Fourier Transform in 4F optical correlator

Fourier optics is used in the field of optical information processing, the staple of which is the classical 4F processor and the plane wave spectrum concept is the basic foundation of Fourier Optics. A general solution to the homogeneous electromagnetic wave equation in rectangular coordinates is formed as a weighted superposition of all possible elementary plane wave solutions as:

\[
E_u(x,y,z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_u(k_x,k_y) e^{j(k_x x + k_y y)} e^{\pm jz \sqrt{k_x^2 - k_y^2}} dk_x dk_y
\]

The plane wave spectrum representation of the electromagnetic field is the basic foundation of Fourier Optics, if \( z=0 \), the equation above simply becomes a Fourier transform (FT) relationship between the field and its plane wave content, hence the name, Fourier optics. All spatial dependence of the individual plane wave components is described explicitly via the exponential functions.

When the plane wave spectrum representation of the electric field is combined with the Fourier transforming property of quadratic lenses, it leads naturally to the development of numerous 2D image processing devices. One of the primary applications of Fourier Optics is in the mathematical operations of cross-correlation and convolution, and these have historically been done with a device known as a 4F correlator, shown in the figure below. The 4F correlator is based on the convolution theorem from Fourier transform theory, which states that convolution in the spatial \((x,y)\) domain is equivalent to direct multiplication in the spatial frequency \((k_x, k_y)\) domain.

Once again, a plane wave is assumed incident from the left and a transparency containing one 2D function, \( f(x,y) \), is placed in the input plane of the correlator, located one focal length in front of the first lens. The transparency spatially modulates the incident plane wave in magnitude and phase, like on the left-hand side of equation, and in so doing, produces a spectrum of plane waves corresponding to the FT of the transmittance function, like on the right-hand side of equation. That spectrum is then formed as an image one focal length behind the first lens, as shown. A transmission mask containing the FT of the second function, \( g(x,y) \), is placed in this same plane, one focal length behind the first lens, causing the transmission through the mask to be equal to the product, \( F(k_x, k_y) \times G(k_x, k_y) \). This product now lies in the input plane of the second lens, so that the FT of this product, is formed in the back focal plane of the second lens.

![4F optical correlator system](image.png)