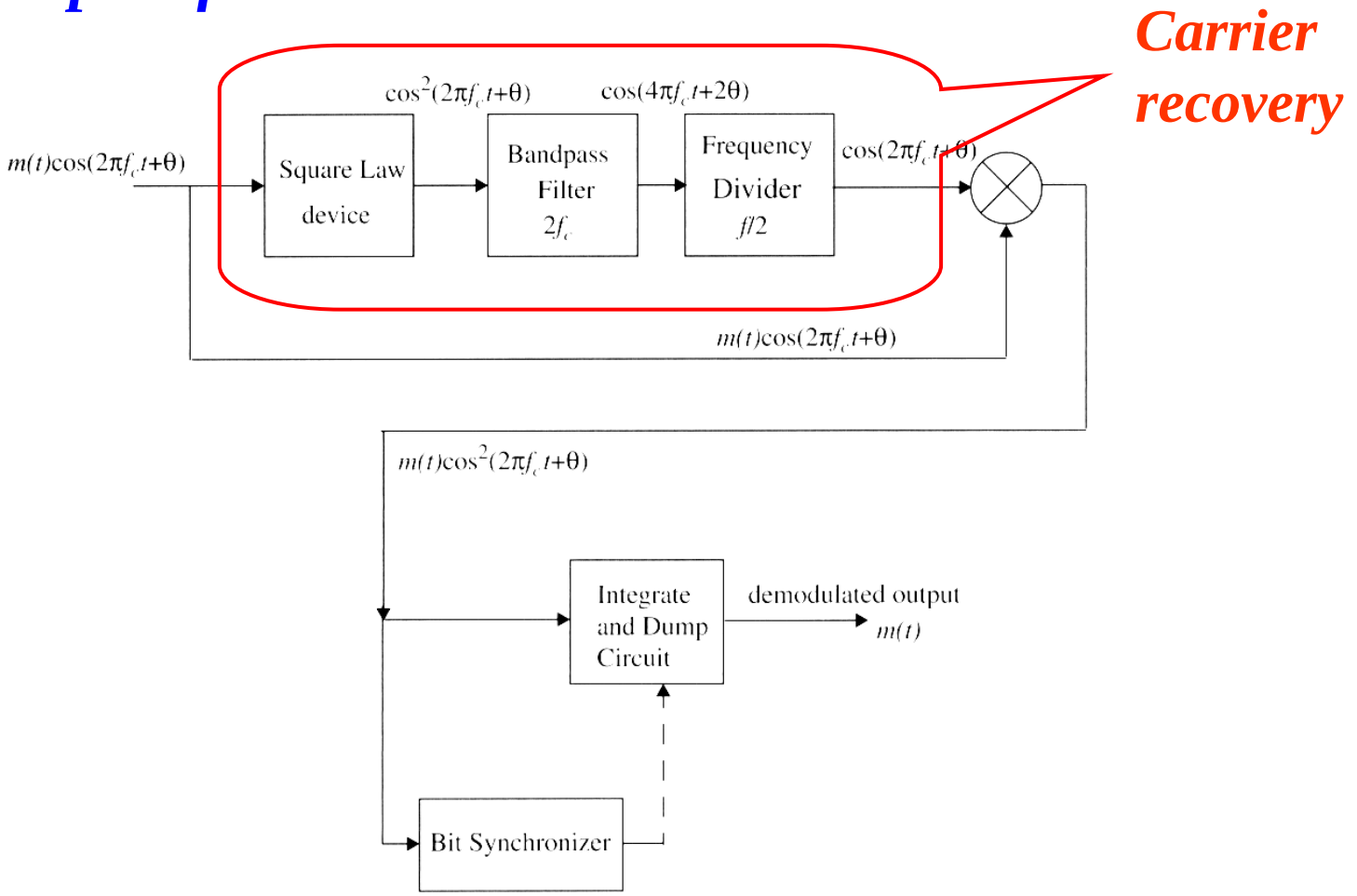



**Figure 6.23** BPSK receiver with carrier recovery circuits.

# Demodulation (3)

- ***Non-coherent demodulation:*** does not require a reference carrier wave
  - *It is less complex than coherent demodulation (easier to implement), but has worse performance*
  - Applicable to: DPSK, FSK, etc.
  - Example: FSK non-coherent demodulator

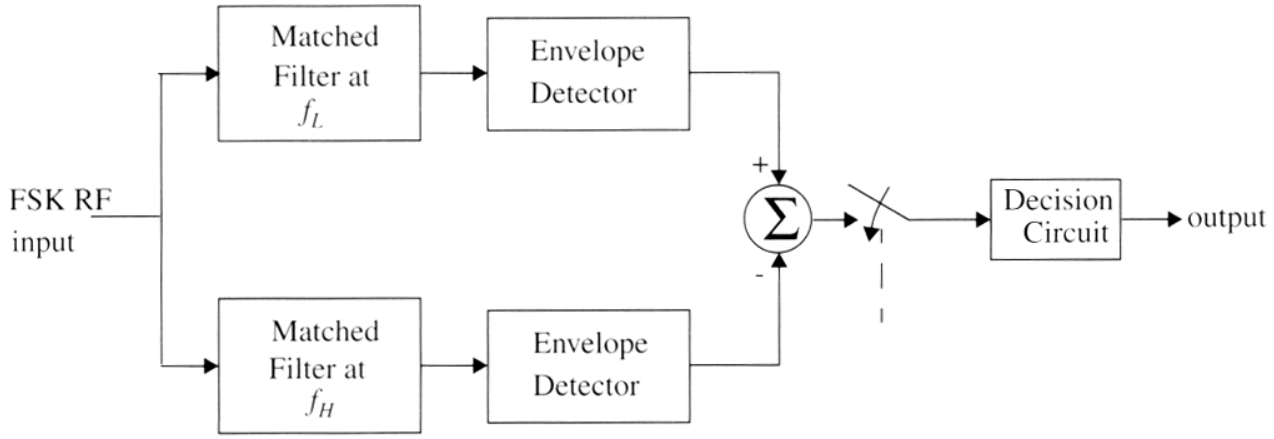



**Figure 6.37** Block diagram of noncoherent FSK receiver.

# Part 3.1 Basic Modulation

# Modulation Process

$$f = f\left(a_1, a_2, a_3, \ldots a_n, t\right) \ (\rightarrow \text{carrier})$$

$a_1, a_2, a_3, \ldots a_n$ ($\rightarrow$ modulation parameters)

$t$ ($\rightarrow$ time)

- Modulation implies varying one or more characteristics (modulation parameters $a_1$, $a_2$, … $a_n$) of a carrier $f$ in accordance with the information-bearing (modulating) baseband signal.
- Sinusoidal waves, pulse train, square wave, etc. can be used as carriers

# Continuous Carrier

***Carrier:*** $A\cos[\omega t + \varphi]$

- A = const
- ω = const
- φ = const

❑ ***Amplitude modulation (AM)***

- A = A(t) – carries information
- ω = const
- φ = const

❑ ***Frequency modulation (FM)***

- A = const
- ω = ω(t)– carries information
- φ = const

❑ ***Phase modulation (PM)***

- A = const
- ω = const
- φ = φ(t) – carries information

*Modulation methods: using* ***amplitude, phase or frequency*** *of the carrier.*

# Basic Modulation

- Modulation involves operations on one or more of the three characteristics of a carrier signal: ***amplitude, frequency and phase***.
- The three basic modulation methods are:
  - *Amplitude Shift Keying (ASK)*
  - *Phase Shift Keying (PSK)*
  - *Frequency Shift Keying (FSK)*
- These could be applied to binary or M-ary signals.
- There are other variants as well.

# Amplitude Shift Keying (ASK) (1)

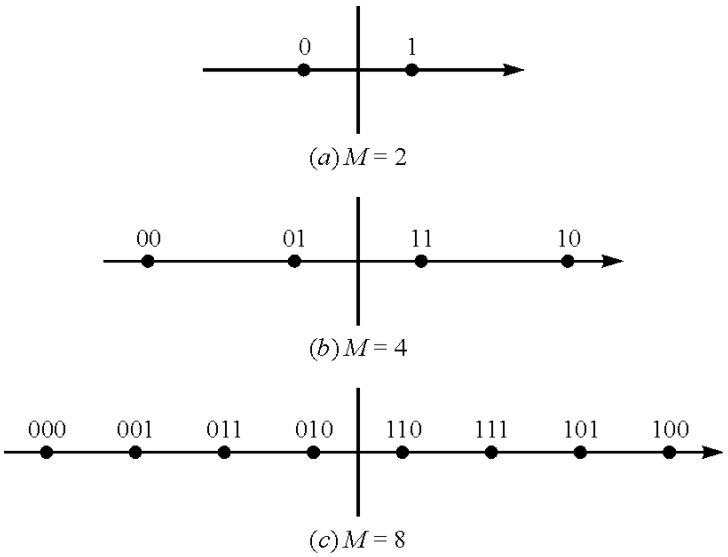
- The modulation signal set is

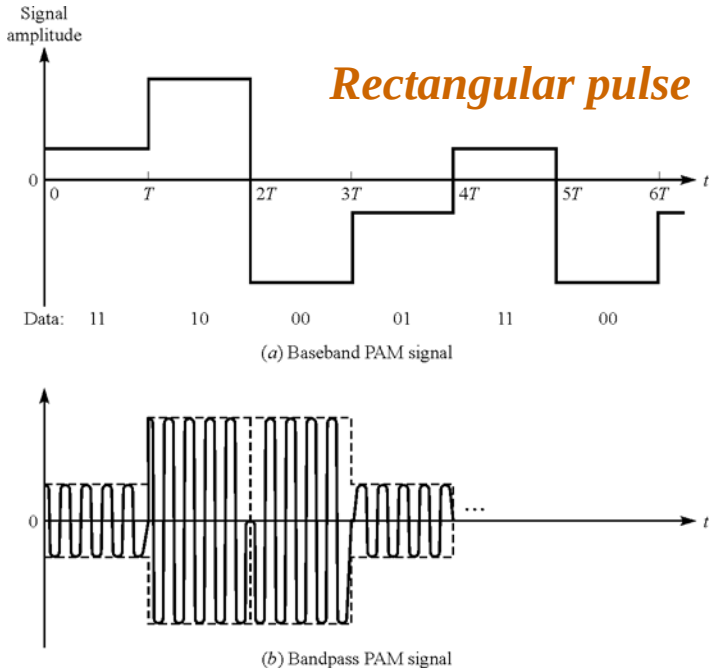
$$s_i(t) = A_i g(t)\cos\left[2\pi f_c t + \theta_c\right], \quad \begin{matrix} i = 1, 2, \cdots, M \\ 0 \le t \le T_s \end{matrix}$$

  - $T_s$ is the symbol period
  - $f_c$ is the carrier frequency, $\theta_c$ is the carrier initial phase
  - *g(t) is a real-value signal pulse whose shape influences the spectrum of the transmitted signal;* →***Pulse shaping***
    - Used to simultaneously reduce the intersymbol effects and the spectral width of a modulated digital signal
    - Example: rectangular pulse, Nyquist pulse shaping, raised cosine pulse shaping, Gaussian pulse shaping, etc.
  - $A_i=(2i-1-M)d$, each symbol represents $\log_2 M$ bits

# Amplitude Shift Keying (ASK) (2)

- The single basis signal is $\phi_1(t) = \sqrt{\frac{2}{\varepsilon_g}} g(t) \cos\left[2\pi f_c t + \theta_c\right]$
- The modulated signal: $s_i(t) = s_i \phi_1(t), \quad s_i = A_i \sqrt{\varepsilon_g / 2}$




- *ASK demonstrates poor performance, as it is heavily affected by noise, fading, and interference. It is rarely used on its own.*

# Phase Shift Keying (PSK)

- The modulation signal set is

$$s_i(t) = A_c g(t)\cos\left[2\pi f_c t + \theta_c + \varphi_i\right], \quad i = 1, 2, \cdots, M$$
$$\varphi_i = \frac{2\pi}{M}(i-1), \quad 0 \le t \le T_s$$

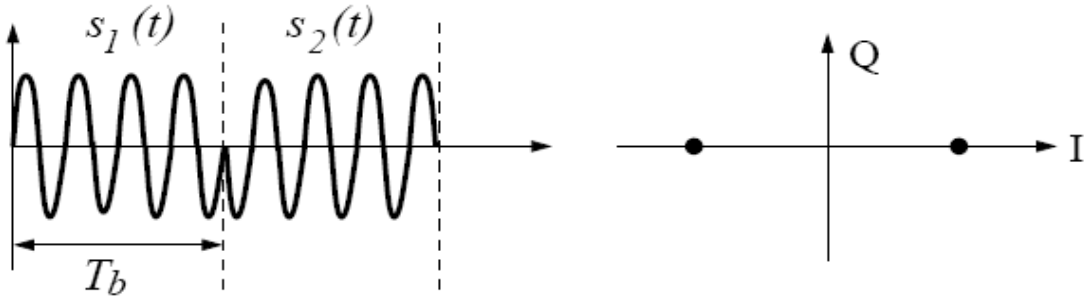
  - $A_c$ is the carrier amplitude,
  - $\varphi_i$ carries information, each symbol represents $\boldsymbol{\log_2 M}$ bits

# Binary Phase Shift Keying (BPSK)

- *M=2: minimum phase separation: 180 °*

$$s_i(t) = A_c g(t)\cos\left[2\pi f_c t + \theta_c + (i-1)\pi\right], \quad i = 1,2, \ 0 \le t \le T_b$$

- $s_1(t)$ and $s_2(t)$ represent bit 0 and bit 1,respectively
- The single basis: $\phi_1(t) = \sqrt{\frac{2}{\varepsilon_g}} g(t)\cos\left[2\pi f_c t + \theta_c\right]$
- The set : $S = \left\{ A_c\sqrt{\frac{\varepsilon_g}{2}}\phi_1(t), -A_c\sqrt{\frac{\varepsilon_g}{2}}\phi_1(t) \right\}$



# Quadrature Phase Shift Keying (QPSK)

- *M=4: symbol period $T_s=2T_b$, minimum phase separation: 90 $^o$*

$$s_i(t) = A_c g(t)\cos\left[2\pi f_c t + \theta_c + (i-1)\frac{\pi}{2}\right], \quad i = 1,2,3,4, \ 0 \le t \le T_s$$

- The basis signals:

$$\phi_1(t) = \sqrt{\frac{2}{\varepsilon_g}} g(t)\cos\left[2\pi f_c t + \theta_c\right], \quad \phi_2(t) = -\sqrt{\frac{2}{\varepsilon_g}} g(t)\sin\left[2\pi f_c t + \theta_c\right]$$

- Constellation diagram:

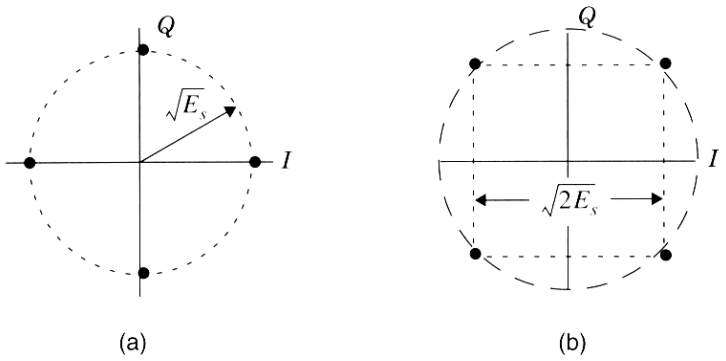



**Figure 6.26** (a) QPSK constellation where the carrier phases are 0, π/2, π, 3π/2; (b) QPSK constellation where the carrier phases are π/4, 3π/4, 5π/4, 7π/4.

# PSK: Bandwidth vs. Power Efficiency

*The system using ideal Nyquist pulse shaping is operated in AWGN channel.*

**Table 6.4** Bandwidth and Power Efficiency of M-ary PSK Signals

| M | 2 | 4 | 8 | 16 | 32 | 64 |
|---|---|---|---|---|---|---|
| $\eta_B = R_b/B^*$ | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| $E_b/N_o$ for BER=$10^{-6}$ | 10.5 | 10.5 | 14 | 18.5 | 23.4 | 28.5 |

* $B$: First null bandwidth of M-ary PSK signals

QPSK can be interpreted as two independent BPSK systems (one on the I-channel and the other on Q-channel), and thus ***the same performance but twice the bandwidth efficiency***.

# Quadrature Amplitude Modulation (QAM) (1)

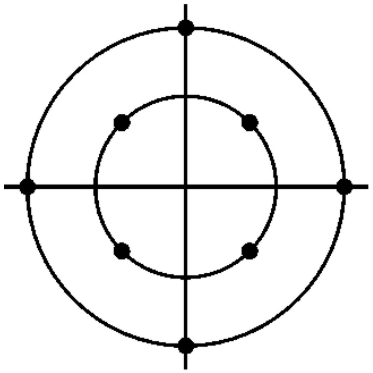
➢ ***Combined amplitude/phase shift keying***

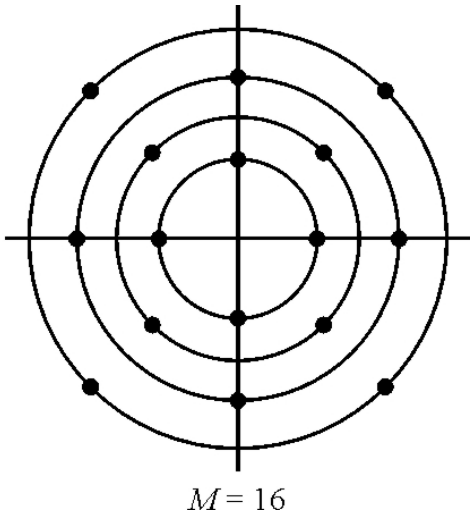
$$s_i(t) = A_i g(t)\cos\left[2\pi f_c t + \theta_c + \varphi_j\right],$$
$$i = 1,2,\cdots,M_1, j = 1,2,\cdots,M_2, 0 \le t \le T_s$$

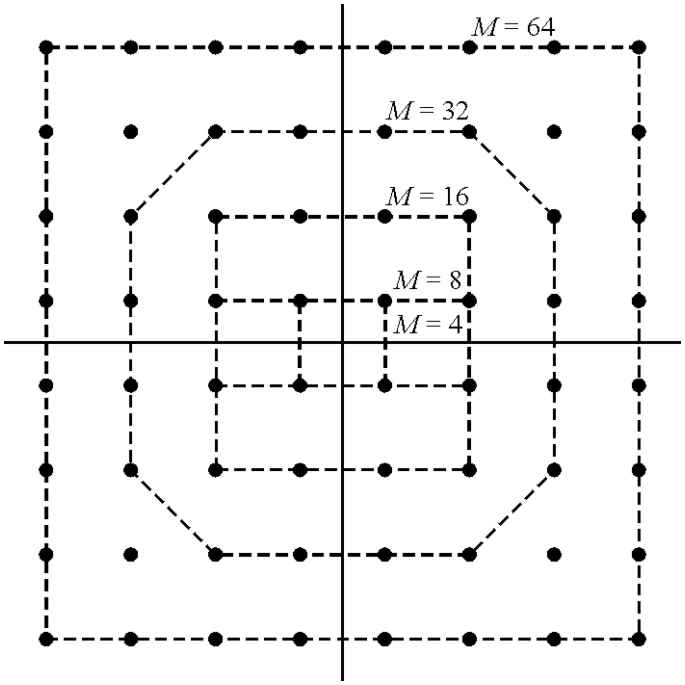
- As both amplitude and phase are used to carry symbol information, it is very bandwidth efficient
- Signal set size $M=M_1M_2$: $2^1 \times 2^1 = 4$, $2^2 \times 2^2 = 16$, $2^3 \times 2^3 = 64$, etc → 4QAM, 16QAM, 64QAM
- The larger M is, *the better bandwidth efficiency* but *lower robustness against noise and fading*

# Quadrature Amplitude Modulation (QAM) (2)

➢ *Examples of constellation*:




***Rectangular constellation***

# QAM: Bandwidth vs. Power Efficiency

*The system using optimum raised cosine pulse shaping is operated in AWGN channel.*

**Table 6.5** Bandwidth and Power Efficiency of QAM [Zie92]

| M | 4 | 16 | 64 | 256 | 1024 | 4096 |
|---|---|---|---|---|---|---|
| $\eta_B$ | 1 | 2 | 3 | 4 | 5 | 6 |
| $E_b/N_o$ for BER = $10^{-6}$ | 10.5 | 15 | 18.5 | 24 | 28 | 33.5 |

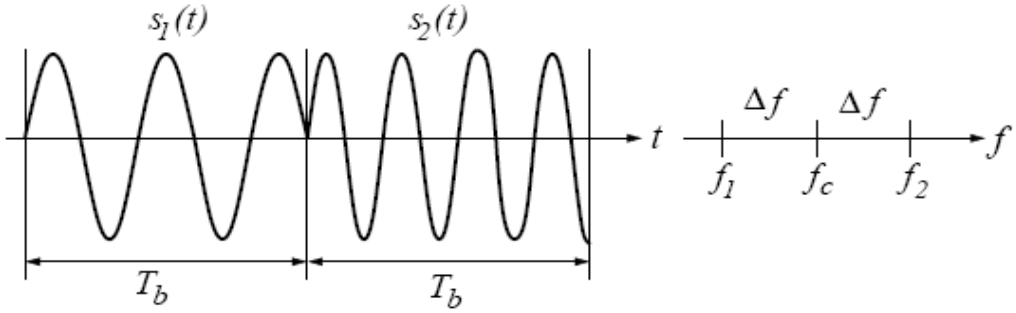
In terms of power efficiency, QAM is superior to M-ary PSK.

# Frequency Shift Keying (FSK)

- The modulation signal set is

$$s_i(t) = A_c \cos\left[2\pi f_i t + \theta_c\right], \quad i = 1, 2, \cdots, M, 0 \le t \le T_s$$

- ***BFSK***: $M=2$
  - Bit 0: $f_1 = f_c - \Delta f, \quad s_1(t) = \sqrt{\frac{2\varepsilon_b}{T_b}} \cos(2\pi(f_c - \Delta f)t + \theta_c)$
  - Bit 1: $f_2 = f_c + \Delta f, \quad s_2(t) = \sqrt{\frac{2\varepsilon_b}{T_b}} \cos(2\pi(f_c + \Delta f)t + \theta_c)$



# FSK: Bandwidth vs. Power Efficiency

**Table 6.6** Bandwidth and Power Efficiency of Coherent M-ary FSK [Zie92]

| M | 2 | 4 | 8 | 16 | 32 | 64 |
|---|---|---|---|---|---|---|
| $\eta_B$ | 0.4 | 0.57 | 0.55 | 0.42 | 0.29 | 0.18 |
| $E_b/N_o$ for BER = $10^{-6}$ | 13.5 | 10.8 | 9.3 | 8.2 | 7.5 | 6.9 |

•Nonlinear modulation: *bandwidth inefficient but power efficient, no need for expensive linear amplifiers*

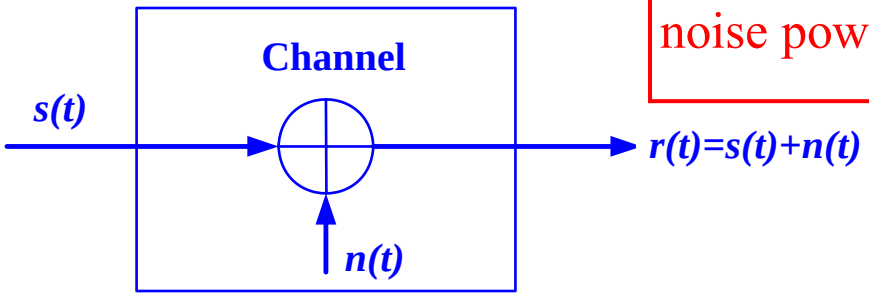
# Other Modulations

- ***Differential phase shift keying (DPSK)***
  - The input binary sequence is differentially encoded before BPSK modulation $\left(d_k = \overline{m_k \oplus d_{k-1}}\right)$
  - Avoids the need for a coherent reference signal at the receiver
- ***Offset QPSK***
  - The phase transitions are limited to $90^0$, the transitions on the I and Q channels are staggered.
- ***π/4 QPSK***
  - The phase transitions are limited to $135^0$
- ***Continuous-phase FSK (CPFSK)***
  - Avoids sudden change in the signal frequency, i.e., large spectral side lobes outside of the main spectral band
  - Minimum shift keying (MSK), Gaussian MSK(GMSK)

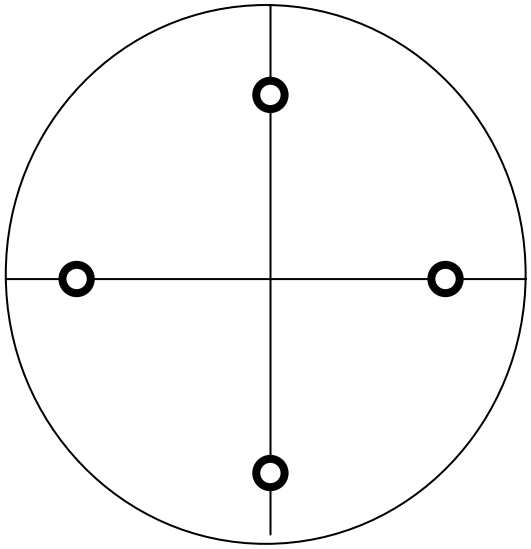
# Performance in AWGN Channel (1)

- The channel is assumed to corrupt the signal by the additive white Gaussian noise.

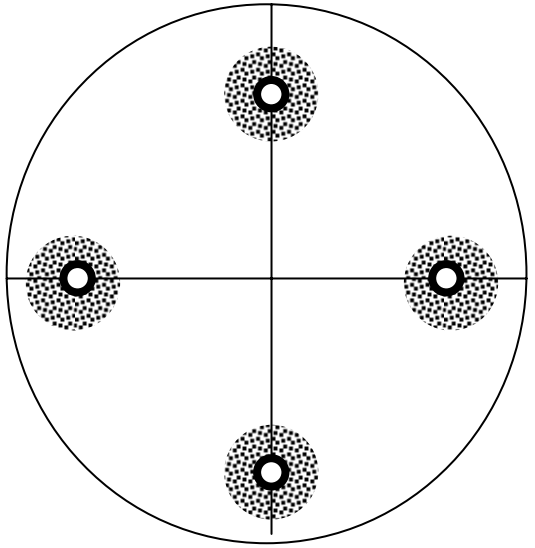
noise power: $\sigma_n^2 = \frac{N_0}{2}$




- *Distortion*



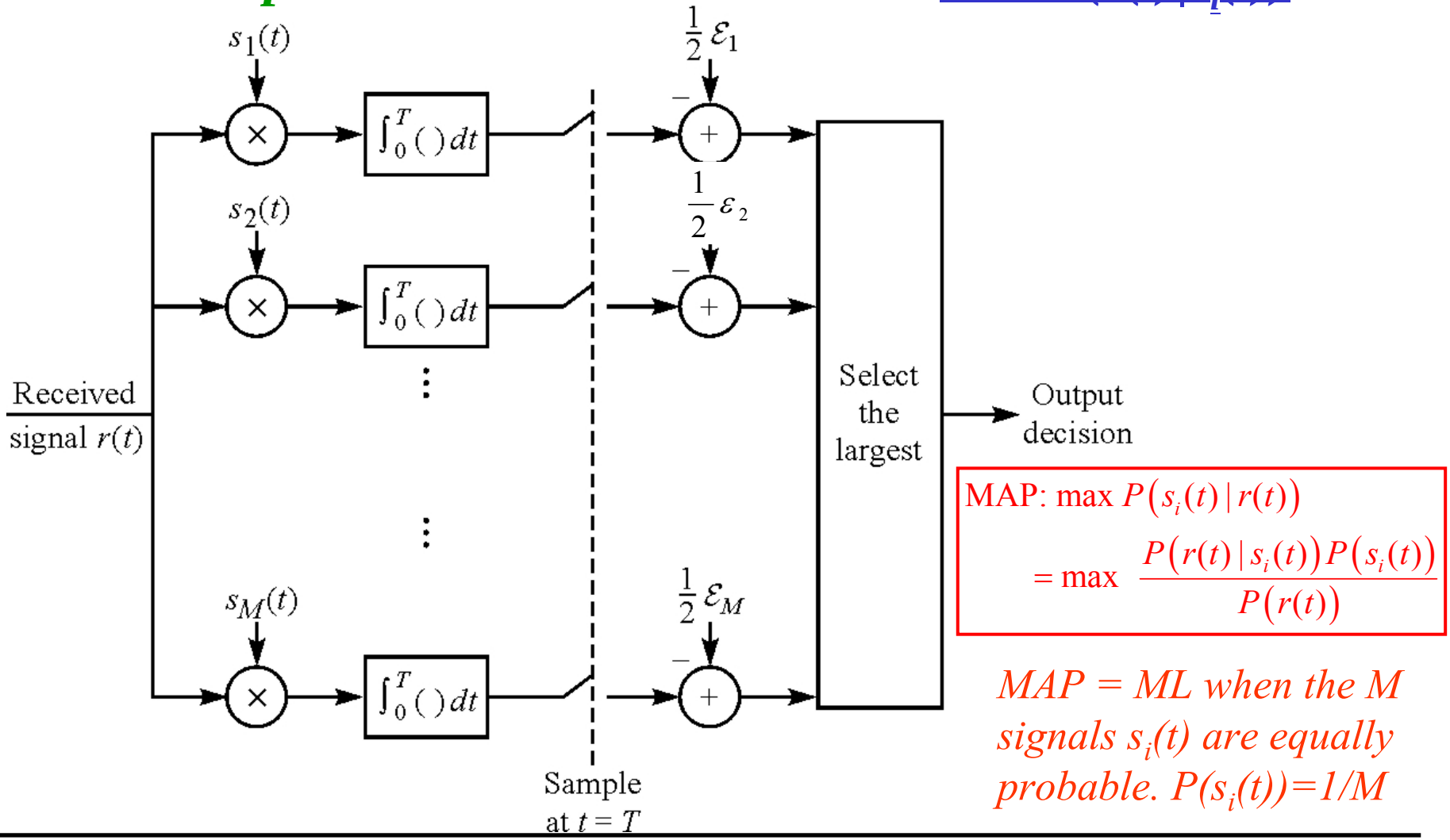
Perfect channel



White noise

# Performance in AWGN Channel (2)

➢ ***The optimum ML AWGN receiver: max P(r(t)|$s_i$(t))***




MAP: $\max P(s_i(t)\,|\,r(t)) = \max \dfrac{P(r(t)\,|\,s_i(t))P(s_i(t))}{P(r(t))}$

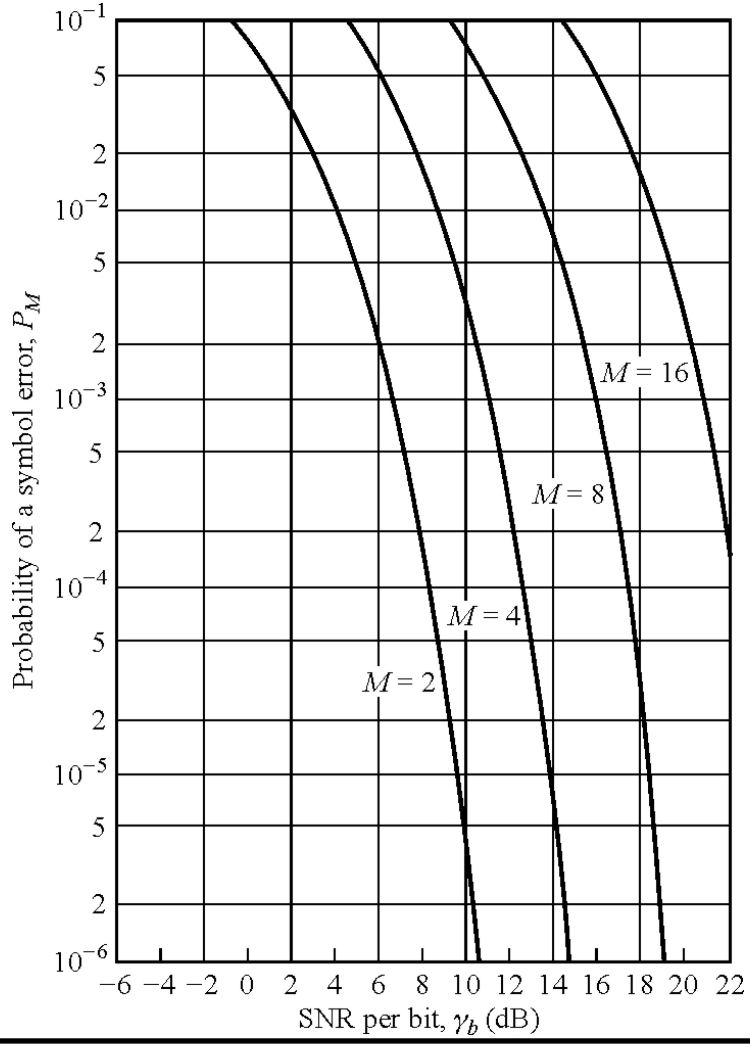
*MAP = ML when the M signals $s_i(t)$ are equally probable. $P(s_i(t))=1/M$*

# Performance in AWGN Channel (3)

- ***ASK:*** symbol error probability

$$P_M = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{(6\log_2 M)\varepsilon_{b,av}}{(M^2-1)N_0}}\right)$$

$\varepsilon_{b,av}$ is the average bit energy



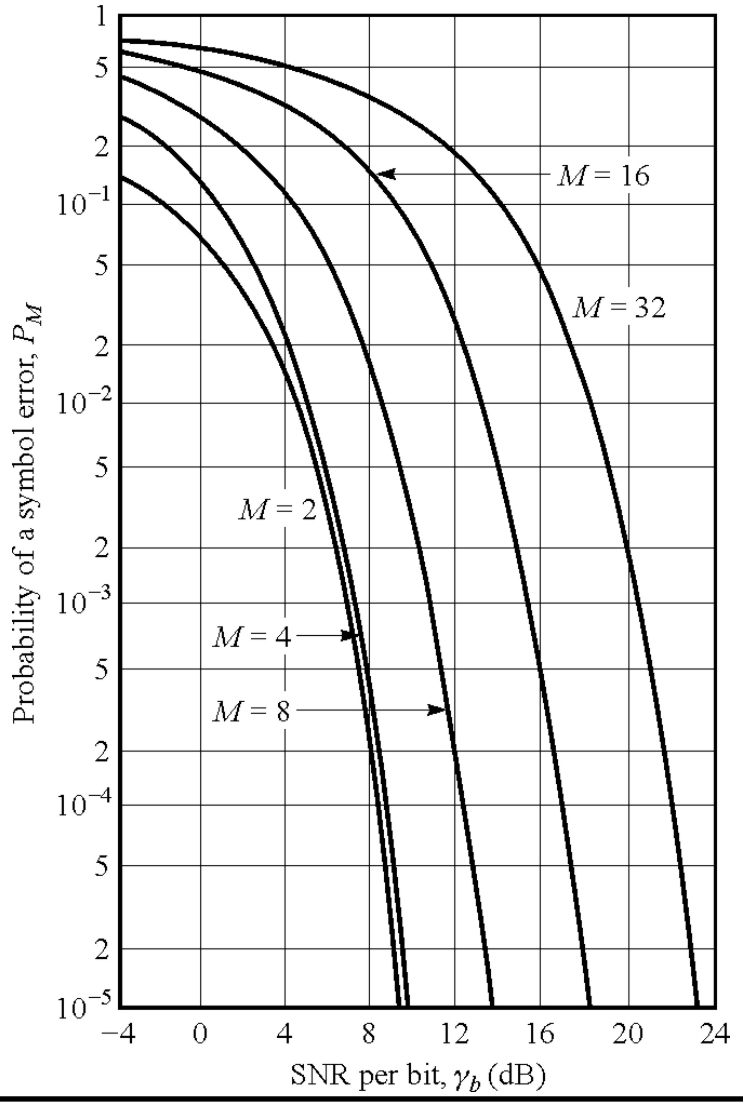
# Performance in AWGN Channel (4)

- ***PSK:*** symbol error probability

$$P_M \approx 2Q\left(\sqrt{\frac{2\varepsilon_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right), \quad M > 4$$

$\varepsilon_s = (\log_2 M)\varepsilon_b$ is the symbol energy

BPSK & QPSK: $P_M = Q\left(\sqrt{\frac{2\varepsilon_b}{N_0}}\right)$



# Performance in AWGN Channel (5)

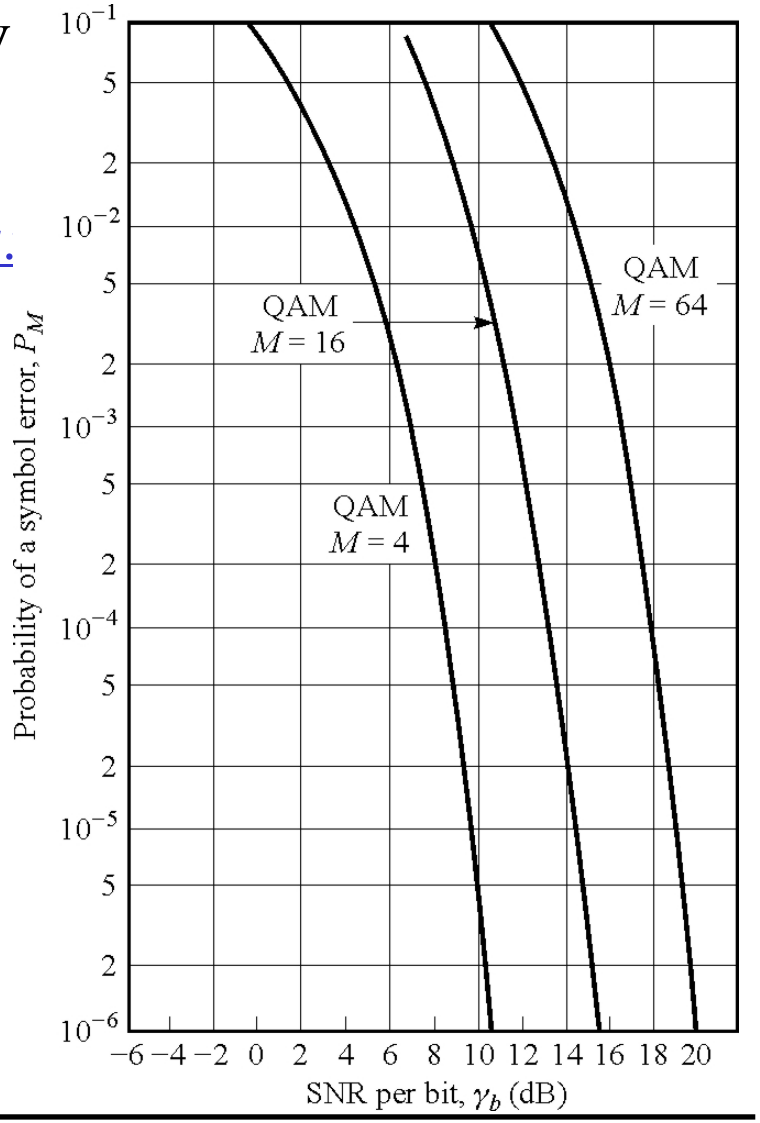
➢ ***QAM:*** symbol error probability

*For rectangular QAM constellations.*

$$P_M \le 4Q\left(\sqrt{\frac{3\varepsilon_{s,av}}{(M-1)N_0}}\right),$$

The average symbol energy:

$$\varepsilon_{s,av} = (\log_2 M)\,\varepsilon_{b,av}$$

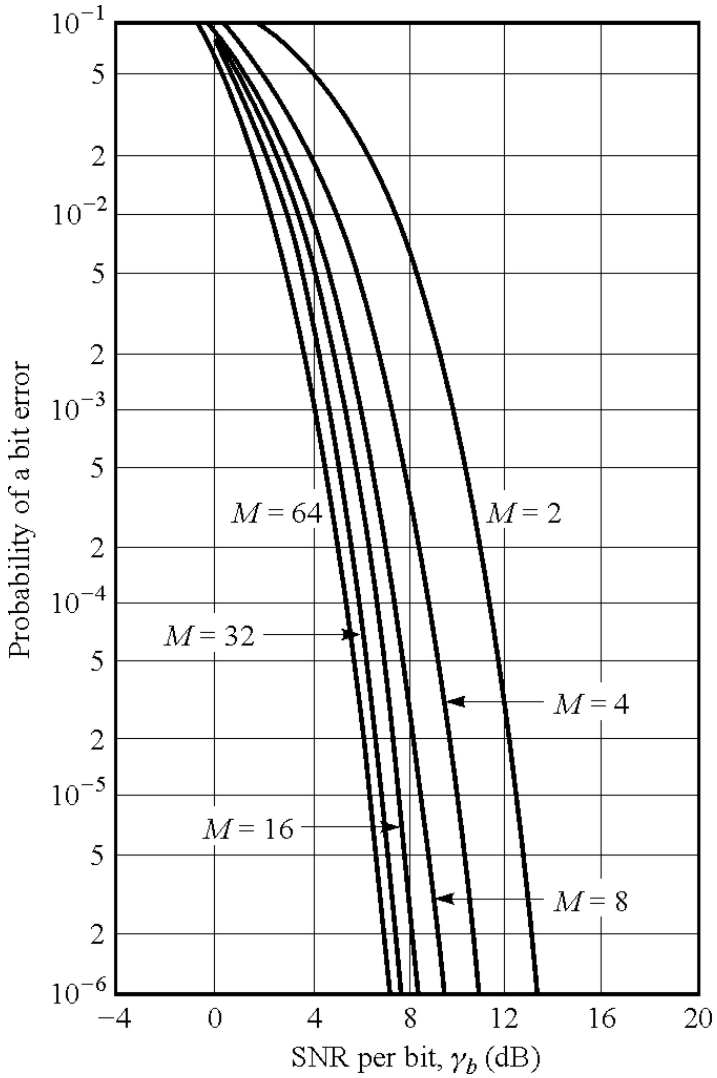


# Performance in AWGN Channel (6)

- ***FSK:*** symbol error probability

$$P_M \le (M-1)Q\left(\sqrt{\frac{\varepsilon_s}{N_0}}\right),$$

$$\varepsilon_s = \left(\log_2 M\right)\varepsilon_b$$



# Part 3.2 Trellis Coded Modulation

# Overview of TCM (1)

- Conventional coding
  - Separate from modulation, performed at the digital level before modulation
  - *The insertion of redundant bits*
    - Given the same information transmission rate, the symbol rate must be (n/k) times that of the uncoded system.
    - The redundancy provides coding gain, however, ***requires extra bandwidth***.
  - ***In a band-limited channel***, the required additional bandwidth is unavailable.

# Overview of TCM (2)

- ***Solution:*** *Trellis coded modulation (TCM)*
  - The combination of coding and modulation
  - Coding gain without expanding bandwidth
    - *Using a constellation with more points* than that required without coding
    - Typically, the number of points is doubled
    - The symbol rate is unchanged and *the bandwidth remains unchanged*.

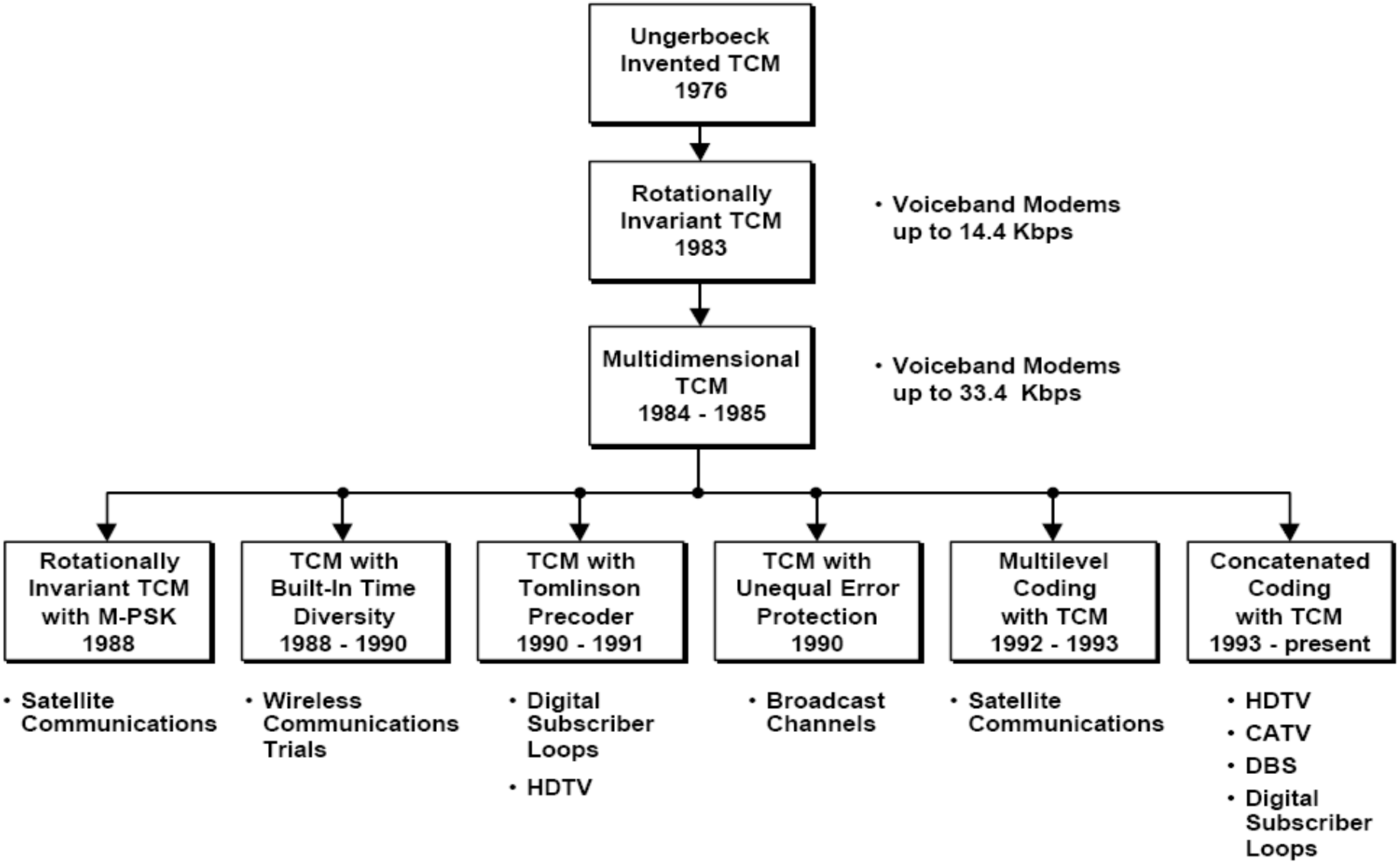
# Overview of TCM (3)

- ***How to achieve the coding gain by TCM?***
  - *Introducing dependancy between every successive symbols*
    - Only certain sequences of successive constellation points are allowed
  - *Maximizing the Euclidean distance between possible sequences of transmitted symbols*
    - Minimum distance between the possible **sequences** of transmitted symbols in signal space ($d_{min}$) determines the performance:

$$P_e \sim e^{-d_{\min}^2 / \sigma_n^2}$$

    - It actually decreases the error probability for a given SNR, thus achieving coding gain

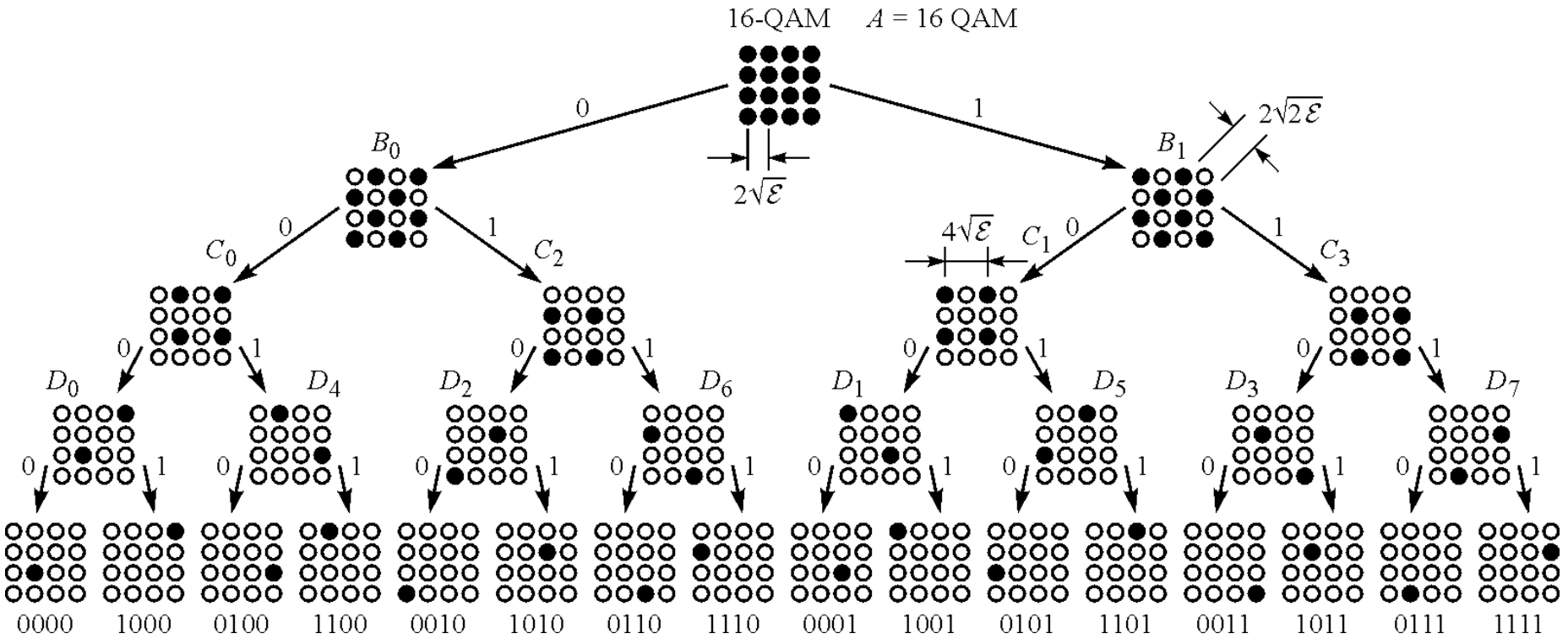
# History of TCM



# Basic Principles of TCM (1)

- ***TCM is to devise an effective method for mapping the coded bits into signal points such that the minimum Euclidean distance is maximized.***
- ***Ungerboek idea: mapping by set partitioning***
  - The signal constellation is partitioned in a systematic manner to form a series of smaller subsets.
  - The resulting subsets have a larger minimum distance than their "parent".
  - *The goal of partitioning: each partition should produce subsets with increased minimum distance.*

# Example of Set Partitioning

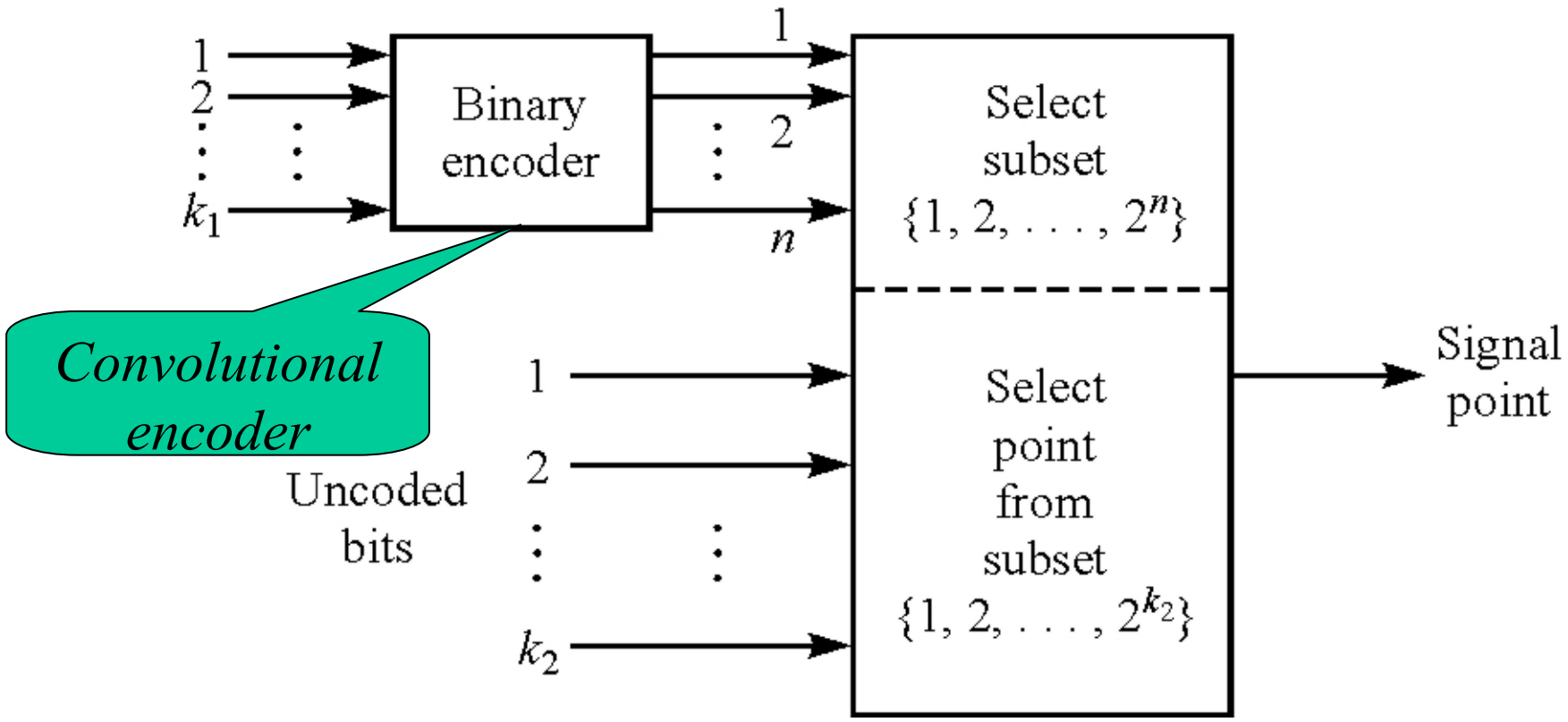


# Basic Principles of TCM (2)

- ***In general, the encoding is performed as follows:***
  - A block of $m$ information bits is separated into two groups of length $k_1$ and $k_2$, respectively.
  - The $k_1$ bits are encoded into $n$ bits, while the $k_2$ bits are left uncoded.
  - The $n$ bits from the encoder are used to select one of the possible subsets in the partitioned signal set, while the $k_2$ bits are used to select one of $2^{k_2}$ signal points in each subset.
- The coder need not code all the incoming bits. When $k_2=0$, all $m$ information bits are encoded.
- There are many ways to map the coded bits into symbols. The choice of mapping will drastically affect the performance of the code.

# Basic Principles of TCM (3)

➢ ***General structure of encoder:***

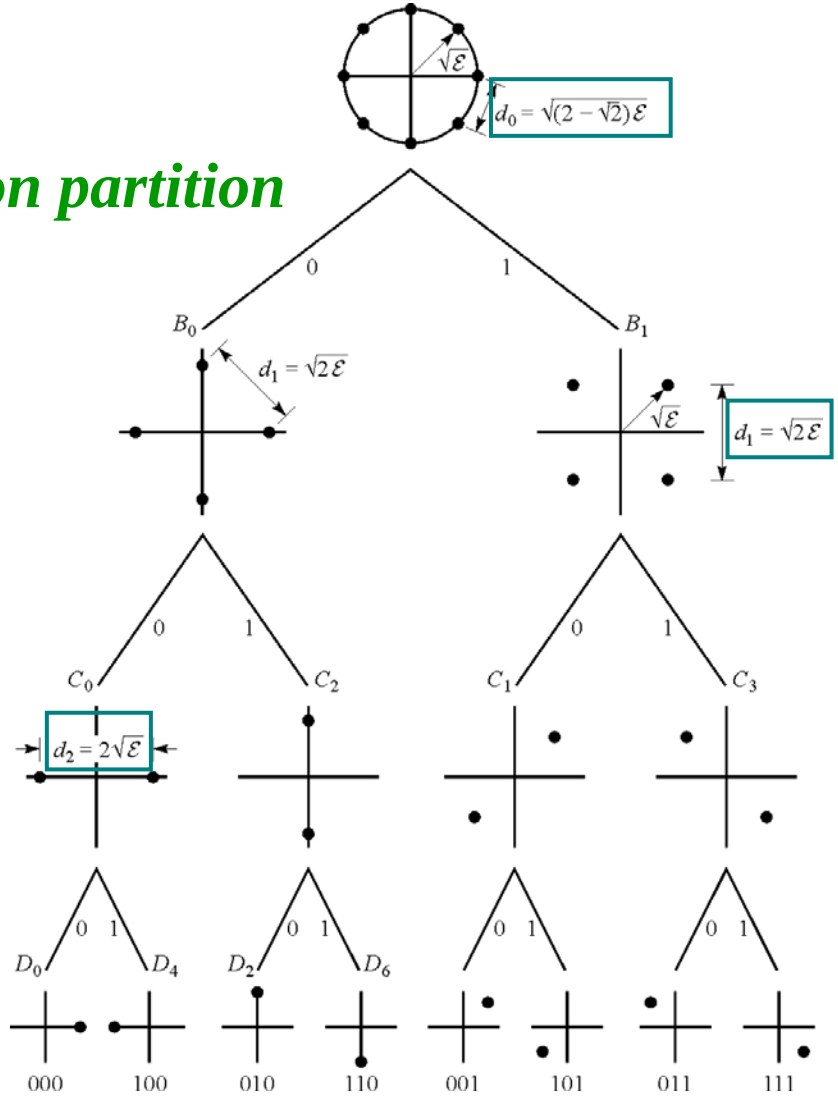


# Basic Principles of TCM (4)

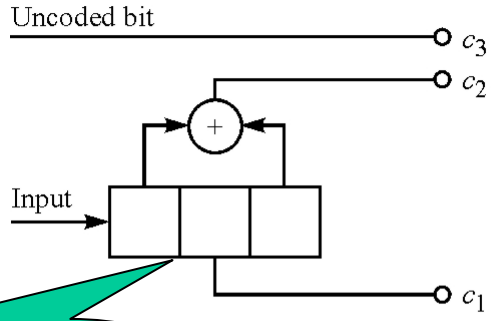
- ***The basic rules for the assignment of signal subsets to state transitions in the trellis***
  - Use all subsets with equal frequency in the trellis
  - *Transitions originating from the same state or merging into the same state in the trellis* are assigned subsets that are separated by the largest Euclidean distance
  - *Parallel state transitions* (when they occur) are assigned signal points separated by the largest Euclidean distance.
    - Parallel transitions in the trellis are characteristic of TCM that contains one or more uncoded information bits.

# Examples of TCM (1)

## *8-PSK constellation partition*

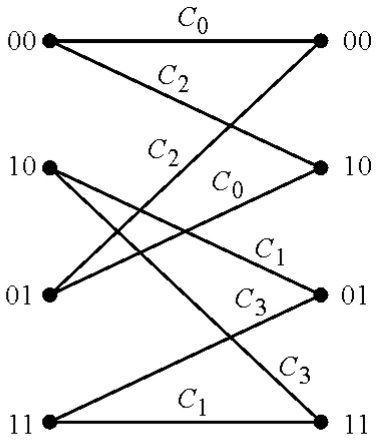


# Examples of TCM (2)

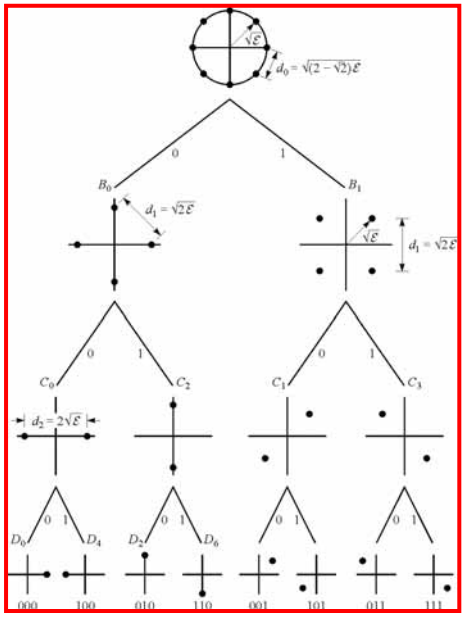



½ convoluntional encoder with 4 states

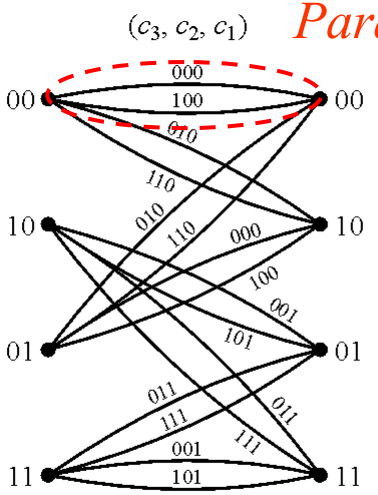
(*a*) Encoder



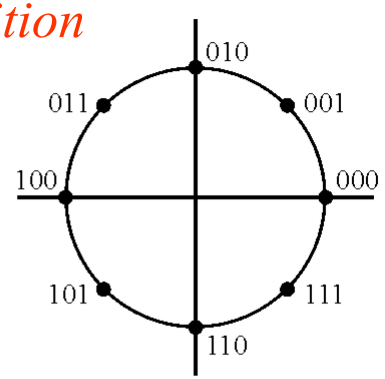

(*b*) Four-state trellis



*Parallel transition*




(*c*) Mapping of bits to state transitions

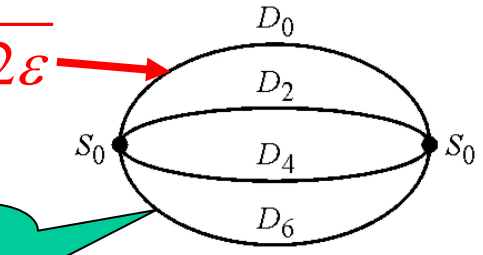



(*d*) Mapping of bits $(c_3, c_2, c_1)$ to signal points corresponding to partition in Fig. 8.3-1 (note nonuniqueness of this mapping)

# Examples of TCM (3)

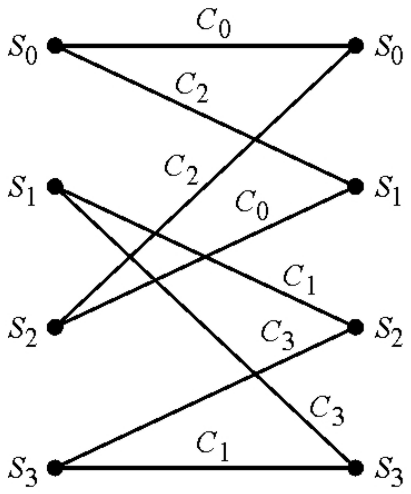
Minimum Euclidean distance:

$$d_{\min,uncoded} = \sqrt{2\varepsilon}$$



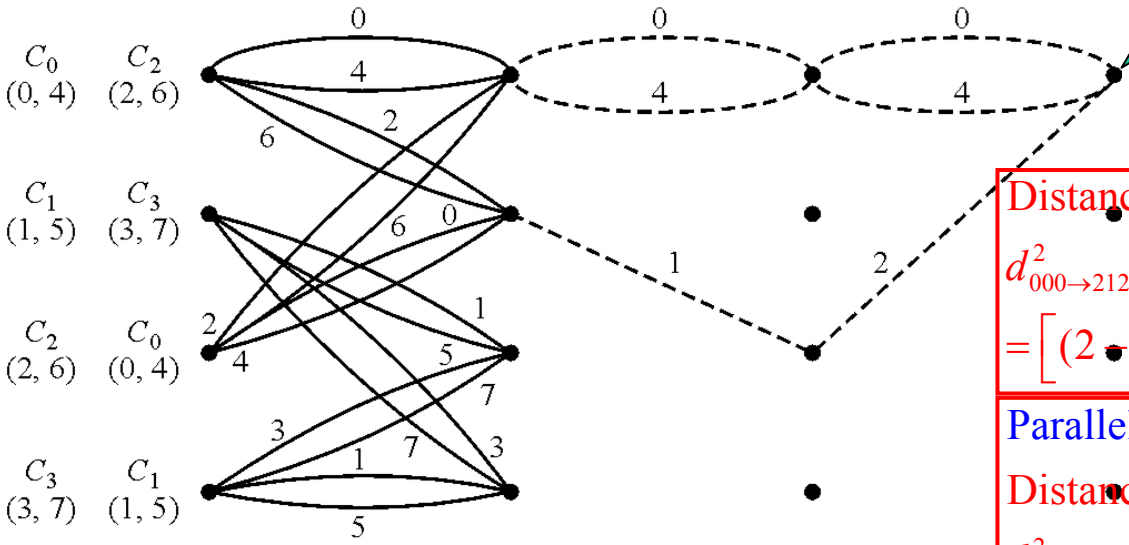

(*a*) One-state trellis

*Uncoded QPSK*




(*b*) Four-state trellis

*Trellis coded 8PSK modulation*




(*c*) Four-state trellis

Distance : $(0, 0, 0) \rightarrow (2, 1, 2)$

$$d^2_{000\rightarrow 212} = d_0^2 + 2d_1^2$$

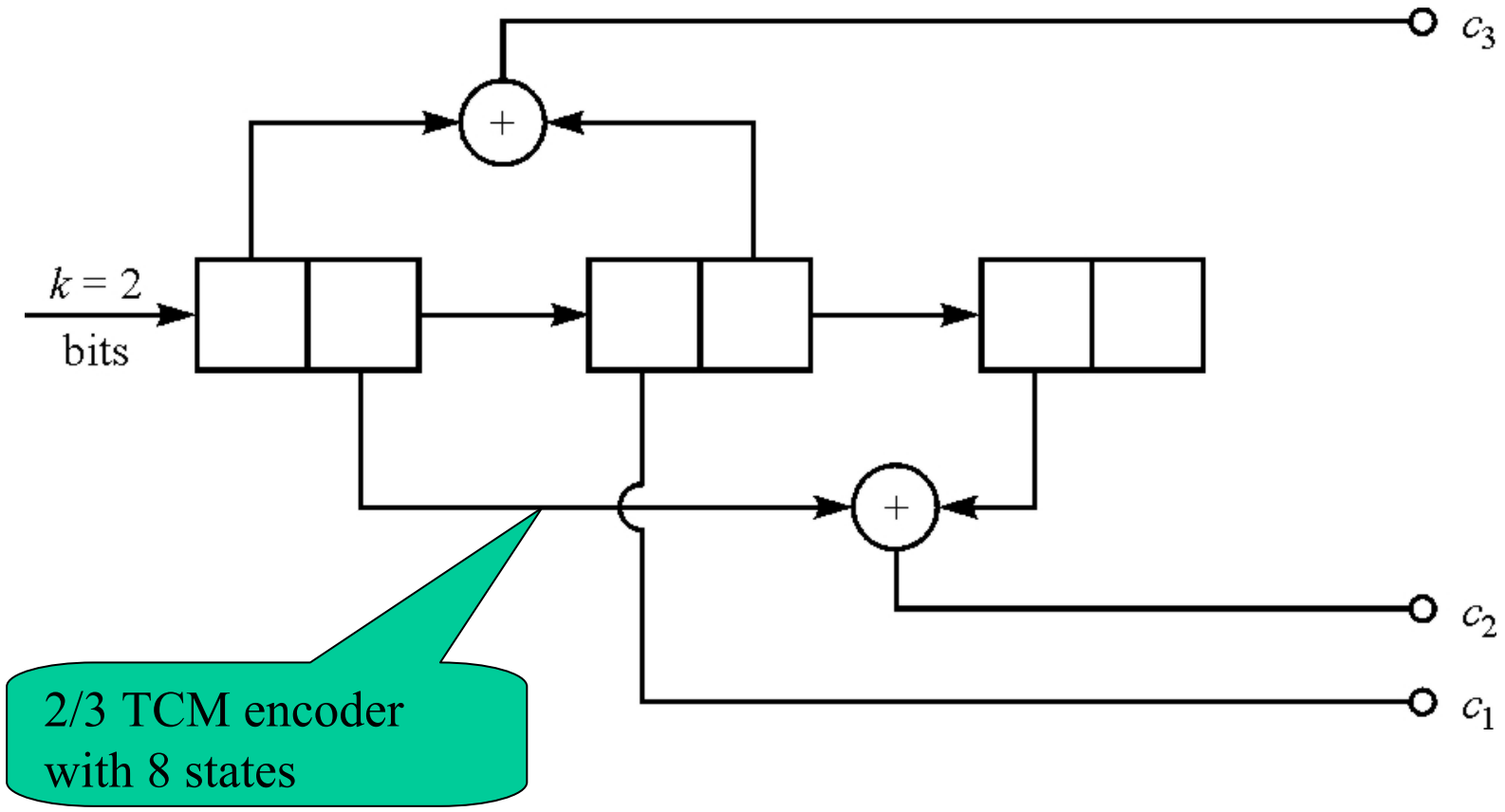
$$= \left[(2-\sqrt{2})\varepsilon + 4\varepsilon\right] = 4.585\varepsilon$$

Parallel transition:

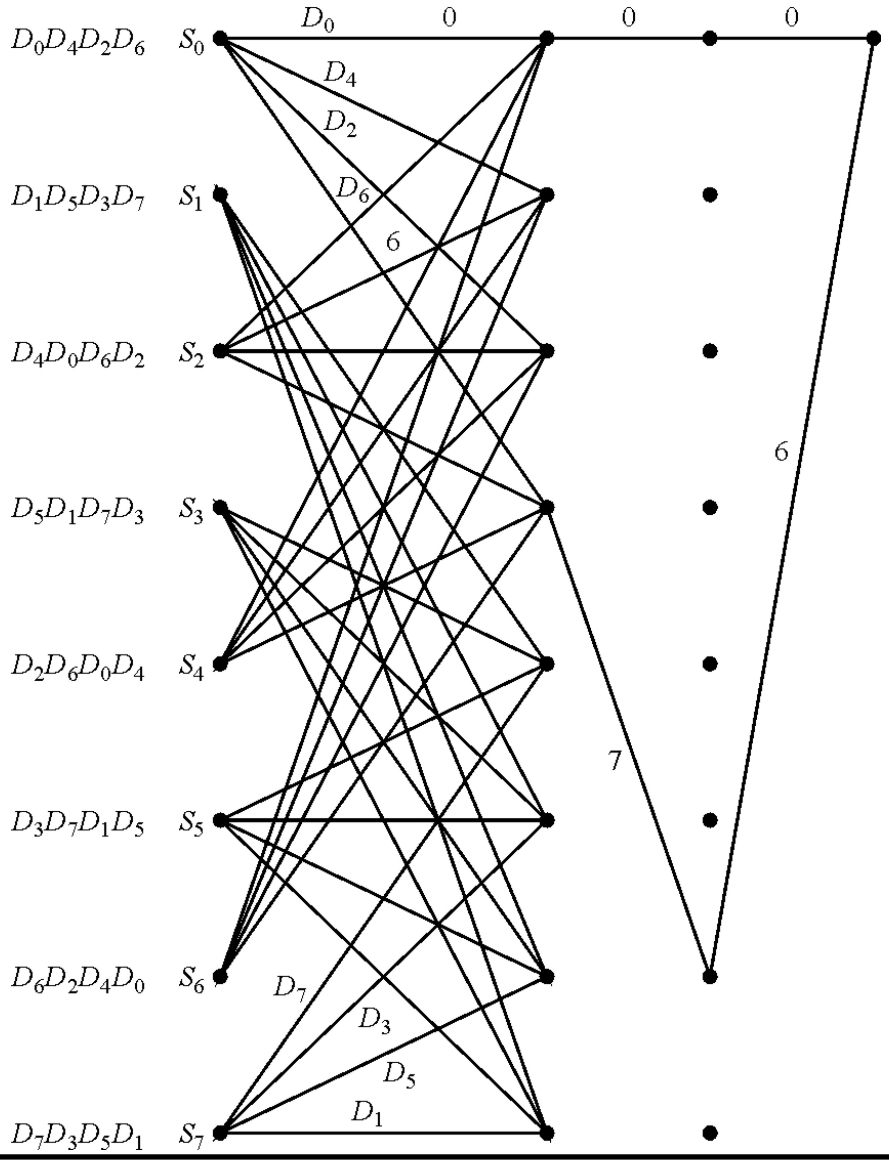
Distance : $(0, 0, 0) \rightarrow (4, 0, 0)$

$$d^2_{000\rightarrow 400} = d_2^2 = 4\varepsilon$$

# Examples of TCM (4)



# Examples of TCM (5)

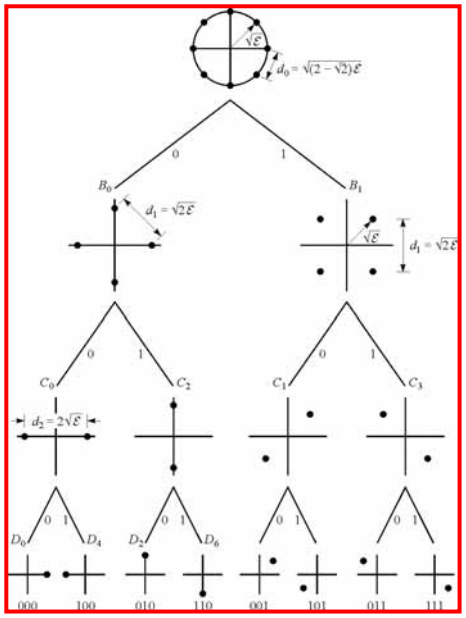



*No parallel transition*

Distance : $(0, 0, 0) \rightarrow (6, 7, 6)$

$$d^2_{000\rightarrow 676} = d_0^2 + 2d_1^2$$

$$= \left[ (2-\sqrt{2})\varepsilon + 4\varepsilon \right] = 4.585\varepsilon$$



# Coding Gain (1)

- *The minimum Euclidean distance between paths that diverge from any state and remerge at the same state in the trellis code is called* ***free Euclidean distance*** $D_{fed}$
- ***Asymptotic coding gain:***

$$\gamma = \left( \frac{E_{uncoded}}{d^2_{\min,uncoded}} \right) \Big/ \left( \frac{E_{coded}}{D^2_{fed,coded}} \right)$$

where E is the normalized average received engergy

when $E_{uncoded} = E_{coded}$, $\gamma = \dfrac{D^2_{fed,coded}}{d^2_{\min,uncoded}}$

In the 4-state example, $D_{fed} = 2\sqrt{\varepsilon}, d_{\min,uncoded} = \sqrt{2\varepsilon}$

$\gamma$=2 $\Rightarrow$ 3dB coding gain

# Coding Gain (2)

- ***Asymptotic coding gain*** *can be increased by* ***increasing the number of states and the rate of the convolutional encoder.***

In the 8-state example,

$$D_{fed} = \sqrt{4.585\varepsilon}, d_{\min,uncoded} = \sqrt{2\varepsilon}$$

$\gamma$=2.2925 $\Rightarrow$ 3.6dB coding gain

# Coding Gain (3)

CODING GAINS FOR TRELLIS-CODED 16-PSK MODULATION

| Number of states | $k_1$ | Code rate $\frac{k_1}{k_1+1}$ | $m=3$ coding gain (dB) of 16-PSK versus uncoded 8-PSK | $m \to \infty$ $N_{free}$ |
|---|---|---|---|---|
| 4 | 1 | 1/2 | 3.54 | 4 |
| 8 | 1 | 1/2 | 4.01 | 4 |
| 16 | 1 | 1/2 | 4.44 | 8 |
| 32 | 1 | 1/2 | 5.13 | 8 |
| 64 | 1 | 1/2 | 5.33 | 2 |
| 128 | 1 | 1/2 | 5.33 | 2 |
| 256 | 2 | 2/3 | 5.51 | 8 |

*Source:* Ungeboeck (1987).

# Viterbi Decoding (1)

- ***Two steps:***
  - ***Step 1:*** At each branch in the trellis,
    - Compare the received signal to each of the siganls allowed for that branch.
    - Save the signal closest to the received signal
    - Label the branch with a metric proportional to the Euclidean distance between the two signals.
    - ***Branch metric calculation***

Determining the best signal within each subset, i.e., the signal with the smallest distance to the received signal → *subset decoding*

# Viterbi Decoding (2)

- ***Two steps:***
  - ***Step 2:***
    - Apply the Viterbi algorithm to the trellis, with surviving partial paths corresponding to partial signal sequences that are closest to the received sequences.
    - Select the ML path (the complete signal sequence closest in Euclidean distance to the received sequence) at the end of the trellis.
    - ***Path metric calculation***
    - ***Trellis update***

# Error Rate Performance

- An error event happens when an erroneous path is selected at the decoder
- *Error-event probability in AWGN channel*:

$$P_e \approx N_{fed} Q\left(\sqrt{\frac{D_{fed}^2}{2N_0}}\right) \quad \text{under high SNR}$$

$N_{fed} \rightarrow$ the number of signal sequences with distance $D_{fed}$ that diverge at any state and remerge at that state after one or more transitions

# Announcement

- **Dr. Yuk's lecture notes could be downloaded from the department course material web page under "ELEC7073(tiyuk)".**

https://www.eee.hku.hk/courses.msc/elec7073a/