Fourier Transform in Cerebral Hemodynamics

As we know, most of the cerebral blood vessels are surrounded by skull, the geometrical feature and hemodynamic status of which are very complicated. There are many people trying to get some indices to reflect the function of cerebral circulation, especially the Wills circle, because it is very difficult to detect those indices directly. And Fourier transform plays an important role in this area, for the blood flow in the artery is frequency dependent. The cerebral circulation seems like a network with four inputs, including two carotid arteries and two vertebral arteries, according to its anatomical structure. Here, let’s take the carotid artery for example to show how the Fourier transform is used in cerebral hemodynamics.

Based on harmonic analysis, the observed blood pressure and flow waves can be expressed as Fourier series. The pressure and flow can be represented respectively as functions of time 

\[ P(t) = P_0 + \sum_{n=1}^{N} P_n \sin(n\omega t + \phi_{pn}) \] (1)

\[ Q(t) = Q_0 + \sum_{n=1}^{N} Q_n \sin(n\omega t + \phi_{qn}) \] (2)

Then the impedance modulus \( Z_n \) is at a frequency of \( \omega \).

\[ Z_n = \frac{P_n}{Q_n} \]

And the characteristic impedance \( Z_c = \lim_{\omega \to 0} |z_n| \), which reflect the propagation property of pulse wave in vascular bed and the physical character of vascular bed itself. In clinic, arteriosclerosis, hypertension and growing old can cause the \( Z_c \) increases abnormally. In the further study, the hydraulic energy of the blood also could be analyzed by Fourier series. The total energy per unit time can be calculated by

\[ W_c = P_0Q_0 + \frac{1}{2} \sum_{n=1}^{N} P_nQ_n \cos(\phi_{pn} - \phi_{qn}) \] (3)

And the average steady energy per unit time is \( W_s = P_0Q_0 \). The average oscillate energy per unit time is \( W_o \).

\[ W_o = \frac{1}{2} \sum_{n=1}^{N} P_nQ_n \cos(\phi_{pn} - \phi_{qn}) = \frac{1}{2} \sum_{n=1}^{N} Q_n^2 |Z_n| \cos(\phi_{pn} - \phi_{qn}) \] (4)

In clinic trials, we try to evaluate the effect of parallel scalp acupuncture therapy on hemodynamic function of cerebral circulation in post-stroke patients through the changes of cerebral circulation hemodynamic indices before and after the therapy. And we find that the \( W_c \) and \( W_s \) increase significantly as immediate effect after therapy, the ratio of oscillatory kinetic energy to total kinetic energy decreases.[1]

Reference