Fourier Transform in solving nonlinear Schrödinger equation

My current research is related to the supercontinuum (SC) generation which requires to generate a temporal stable and broadband SC source such that it can be used as the light source in real-time ultra-fast imaging. Ultra-broadband (SC) at the 1-m wavelength range, which is regarded as a diagnostic window in biophotonics, represents a versatile light source for a wide range of bioimaging and spectroscopy applications. In particular, applications which require high-speed and high-throughput operations, such as real-time high speed spectroscopy based on dispersive Fourier transform (DFT) and serial time-encoded amplified microscopy (STEAM), demand for the robust SC source with not only a broadband spectrum but also a good temporal stability. In order to have a better understanding of the SC generation process, Nonlinear Schrödinger equation is used to simulated the SC generation.

The Nonlinear Schrödinger equation (NLS) is a nonlinear partial differential equation which generally does not have analytic solutions, so it cannot be solved easily by analytical method. In order to solve NLS, a numerical approach is necessary for an understanding of the nonlinear effects in optical fibers. One of the most common methods to solve NLS is using split-step Fourier method. The advantage of using this method lies in the fast speed comparing with finite-difference method.

In the Fourier Method, the NLSE can be written in the form of \( \frac{\partial A}{\partial z} = (\hat{D} + \hat{N})A \), where D and N are differential and nonlinear operator respectively, this method obtains an approximate solution the light is propagation along the optical field over a small distance \( h \), and the dispersion and nonlinear are assumed to acts alone in the process. In the equation \( A(z + h, T) = e^{h\hat{D}}e^{h\hat{N}}A(z, T) \). The exponential operator \( e^{h\hat{D}} \) can be evaluated in the Fourier domain and can be approximated as \( e^{h\hat{D}}B(z, T) = F_T^{-1}e^{h\hat{D}(iw)}F_TB(z, T) \). Thus, the evaluation of this equation is very easily to be obtained. By using FFT method, the numerical computation time can be greatly enhanced.

To study on the temporal stability of the SC general process, one pulse evolution in both time and frequency was obtained as we run the NLSE program once. The whole picture of the stability of the SC is obtained by running numerous times of the program such that the pulse-to-pulse variation can be compared.