Subspace Identification for DOA Estimation in Massive / Full-dimension MIMO Systems: Bad Data Mitigation and Automatic Source Enumeration

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Abstract

In this paper, the direction-of-arrival (DOA) estimation problem for massive multiple-input multiple-output (MIMO) systems with a two dimensional (2D) array (also known as full-dimension MIMO) is investigated, assuming no knowledge of path number, noise power, path gain correlations and bad data statistics. Based on the variational Bayesian framework, a novel iterative algorithm for subspace identification operating on tensor represented data is proposed with integrated features of effective bad data mitigation and automatic source enumeration. The subspace recovered from the proposed algorithm not only enables existing 2D DOA estimators to be readily applied, if the number of signal paths is less than the number of horizontal antennas and vertical antennas, the subspaces in elevation and azimuth domains can be separately estimated, from which one dimensional (1D) DOA estimators can be utilized, thus further lowering the complexity. Simulation results are presented to illustrate the
excellent performance of the proposed subspace recovery method and subsequent DOA estimation in terms of accuracy and robustness.

Index Terms

Massive MIMO, Robust Estimation, Subspace Method, Multidimensional Signal Processing

I. INTRODUCTION

As data traffic is growing explosively, a critical challenge in the next generation of wireless systems is to increase the spectral efficiency. Massive multiple-input multiple-output (MIMO) systems, where each base station (BS) is equipped with a very large number of antennas, are a contender for the enabling technologies. With the extensive spatial freedoms offered by large antenna arrays, abundant users are expected to occupy the same set of time and frequency resources with negligible interference, thus circumventing the longstanding bandwidth limitation in wireless communications [1].

To realize the theoretical promise, extensive research has been conducted to tackle the technical challenges in massive MIMO systems, such as interference mitigation [2], optimal pilot beam pattern design [3], joint spatial division and multiplexing [4], small size codebook designs [5] and user scheduling [6]. However, in all the research works mentioned above, knowledge of channel correlations at the BSs is required. To model channel correlations, the geometric stochastic channel model that mimics the underlying radio propagation is widely used [2], [5], [7]–[9], wherein direction of arrivals (DOAs) of signal paths are crucial model parameters. Thus, accurate DOA estimation for dominant signal paths is a prerequisite for channel correlations acquisition.

It is well-known that efficient DOA estimators for one dimensional (1D) arrays, such as MUSIC, ESPRIT and their variants [13], require subspace identification, as the DOA signal does not fully occupy the space generated by the array. While this is also true for DOA estimation in massive MIMO systems, there is a significant difference: massive MIMO systems, also known as full-dimensional MIMO (FD-MIMO) systems, would most likely employ two dimensional (2D) arrays to accommodate the vast amount of antennas at the BSs [10]–[12], leading to a 2D DOA estimation problem [14], [15]. Extensions of 1D DOA estimation results to 2D DOA estimation indeed exist, giving rise to the 2D ESPRIT [16], 2D MUSIC [17] and 2D Matrix Pencil [18]. Nevertheless, as data gathered from 2D arrays are inherently organized in a three-dimensional
structure, it is more natural to represent and manipulate the data using tensors [24]. Pioneering works of [19], [20] noticed this fact and introduced the tensor based higher-order singular value decomposition (HOSVD) for subspace and DOA estimations. It has been proved theoretically [19], [20] that better subspace estimates and subsequent DOA estimates can be obtained via the tensor approach compared to the matrix based counterpart, if the number of signal paths is strictly less than the number of horizontal antennas or vertical antennas, a condition which can be easily satisfied in massive MIMO systems.

However, directly applying the HOSVD subspace estimator from [19] to massive MIMO systems suffers from several problems. First, massive MIMO systems are expected to be built with low-cost hardwares. Therefore, bad data will be very likely to occur due to hardware imperfections [1]. As the HOSVD subspace estimator is least-squares based, it cannot avoid the adverse effect brought by bad data, thereby significant performance degradation would be expected. Secondly, HOSVD must be operated with known signal path number, which cannot be trivially obtained, especially in presence of bad data. Existing approaches require advanced source enumeration methods exploiting robust statistics to be executed before the subspace estimation [21]–[23]. Even though this additional step offers a way to get around the problem of unknown path numbers, it costs extra complexity.

In order to exploit the benefits from tensor algebra while avoiding the disadvantages presented in HOSVD, we devise a new subspace estimator based on the framework of probabilistic inference. Since its inception [25], subspace recovery using probabilistic models (also known as probabilistic principle component analysis) provides a viable alternative to the conventional SVD approach, and it further taps into the vast amount of techniques in Bayesian statistics and machine learning, such as automatic dimensionality determination for the principle subspace [26], and handling of missing data and outliers [27]. The idea is to model the basis vectors of the unknown subspace as Gaussian vectors, and then these unknown vectors are estimated using the expectation-maximization algorithm (or some variants). A recent work on low-rank matrix estimation [39] demonstrates that the Bayesian approach indeed leads to accurate subspace recovery.

In this paper, we take a step further, and derive a probabilistic subspace estimation algorithm for tensor-represented data, with integrated features of effective bad data mitigation and automatic source enumeration. In particular, due to the intractable integrations required for exact Bayesian
inference in identifying the subspace under unknown subspace dimension and the presence of bad data, variational inference [30] is exploited by seeking another distribution that is the closest to the true posterior distribution in Kullback-Leibler (KL) divergence sense. This leads to an iterative algorithm that alternatively reconstructs the subspace, refines the corresponding confidence, and mitigates bad data. Upon convergence, not only the recovered range space of the 2D steering matrix can robustify any conventional 2D DOA estimation algorithm, if the number of signal paths is less than the number of horizontal antennas and vertical antennas, the subspaces in elevation and azimuth domains can be separately estimated. For the latter case, the 2D DOA estimation problem can be decomposed into two 1D DOA estimation problems, which further lowers the complexity. Notice that the probabilistic models of existing works on variational inference with tensor data [33]-[35] are tailored for real-valued data, and cannot process complex-valued data from communication systems. Furthermore, their probabilistic models do not take bad data into account, and thus their results will fail to work in massive MIMO systems with low-cost transceivers.

The remainder of this paper is organized as follows. The system model is described in Section II. In Section III, after linking the tensor algebra to the subspace estimation, the probabilistic model for subspace recovery is established, based on which a robust subspace estimator exploiting the variational inference is derived in Section IV. Using the subspaces recovered from Section IV, the DOA estimators are devised in Section V. Simulation results are presented in Section VI. Finally, conclusions are drawn in Section VII.

Notation: Boldface lowercase and uppercase letters will be used for vectors and matrices, respectively. Tensors are written as Calligraphic letters. \( \mathbb{E}[\cdot] \) denotes the expectation of its argument and \( j \triangleq \sqrt{-1} \). Superscripts \( T, \ast, H \) and \( \dagger \) denote transpose, conjugate, Hermitian and Moore-Penrose pseudoinverse, respectively. The symbols \( \odot, \otimes, \oslash \) and \( \circ \) denote the Hadamard product, Kronecker product, Khatri Rao product (column-wise Kronecker product) and vector outer product, respectively. The operator \( \text{Tr}\{\mathbf{A}\} \) takes the trace of matrix \( \mathbf{A} \) and \( \| \cdot \|_F \) represents the Frobenius norm of the argument. The operator \( \sigma_{\text{min}}\{\mathbf{Q}\} \) denotes the smallest singular value of the matrix \( \mathbf{Q} \) and \( \text{orth}\{\mathbf{Q}\} \) is an orthonormal basis for the subspace spanned by the columns of \( \mathbf{Q} \). The symbol \( \propto \) represents a linear scalar relationship between two real-valued functions. The \( N \times N \) diagonal matrix with diagonal components \( a_1 \) through \( a_N \) is represented as \( \text{diag}\{a_1, a_2, \ldots, a_N\} \), while \( \mathbf{I}_M \) represents the \( M \times M \) identity matrix. An all-zeros \( M \times N \) matrix
is denoted by $0_{M \times N}$, and $1_{M \times N}$ denotes an all-ones $M \times N$ matrix. The $i^{th}$ row and $j^{th}$ column of matrix $A$ is represented by $A_{i,:}$ and $A_{:,j}$, respectively. For a third-order tensor $A \in \mathbb{C}^{M \times N \times T}$, its $(i, j, k)^{th}$ element is denoted by $A_{i,j,k}$, while $A_{i,:,:}$, $A_{:,j,:}$ and $A_{:,,:,k}$ represent the row vectors $[A_{i,j,1}, A_{i,j,2}, \ldots, A_{i,j,T}]$, $[A_{i,1,k}, A_{i,2,k}, \ldots, A_{i,N,k}]$ and $[A_{1,j,k}, A_{2,j,k}, \ldots, A_{M,j,k}]$, respectively.

II. SYSTEM MODEL

We consider a massive MIMO system with antennas arranged in a uniform rectangular array (URA) at the BS, as shown in Fig. 1. There are totally $M \times N$ antennas, with vertical inter-antenna spacing $d_1$ and horizontal inter-antenna spacing $d_2$. Obviously, the URA degenerates to the conventional uniform linear array when $M$ or $N$ equals 1. For the mobile terminals, each is equipped with one antenna, and the uplink signal goes through a channel with $L$ dominant paths, each has a corresponding elevation angle $\theta_l$ and azimuth angle $\phi_l$. The discrete-time complex baseband signal received by the $(m, n)^{th}$ antenna at BS can be expressed as

$$y_{m,n}(k) = \sum_{l=1}^{L} s(k)\alpha_l(k)\exp\{-j[(m-1)u_l + (n-1)v_l]\} + w_{m,n}(k) + e_{m,n}(k) \quad (1)$$

where $u_l = \frac{2\pi d_1}{\lambda} \cos \theta_l$, $v_l = \frac{2\pi d_2}{\lambda} \sin \theta_l \cos \phi_l$ with $\lambda$ being the wavelength of the carrier signal; $s(k)$ is the transmitted symbol at time $k$; the path gain $\alpha_l(k)$ follows the zero-mean wide-sense stationary random process with correlation $\mathbb{E}[\alpha_l(k)\alpha_l^*(k+\tau)] = \delta(l-l')r_l(\tau)$; the additive noise $w_{m,n}(k) \sim \mathcal{C}\mathcal{N}(0, \beta^{-1})$ is assumed to be independent over $m$, $n$ and $k$; the potential bad datum $e_{m,n}(k)$ takes an unknown value if hardwares associated with the $(m, n)^{th}$ antenna fail to work normally at time $k$, and otherwise takes the zero value.

Our goal is to estimate $\{\phi_l\}_{l=1}^{L}$ and $\{\theta_l\}_{l=1}^{L}$ with correct paring, assuming no knowledge of path number $L$, noise power $\beta^{-1}$, path gain correlation $r_l(\tau)$ and bad data statistics. In conventional 2D DOA estimation, after collecting $T$ measurement snapshots, (1) is formulated into a matrix representation as

$$Y = (A[u] \diamond A[v])Z^T + W + E \quad (2)$$

where the $k^{th}$ column of $Y \in \mathbb{C}^{MN \times T}$ is obtained by stacking the data sampled at time $k$ into a long $MN \times 1$ vector, and $W, E$ are stacked in the same way as $Y$. Furthermore, in (2), $A[u] \in \mathbb{C}^{MN \times L}$, $A[v] \in \mathbb{C}^{N \times L}$ and $Z \in \mathbb{C}^{T \times L}$ are matrices with the $l^{th}$ column defined as $a(u_l) = [1, \exp(-ju_l), \ldots, \exp(-j(M-1)u_l)]^T$, $a(v_l) = [1, \exp(-jv_l), \ldots, \exp(-j(N-1)v_l)]^T$. 

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and \(z_l = [s(1)\alpha_l(1), s(2)\alpha_l(2), \ldots, s(T)\alpha_l(T) ]^T\), respectively. Since the DOA pairs \(\{\phi_l, \theta_l\}_{l=1}^L\) are encoded in the columns of \(A[u] \odot A[v]\), conventional 2D DOA estimators can be applied after the signal subspace is obtained from SVD of \(Y\).

However, it is obvious in (1) that the received data \(\{y_{m,n}(k)\}_{m=1,n=1,k=1}^{M,N,T}\) inherently constitute a 3D structure, thereby it is more natural to represent them using a third-order tensor \(Y \in \mathbb{C}^{M \times N \times T}\).

From the definition of a tensor [24], the system model in (1) can be expressed as

\[Y = J_{A[u], A[v]} Z_K + W + \mathcal{E} (3)\]

where \([A[u], A[v], Z]\triangleq \sum_{l=1}^L a(u_l) \odot a(v_l) \odot z_l\). With the tensor model (3), the HOSVD can be applied to \(Y\) to recover the range space of \(A[u] \odot A[v]\) [19]. It is proved theoretically [20] that when the path number \(L\) is strictly less than \(M\) or \(N\), the performance of the tensor approach is better than that of the matrix counterpart. Unfortunately, as mentioned in the Introduction, HOSVD suffers from problems such as being susceptible to bad data and requiring prior knowledge of the path number.

### III. Subspace Estimation and Bayesian Modelling

#### A. Property on Subspace Estimation

To recover the subspaces from the tensor represented data in (3) while avoiding the practical problems that HOSVD faces, the following property that links the tensor algebra to the subspace estimation is introduced first.

**Property 1.** For any matrices \(\Xi \in \mathbb{C}^{M \times L}\), \(\Upsilon \in \mathbb{C}^{N \times L}\) and \(\Pi \in \mathbb{C}^{T \times L}\) with \(L \leq \min\{M,N,T\}\), if \([A[u], A[v], Z] = [\Xi, \Upsilon, \Pi]\) (i.e., \(\sum_{l=1}^L a(u_l) \odot a(v_l) \odot z_l = \sum_{l=1}^L \Xi_{:,l} \odot \Upsilon_{:,l} \odot \Pi_{:,l}\)), the following statements hold:

- **P1)** If \(\{\Xi \odot \Upsilon, \Pi\}\) are of full column rank, the columns of \(\Xi \odot \Upsilon\) span the same subspace as the range space of \(A[u] \odot A[v]\);
- **P2)** If \(\{\Xi, \Upsilon, \Pi\}\) are of full column rank, the columns of \(\Xi\) and \(\Upsilon\) span the same subspaces as those spanned by the columns of \(A[u]\) and \(A[v]\) respectively.

**Proof:** If \([A[u], A[v], Z] = [\Xi, \Upsilon, \Pi]\), after tensor unfolding operations [24], it can be shown that

\[(\Xi \odot \Upsilon) \Pi^T = (A[u] \odot A[v]) Z^T (4)\]
$$\Xi (\Upsilon \odot \Pi)^T = A [u] (A [v] \odot Z)^T$$

$$\Upsilon (\Xi \odot \Pi)^T = A [v] (A [u] \odot Z)^T.$$  

For $P1)$, as $\Pi^T$ has a full row rank, the right pseudoinverse $(\Pi^T)^\dagger$ exists. From (4), it can be shown that the matrix $T_1 = Z^T (\Pi^T)^\dagger$ would make $\Xi \odot \Upsilon = (A [u] \odot A [v]) T_1$. As $\Xi \odot \Upsilon$ and $A [u] \odot A [v]$ are of full column rank, $T_1$ must be a non-singular matrix, and therefore $P1)$ holds. On the other hand, if $\Xi$, $\Upsilon$ and $\Pi$ are of full column rank, the right pseudoinverse $((\Upsilon \odot \Pi)^T)^\dagger$ and $((\Xi \odot \Pi)^T)^\dagger$ exist. From (5) and (6), it can be shown that $T_2 = (A [v] \odot Z)^T ((\Upsilon \odot \Pi)^T)^\dagger$ and $T_3 = (A [u] \odot Z)^T ((\Xi \odot \Pi)^T)^\dagger$ would make $\Xi = A [u] T_2$ and $\Upsilon = A [v] T_3$ hold. As $\Xi$ and $A [u]$ are of full column rank, $T_2$ must be non-singular. Similarly, $T_3$ is also non-singular. Therefore, $P2)$ holds.

**Property 1** points out that subspaces recovery is equivalent to the matrices estimation for $\Xi \in \mathbb{C}^{M \times L}$, $\Upsilon \in \mathbb{C}^{N \times L}$ and $\Pi \in \mathbb{C}^{T \times L}$ subject to the constraints $[\Xi, \Upsilon, \Pi] = [A [u], A [v], Z]$. The only difference between the two cases in **Property 1** is that for the first case, we form $\Xi \odot \Upsilon$ before DOA estimation, while for the latter case, DOA estimation can be directly performed on $\Xi$ and $\Upsilon$. Furthermore, the condition in $P2)$ is more stringent, thus if $\{\Xi, \Upsilon, \Pi\}$ are of full column rank, we also have the columns of $\Xi \odot \Upsilon$ spanning the same subspace as the range space of $A [u] \odot A [v]$. To estimate $\Xi$, $\Upsilon$ and $\Pi$ based on the framework of probabilistic inference, a probabilistic model that encodes our knowledge about observations and the unknowns is established in the following.

**B. Bayesian Modelling**

With the noise component $W$ being white and zero-mean circularly-symmetric complex Gaussian, the conditional likelihood of $Y$ is

$$p (Y | [\Xi, \Upsilon, \Pi], \mathcal{E}, \beta) \propto \exp (\beta \| Y - [\Xi, \Upsilon, \Pi] - \mathcal{E} \|^2_F).$$  

(7)

For $\beta$, since it is unknown, we treat it as a random variable and impose the non-informative Jeffrey’s prior [28] as

$$p (\beta) \propto \beta^{-1}.$$

(8)

Furthermore, the generative model for bad datum $\mathcal{E}_{i,j,k}$ is also unknown, and the only information at hand is that $\mathcal{E}_{i,j,k}$ is very likely to be zero due to rare occurrence of hardware instabilities.
As the student-t distribution is capable of modeling outliers, we take the prior of $E_{i,j,k}$ to be $p(E_{i,j,k}) = T(E_{i,j,k} \mid 0, a_{i,j,k}, b_{i,j,k})$. In addition, as the potential bad data $E$ is attributed to various hardware imperfections in RF chains and A/D-D/A converters, the statistics of bad data such as mean and correlations are hard to obtain. Therefore, we set $a_{i,j,k}$ and $b_{i,j,k}$ to be $10^{-6}$ to produce a non-informative prior on $E_{i,j,k}$ \[28\], and assume the $E_{i,j,k}$ are independent of each other, i.e.,

\[
p(E) = \prod_{i=1}^{M} \prod_{j=1}^{N} \prod_{k=1}^{T} T(E_{i,j,k} \mid 0, 10^{-6}, 10^{-6}) . \tag{9}
\]

To facilitate the Bayesian inference procedure, the student-t distribution can be equivalently represented as a Gaussian scale mixture \[36\] as follows:

\[
T(E_{i,j,k} \mid 0, a_{i,j,k}, b_{i,j,k}) = \int CN(E_{i,j,k} \mid 0, \zeta_{i,j,k}^{-1}) \Gamma(\zeta_{i,j,k} \mid a_{i,j,k}, b_{i,j,k}) \, d\zeta_{i,j,k}. \tag{10}
\]

This means that the student-t prior can be obtained by mixing up infinite zero-mean circularly-symmetric complex Gaussian distributions where the mixing distribution on the precision $\zeta_{i,j,k}$ is the Gamma distribution having parameters $a_{i,j,k}$ and $b_{i,j,k}$.

For $\Xi$, $\Upsilon$ and $\Pi$, as the matrix second dimension $L$ (which represents path number) is not known, we take $R > L$ as an upper bound for the common column number. To model this knowledge, the columns of $\Xi$, $\Upsilon$ and $\Pi$ are assumed to be independent zero-mean circularly-symmetric complex Gaussian vectors with the $l$th column of the three matrices having a common precision $\gamma_l$. That is,

\[
p(\Xi, \Upsilon, \Pi) = p(\Xi) p(\Upsilon) p(\Pi)
= \prod_{l=1}^{R} \left\{ CN(\Xi_{:,l} \mid 0_{M \times 1}, \gamma_l^{-1}I_M) CN(\Upsilon_{:,l} \mid 0_{N \times 1}, \gamma_l^{-1}I_N) CN(\Pi_{:,l} \mid 0_{T \times 1}, \gamma_l^{-1}I_T) \right\} \tag{11}
\]

and the precision $\gamma_l$ follows Gamma distribution

\[
p(\gamma_l) = \Gamma(\gamma_l \mid c_l, d_l) \tag{12}
\]

where $c_l$ and $d_l$ are the hyper-parameters. If the power distribution of a certain channel path is known, we can adjust $c_l$ and $d_l$ to match this prior knowledge. Otherwise, $c_l$ and $d_l$ can be chosen as small values (e.g., $10^{-6}$) to produce the non-informative prior on $\gamma_l$. 
IV. VARIATIONAL PARAMETER INFERENCE

Let $\Theta$ be a set containing the unknown variables $E, \Xi, \Upsilon, \Pi, \beta, \{\gamma_l\}_{l=1}^{R}$ and latent variables $\{\zeta_{i,j,k}\}_{i=1,j=1,k=1}^{M,N,T}$ in the probabilistic model. In the Bayesian framework, inference for the variables in $\Theta$ relies on the posterior distribution $p(\Theta \mid \mathcal{Y}) = p(\Theta, \mathcal{Y}) / p(\mathcal{Y})$. However, the computation of the exact posterior distribution is intractable due to multiple integrations in $p(\mathcal{Y}) = \int p(\Theta, \mathcal{Y}) d\Theta$. To handle this problem, previous Bayesian tensor analysis is mainly based on Monte Carlo statistical methods, such as Markov chain Monte Carlo (MCMC) and Gibbs sampling, where a large number of random samples are generated from the joint distributions and marginalization is approximated by operations on samples [31], [32]. Although sampling methods can approach the true posteriors when the number of samples approaches infinity, it is computationally demanding. More recently, variational inference has been explored to give deterministic approximations of posteriors with high accuracy and low complexity [28], [30].

In the variational inference approach, we find a variational distribution $Q(\Theta)$ that closely approximates the true posterior distribution $p(\Theta \mid \mathcal{Y})$ by minimizing the KL divergence:

$$\text{KL}(Q(\Theta) \| p(\Theta \mid \mathcal{Y})) \equiv -\mathbb{E}_{Q(\Theta)} \left\{ \ln \frac{p(\Theta \mid \mathcal{Y})}{Q(\Theta)} \right\}.$$  \hspace{1cm} (13)

The KL divergence vanishes when $Q(\Theta) = p(\Theta \mid \mathcal{Y})$ if no constraint is imposed on $Q(\Theta)$, which however leads us back to the original intractable posterior distribution. In order to avoid this problem, we invoke the commonly used mean-field approximation, which assumes the variational distribution takes a fully factorized form as $Q(\Theta) = \prod Q(\Theta_k)$ where $\Theta_k \in \Theta$. Under this approximation, $Q(\Theta_k)$ that minimizes the KL divergence (13) can be found using [28]

$$Q(\Theta_k) \propto \exp \left\{ \mathbb{E}_{\prod_{j \neq k} Q(\Theta_j)} \left[ \ln p(\mathcal{Y}, \Theta) \right] \right\}$$  \hspace{1cm} (14)

where

$$p(\mathcal{Y}, \Theta) \propto \exp \left\{ MNT \ln \beta - \beta \| \mathcal{Y} - [\Xi, \Upsilon, \Pi] - \mathcal{E} \|^2_F + (M + N + T) \sum_{l=1}^{R} \ln \gamma_l \right. \left. - \operatorname{Tr} \left\{ \Gamma (\Xi^H \Xi + \Upsilon^H \Upsilon + \Pi^H \Pi) \right\} + \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{T} (\ln \zeta_{i,j,k} - \zeta_{i,j,k} \mathcal{E}_{i,j,k}^* \mathcal{E}_{i,j,k}) \right. \left. + \sum_{i=1}^{R} [(c_l - 1) \ln \gamma_l - d_l \gamma_l] - \ln \beta + \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{T} [(a_{i,j,k} - 1) \ln \zeta_{i,j,k} - b_{i,j,k} \zeta_{i,j,k}] \right\}$$  \hspace{1cm} (15)
with $\Gamma = \text{diag} \{ \gamma_1, \gamma_2, ..., \gamma_R \}$. Using (14) and (15), the distributions $Q(\Theta_k)$ for different $\Theta_k \in \Theta$ are evaluated in the following.

### A. Derivations for $Q(\Xi)$, $Q(\Upsilon)$ and $Q(\Pi)$

Using (14) and (15), $Q(\Xi)$ is derived in Appendix A to be

$$Q(\Xi) \propto \exp \left\{ - \text{Tr} \left\{ \Xi^* (\Sigma^\Xi)^{-1} \Xi^T - (M^\Xi)^* (\Sigma^\Xi)^{-1} \Xi^T - \Xi^* (\Sigma^\Xi)^{-1} (M^\Xi)^T \right\} \right\} \tag{16}$$

with

$$M^\Xi \triangleq \mathbb{E}_{Q(\beta)}[\beta] \left( \mathbb{E}_{Q(\Upsilon)} \left[ \Upsilon^{(1)} [\Upsilon - \mathcal{E}] \right] \right) \left( \mathbb{E}_{Q(\Upsilon)} \left[ \Upsilon \right] \circ \mathbb{E}_{Q(\Pi)} \left[ \Pi \right] \right)^* (\Sigma^\Xi)^* \tag{17}$$

$$[\Sigma^\Xi]^{-1} \triangleq \mathbb{E}_{Q(\beta)}[\beta] \mathbb{E}_{Q(\Upsilon)Q(\Pi)} \left[ (\Upsilon \circ \Pi)^H (\Upsilon \circ \Pi) \underbrace{\begin{bmatrix} \Upsilon \circ \Pi \end{bmatrix}}_{\mathcal{H}_1} \right] + \text{diag} \left\{ \mathbb{E}_{Q(\gamma_1)} \left[ \gamma_1 \right], ..., \mathbb{E}_{Q(\gamma_R)} \left[ \gamma_R \right] \right\} \tag{18}$$

where $\{\Upsilon^{(i)} [\mathcal{A}]\}_{i=1,2,3}$ denotes the unfolding operation on a third-order tensor $\mathcal{A}$ along the $i^{th}$ mode [24]. After completing the square over $\Xi$, we see that (16) can be expressed as a circularly-symmetric complex matrix normal distribution. More specifically, with $\mathcal{CMN}(X \mid M, \Sigma_r, \Sigma_c)$ representing the distribution $p(X) \propto \exp\left\{ - \text{Tr} \left\{ \Sigma_c^{-1} (X - M)^H \Sigma_r^{-1} (X - M) \right\} \right\}$ [38], we have $Q(\Xi) = \mathcal{CMN}(\Xi \mid M^\Xi, I_M, (\Sigma^\Xi)^*)$.

From (16)-(18), it is obvious that evaluation of $Q(\Xi)$ requires some expectation calculations. For those with the form $\mathbb{E}_{Q(\Theta_k)}[\Theta_k]$ where $\Theta_k \in \Theta$, the value can be easily obtained if the corresponding $Q(\Theta_k)$ is available. On the other hand, the term $a_1$ can be computed using the property below, which is the complex-valued extension to the Theorem 3.1 in [35].

**Property 2.** For any two matrices $A \in \mathbb{C}^{k_1 \times k_3} \sim \mathcal{CMN}(A \mid M^A, I_{k_1}, \Sigma^A)$ and $B \in \mathbb{C}^{k_2 \times k_3} \sim \mathcal{CMN}(B \mid M^B, I_{k_2}, \Sigma^B)$, $\mathbb{E}_{p(A)p(B)}[ (A \circ B)^H (A \circ B) ]$ can be computed as

$$\mathbb{E}_{p(A)p(B)}[ (A \circ B)^H (A \circ B) ] = \left[ (M^A)^H (M^A) + k_1 \Sigma^A \right] \circ \left[ (M^B)^H (M^B) + k_2 \Sigma^B \right]. \tag{19}$$

**Proof:** See Appendix B.

Similarly, $Q(\Upsilon)$ and $Q(\Pi)$ can be found to be

$$Q(\Upsilon) = \mathcal{CMN} \left( \Upsilon \mid M^\Upsilon, I_N, (\Sigma^\Upsilon)^* \right) \tag{20}$$

$$Q(\Pi) = \mathcal{CMN} \left( \Pi \mid M^\Pi, I_I, (\Sigma^\Pi)^* \right) \tag{21}$$
with the means and covariances expressed as

\[
M_\beta \triangleq E_{Q(\beta)}[\beta] \left( E_{Q(\beta)} \left[ M^{(2)} [Y - \mathcal{E}] \right] \right) \left( E_{Q(\Pi)}[\Pi] \odot E_{Q(\Xi)}[\Xi] \right)^* (\Sigma^\beta)^* \tag{22}
\]

\[
[\Sigma^\beta]^{-1} \triangleq E_{Q(\beta)}[\beta] \left[ \left( E_{Q(\Xi)}[\Pi] \odot E_{Q(\Xi)}[\Xi] \right)^H \left( \Pi \odot \Xi \right) \right] + \text{diag}\left\{ E_{Q(\gamma_1)}[\gamma_1], \ldots, E_{Q(\gamma_R)}[\gamma_R] \right\} \tag{23}
\]

\[
M_\Xi \triangleq E_{Q(\beta)}[\beta] \left( E_{Q(\beta)} \left[ M^{(3)} [Y - \mathcal{E}] \right] \right) \left( E_{Q(\Xi)}[\Xi] \odot E_{Q(\gamma)}[\gamma] \right)^* (\Sigma^\Xi)^* \tag{24}
\]

\[
[\Sigma^\Xi]^{-1} \triangleq E_{Q(\beta)}[\beta] \left[ \left( E_{Q(\Xi)}[\Xi] \odot E_{Q(\gamma)}[\gamma] \right)^H \left( \Xi \odot \gamma \right) \right] + \text{diag}\left\{ E_{Q(\gamma_1)}[\gamma_1], \ldots, E_{Q(\gamma_R)}[\gamma_R] \right\} \tag{25}
\]

where \(a_2\) and \(a_3\) can be computed using Property 2.

B. Derivation for \(Q(\mathcal{E})\)

By substituting (15) into (14) and only taking the terms relevant to \(\mathcal{E}\), \(Q(\mathcal{E})\) is expressed as

\[
Q(\mathcal{E}) \propto \prod_{i=1}^{M} \prod_{j=1}^{N} \prod_{k=1}^{T} \exp \left\{ E_{\Pi_{\theta_i \neq \theta_j}} Q(\theta_j) \right\} \left[ -\beta \left| Y_{i,j,k} - \sum_{l=1}^{R} \Xi_{i,l} Y_{j,l} \Pi_{k,l} - \mathcal{E}_{i,j,k} \right|^2 - \zeta_{i,j,k} \mathcal{E}_{i,j,k}^* \mathcal{E}_{i,j,k} \right\}. \tag{26}
\]

Taking expectations, the term inside the exponent of (26) is

\[
- \mathcal{E}_{i,j,k}^* \left( E_{Q(\beta)}[\beta] + E_{\zeta_{i,j,k}}[\zeta_{i,j,k}] \right) \mathcal{E}_{i,j,k} \equiv p_{i,j,k}
\]

\[
+ 2\Re \left[ \mathcal{E}_{i,j,k}^* p_{i,j,k} E_{Q(\beta)}[\beta] p_{i,j,k}^{-1} \left( Y_{i,j,k} - \sum_{l=1}^{R} E_{Q(\Xi)}[\Xi]_{i,l} E_{Q(\gamma)}[\gamma]_{j,l} E_{Q(\Pi)}[\Pi]_{k,l} \right) \right]. \tag{27}
\]

From the expressions above, it is easy to conclude that

\[
Q(\mathcal{E}) = \prod_{i=1}^{M} \prod_{j=1}^{N} \prod_{k=1}^{T} Q(\mathcal{E}_{i,j,k}) \tag{28}
\]

where \(Q(\mathcal{E}_{i,j,k}) = \mathcal{CN} \left( \mathcal{E}_{i,j,k} \mid m_{i,j,k}, p_{i,j,k}^{-1} \right)\).

Notice that from (27), the computation of bad data mean \(m_{i,j,k}\) can be rewritten as \(m_{i,j,k} = n_1 n_2\), where \(n_1 = \left( E_{Q(\zeta_{i,j,k})}[\zeta_{i,j,k}] \right)^{-1} \) and \(n_2 = \left( Y_{i,j,k} - \sum_{l=1}^{R} E_{Q(\Xi)}[\Xi]_{i,l} E_{Q(\gamma)}[\gamma]_{j,l} E_{Q(\Pi)}[\Pi]_{k,l} \right).\) From the system model in (3), it can be seen that \(n_2\) is the estimated bad data plus noise. On the other hand, since \(E_{Q(\zeta_{i,j,k})}[\zeta_{i,j,k}]^{-1}\) and \(E_{Q(\beta)}[\beta]^{-1}\) can be interpreted as the
estimated power of the bad data and the noise respectively, \( n_1 \) represents the proportion of the bad data in the estimated bad data plus noise. Therefore, if the estimated power of the bad data \( (\mathbb{E}_Q(\zeta_{i,j,k})[\zeta_{i,j,k}])^{-1} \) goes to zero, the bad data mean \( m_{i,j,k} \) becomes zero accordingly, indicating that the hardwares associated with the \((i,j)\)\(^{th}\) antenna work well at time \( k \).

C. Derivations for \( Q(\gamma_l) \), \( Q(\zeta_{i,j,k}) \) and \( Q(\beta) \)

Again using (14) and (15), \( Q(\gamma_l) \) is found to be

\[
Q(\gamma_l) \propto \exp \left\{ \left( \bar{c}_l + M + N + T + 1 \right) \ln \gamma_l \right. \\
- \gamma_l \left[ d_l + \left( \mathbb{E}_Q(\Xi) \left[ \Xi \right] + \mathbb{E}_Q(\Upsilon) \left[ \Upsilon \right] + \mathbb{E}_Q(\Pi) \left[ \Pi \right] \right)_l \right] \right\} 
\]

(29)

which has the same functional form as the Gamma distribution, i.e., \( Q(\gamma_l) = \text{Gamma}(\gamma_l | \bar{c}_l, \bar{d}_l) \). Since \( \mathbb{E}_Q(\gamma_l)[\gamma_l] = \bar{c}_l/\bar{d}_l \) is required for updating other variables in \( \Theta \), we need to compute \( \bar{c}_l \) and \( \bar{d}_l \). While computation of \( \bar{c}_l \) is straightforward, \( \mathbb{E}_Q(\Xi)[\Xi] \), \( \mathbb{E}_Q(\Upsilon)[\Upsilon] \) and \( \mathbb{E}_Q(\Pi)[\Pi] \) are required for \( \bar{d}_l \), and they can be computed using Property 2 by simply setting

\[ \mathbf{B} \sim \mathcal{CMN}(\mathbf{B} | \mathbf{1}_{1 \times N_3}, 1, \mathbf{0}_{R \times R}) \].

More specifically, the resultant expressions are \( \mathbb{E}_Q(\Xi)[\Xi] = [\mathbf{M}^\Xi]^H \mathbf{M} + \mathbf{M}(\Sigma^\Xi)^* \), \( \mathbb{E}_Q(\Upsilon)[\Upsilon] = [\mathbf{M}^\Upsilon]^H \mathbf{M}^\Upsilon + \mathbf{N}(\Sigma^\Upsilon)^* \) and \( \mathbb{E}_Q(\Pi)[\Pi] = [\mathbf{M}^\Pi]^H \mathbf{M}^\Pi + \mathbf{T}(\Sigma^\Pi)^* \).

Also, \( Q(\zeta_{i,j,k}) \) and \( Q(\beta) \) can be derived to be Gamma distributions

\[
Q(\zeta_{i,j,k}) = \text{Gamma} \left( \zeta_{i,j,k} | \bar{a}_{i,j,k}, \bar{b}_{i,j,k} \right) 
\]

(30)

\[
Q(\beta) = \text{Gamma} \left( \beta | \bar{e}, \bar{f} \right) 
\]

(31)

with parameters defined as

\[
\bar{a}_{i,j,k} \triangleq a_{i,j,k} + 1
\]

(32)

\[
\bar{b}_{i,j,k} \triangleq b_{i,j,k} + (m_{i,j,k})^* m_{i,j,k} + \frac{1}{p_{i,j,k}}
\]

(33)

\[
\bar{e} \triangleq MNT
\]

(34)

\[
\bar{f} \triangleq \mathbb{E}_Q(\Xi)Q(\Upsilon)Q(\Pi)Q(\varepsilon) \left[ \| \Upsilon - [\Xi, \Upsilon, \Pi] - \mathcal{E} \|_F^2 \right].
\]

(35)
Similar to $Q(\gamma_l)$, $\mathbb{E}_{Q(\zeta_{i,j,k})}[\zeta_{i,j,k}]$ and $\mathbb{E}_{Q(\beta)}[\beta]$ can be computed by $\tilde{a}_{i,j,k}/\tilde{b}_{i,j,k}$ and $\tilde{e}/\tilde{f}$. For $\tilde{a}_{i,j,k}, \tilde{b}_{i,j,k}$ and $\tilde{e}$, the computations are straightforward. For $\tilde{f}$, it is derived in Appendix C to be

$$
\tilde{f} = \| \mathcal{Y} - [M^\Xi, M^\Upsilon, M^\Pi] - \mathbb{E}_{Q(\mathcal{E})}[\mathcal{E}] \|_F^2 + T \left\{ M \left( (M^\Upsilon)^T M^\Upsilon \right) \circ (M^\Pi)^T M^\Pi \right\} \Sigma^\Xi + N \left( (M^\Xi)^T M^\Xi \right) \circ (M^\Pi)^T M^\Pi \Sigma^\Upsilon + MN \left( (M^\Pi)^T M^\Pi \right) (\Sigma^\Xi \circ \Sigma^\Upsilon) + 2N \left( (M^\Xi)^T M^\Xi \right) (\Sigma^\Xi \circ \Sigma^\Pi) + MNT \left( (M^\Upsilon)^T M^\Upsilon \right) \Sigma^\Pi \right\} + N \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^T p_{i,j,k}^{-1},
$$

(36)

Although the equation (36) for computing $\tilde{f}$ is complicated, its meaning is clear when we refer to its definition in equation (35), from which it can be seen that $\tilde{f}$ represents the estimate of the overall noise power.

D. Iterative Algorithm for Subspace Estimation

From the expressions of $\{Q(\Xi), Q(\Upsilon), Q(\Pi), Q(\mathcal{E}), Q(\gamma_l), Q(\zeta_{i,j,k}), Q(\beta)\}$ evaluated above, it is seen that the calculation of a particular $Q(\Theta_k)$ relies on the statistics of other variables in $\Theta$. As a result, variational distributions for each variable in $\Theta$ should be iteratively updated. Notice that the convergence of variational inference is guaranteed due to the fact that KL divergence in (13) is convex with respect to $Q(\Theta)$ [29]. The iterative algorithm is summarized as follows.

Initializations:

Choose $R > L$ and initial values $\{M^{\Xi,0}, M^{\Upsilon,0}, M^{\Pi,0}, \Sigma^{\Xi,0}, \Sigma^{\Upsilon,0}, \Sigma^{\Pi,0}\}$ and $\{\tilde{c}_l^0, \tilde{d}_l^0, \tilde{a}_{i,j,k}^0, \tilde{b}_{i,j,k}^0, \tilde{e}_l^0, \tilde{f}_l^0\}$ for all $l$, $i$, $j$ and $k$.

Iterations: For the $t^{th}$ iteration,

Update $\{p_{i,j,k}, m_{i,j,k}\}_{i=1,j=1,k=1}^{M,N,T}$

$$
p_{i,j,k}^t = \frac{\tilde{e}_{i,j,k}^{t-1}}{\tilde{f}_{i,j,k}^{t-1}} + \frac{\tilde{a}_{i,j,k}^{t-1}}{\tilde{b}_{i,j,k}^{t-1}},
$$

(37)

$$
m_{i,j,k}^t = \frac{\tilde{e}_{i,j,k}^{t-1}}{\tilde{f}_{i,j,k}^{t-1}} p_{i,j,k}^t \left( \mathcal{Y}_{i,j,k} - \sum_{l=1}^R M^{\Xi,t-1}_{i,l} M^{\Upsilon,t-1}_{j,l} M^{\Pi,t-1}_{k,l} \right);
$$

(38)
Update $M^{\Xi}$, $\Sigma^{\Xi}$

$$\Sigma^{\Xi,t} = \left( \frac{\bar{c}^{t-1}}{\bar{f}^{t-1}} \left[ M^{\Xi,t-1}, M^{\Pi,t-1}, \Sigma^{\Xi,t-1}, \Sigma^{\Pi,t-1} \right] + \text{diag}\left\{ \frac{\bar{c}_1^{t-1}}{\bar{d}_1^{t-1}}, \ldots, \frac{\bar{c}_R^{t-1}}{\bar{d}_R^{t-1}} \right\} \right)^{-1}$$

$$M^{\Xi,t} = \frac{\bar{c}^{t-1}}{\bar{f}^{t-1}} \left( \mathcal{U}(1) \left[ \mathcal{Y} - \mathcal{M}^t \right] \right) \left( M^{\Xi,t-1} \circ M^{\Pi,t-1} \right)^* \left( \Sigma^{\Xi,t} \right)^*$$

where $\mathcal{M}$ is a tensor with its $(i, j, k)^{th}$ elements being $m_{i,j,k}$, and $\mathcal{U} [A, B, G, H]$ operating on $A \in \mathbb{C}^{k_1 \times k_3}$, $B \in \mathbb{C}^{k_1 \times k_3}$ and $\{G, H\} \in \mathbb{C}^{k_3 \times k_3}$ is defined as

$$\mathcal{U} [A, B, G, H] = (A^H A + \kappa_1 G^*) \odot (B^H B + \kappa_2 H^*)$$

Update $M^{\Psi}$, $\Sigma^{\Psi}$

$$\Sigma^{\Psi,t} = \left( \frac{\bar{c}^{t-1}}{\bar{f}^{t-1}} \left[ M^{\Psi,t-1}, M^{\Xi,t}, M^{\Pi,t-1}, \Sigma^{\Xi,t} \right] + \text{diag}\left\{ \frac{\bar{c}_1^{t-1}}{\bar{d}_1^{t-1}}, \ldots, \frac{\bar{c}_R^{t-1}}{\bar{d}_R^{t-1}} \right\} \right)^{-1}$$

$$M^{\Psi,t} = \frac{\bar{c}^{t-1}}{\bar{f}^{t-1}} \left( \mathcal{U}(2) \left[ \mathcal{Y} - \mathcal{M}^t \right] \right) \left( M^{\Pi,t-1} \circ M^{\Xi,t} \right)^* \left( \Sigma^{\Psi,t} \right)^*$$

Update $M^{\Pi}$, $\Sigma^{\Pi}$

$$\Sigma^{\Pi,t} = \left( \frac{\bar{c}^{t-1}}{\bar{f}^{t-1}} \left[ M^{\Xi,t}, M^{\Psi,t}, M^{\Xi,t}, \Sigma^{\Psi,t} \right] + \text{diag}\left\{ \frac{\bar{c}_1^{t-1}}{\bar{d}_1^{t-1}}, \ldots, \frac{\bar{c}_R^{t-1}}{\bar{d}_R^{t-1}} \right\} \right)^{-1}$$

$$M^{\Pi,t} = \frac{\bar{c}^{t-1}}{\bar{f}^{t-1}} \left( \mathcal{U}(3) \left[ \mathcal{Y} - \mathcal{M}^t \right] \right) \left( M^{\Xi,t} \circ M^{\Psi,t} \right)^* \left( \Sigma^{\Pi,t} \right)^*$$

Update $\{\tilde{a}_{i,j,k}, \tilde{b}_{i,j,k}\}_{i=1,j=1,k=1}^{M,N,T}$, $\{\tilde{c}_t, \tilde{d}_t\}_{t=1}^R$, and $\{\tilde{c}, \tilde{f}\}$

$$\tilde{a}_{i,j,k}^t = a_{i,j,k}^0 + 1$$

$$\tilde{b}_{i,j,k}^t = b_{i,j,k}^0 + (m_{i,j,k}^t)^t m_{i,j,k}^t + 1/p_{i,j,k}^t$$

$$\tilde{c}_t^t = c_t^0 + M + N + T$$

$$\tilde{d}_t^t = d_t^0 + b_t^t$$

$$\tilde{c}^t = MNT$$

$$\tilde{f}^t = \| \mathcal{Y} - [M^{\Xi,t}, M^{\Psi,t}, M^{\Pi,t}] - \mathcal{M}^t \|_F^2 + c^t$$
where
\[
\begin{align*}
    b_i^t &= \left( M_{\Xi,i}^t \right)^H M_{\Xi,i}^t + \left( M_{\Pi,i}^t \right)^H M_{\Pi,i}^t + N \sum_{i,j}^t + T \sum_{i,j}^t + M \sum_{i,j}^t + N \sum_{i,j}^t + T \sum_{i,j}^t \\
    c_t^t &= \text{Tr} \left\{ M \left[ \left( M_{\Pi,i}^t \right)^H M_{\Pi,i}^t \right] \circ \left[ \left( M_{\Pi,i}^t \right)^H M_{\Pi,i}^t \right] \right\} + M N \left[ \left( M_{\Pi,i}^t \right)^H M_{\Pi,i}^t \right] \left( \sum_{i,j}^t \circ \sum_{i,j}^t \right) \\
    \end{align*}
\]

\[\text{Until Convergence}\]

During the iterations, some \( \{ \tilde{c}_i/d_i \}_{i=1}^R \) will stay very large, indicating that \( \tilde{c}_i/d_i \) has converged and the corresponding columns of \( \Xi, \Upsilon \) and \( \Pi \) are all-zero with very high probability. Therefore, these redundant columns can be pruned out. After convergence, the subspace spanned by columns in \( A[u] \circ A[v] \) is given by the range space of \( M_{\Xi} \circ M_{\Upsilon} \). Furthermore, if \( M_{\Xi} \) and \( M_{\Upsilon} \) are of full column rank, the subspace spanned by the columns in \( A[u] \) and \( A[v] \) is given by the range space of \( M_{\Xi} \) and \( M_{\Upsilon} \) respectively.

\textbf{Remark 1:} When \( L > \max\{M,N\} \), we have \( \text{rank}(A[u]) < L \) and \( \text{rank}(A[v]) < L \), and therefore the proposed method will recover \( \Xi \) and \( \Upsilon \) both with the column number smaller than \( L \). This violates the dimension assumptions stated in \textbf{Property 1}. However, if the data model is represented as \( \tilde{Y} = [A[u,v],1_{1\times L},Z] + \tilde{W} + \tilde{E} \in \mathbb{C}^{MN \times 1 \times T} \) where \( A[u,v] = A[u] \circ A[v] \) with rank \( L \), the proposed algorithm will recover a \( \Xi \in \mathbb{C}^{MN \times L} \) with independent columns. This leads to a full column rank matrix \( \Xi \circ 1_{1 \times L} \) with the range space identical to the subspace spanned by columns of \( A[u,v] \circ 1_{1 \times L} \) as stated in \textbf{Property 1}. Although this representation is equivalent to the matrix model (2), it allows us to reuse the results from the proposed tensor-based algorithm, rather than re-deriving the matrix factorization from (2) using the methods in [39].

More specifically, the likelihood function (7) is now written as \( p(\tilde{Y} \mid [A[u,v],1_{1\times L},Z], \tilde{E}, \beta) \propto \exp\{ -\beta \| \tilde{Y} - [A[u,v],1_{1\times L},Z] - \tilde{W} - \tilde{E} \|_F^2 \} \) and the prior (11) becomes \( p(\Xi, \Upsilon, \Pi) = \prod_{i=1}^R \left\{ \mathcal{C}\mathcal{N}(\Xi_{i:t} \mid \gamma_{i:1}^{-1} I_M) \mathcal{C}\mathcal{N}(\Upsilon_{i:t} \mid 1, 0) \mathcal{C}\mathcal{N}(\Pi_{i:t} \mid 0_{T \times 1}, \gamma_{i:T}^{-1} I_T) \right\} \). As \( \Upsilon \) is fixed to be \( 1_{1 \times R} \), there is no need to update \( \Upsilon \). The updating equations for \( \Sigma, \Xi, \Pi \) and \( \beta \) can be obtained by directly setting \( M_{\Upsilon} = 1_{1 \times R} \) and \( \Sigma_{\Upsilon} = 0_{R \times R} \) in (38), (39), (40), (44), (45), (51) and (53).
Furthermore, the updating equations for the hyperparameters $\tilde{c}_l$ and $\tilde{d}_l$ are obtained by discarding the terms relevant to $\Upsilon$ in (48), (49) and (52).

E. Complexity Analysis

For each iteration stated above, the complexity is dominated by updating $\{M^\Xi, M^\Upsilon, M^\Pi\}$, which costs $O(3R^3 + (M + N + T)R^2 + 3MNTR)$. Empirically, variational inference shows a fast convergence and $R$ approaches $L$ rapidly in the first few iterations [28], [39]. Thus, the overall complexity is about $O\left(p \left[ 3L^3 + (M + N + T)L^2 + 3MNTL \right] \right)$ where $p$ is the iteration number for convergence. In massive MIMO systems, the number of antennas is expected to be very large and it is easy to have $L \ll MN$. Thus, the main complexity of the proposed algorithm is $O(3pMNTL)$. On the other hand, when HOSVD is used for subspace estimation in [19], it only needs the $r$ leading left singular vectors of each unfolding matrix where $r$ is the rank of the unfolding matrix, and an efficient solution called Orthogonal Iterations (also known as Subspace Iterations) exists [37]. Using the efficient Orthogonal Iterations on the unfolding matrices with sizes $M \times NT$, $N \times MT$ and $T \times MN$, the main complexity of the HOSVD based subspace estimator is $O(kMNTL + kMNT \min\{M, L\} + kMNT \min\{N, L\})$ where $k$ is a constant depending on how the Orthogonal Iterations is implemented. Therefore, it can be seen that the complexity of the proposed algorithm is comparable to that of the subspace estimation using HOSVD.

V. LOW-COMPLEXITY DOA ESTIMATORS

As the subspace spanned by the columns in $A^u \odot A^v$ is given by the range space of $M^\Xi \odot M^\Upsilon$, existing 2D DOA estimators such as 2D ESPRIT [16] and 2D MUSIC [17] can be directly applied to $M^\Xi \odot M^\Upsilon$, and the DOA estimators are not repeated here. On the other hand, when $M^\Xi$ and $M^\Upsilon$ are of full column rank, the range space of $A^u$ and $A^v$ are separately recovered, due to Property $P2$). This is possible even in the existence of bad data and demand of determination of number of sources, due to novel features of the proposed algorithm. Therefore, there is no need to reconstruct the range space of the 2D steering matrix for DOA estimation. For completeness, in the following, we summarize how 1D ESPRIT and MUSIC [40] can be used to estimate $\{\theta_l\}_{l=1}^L$ and $\{\phi_l\}_{l=1}^L$. 
For 1D ESPRIT, we first compute $\Psi_u = (J_1^uM^\Xi)\dagger J_2^uM^\Xi$ and $\Psi_v = (J_1^vM^\Upsilon)\dagger J_2^vM^\Upsilon$ where $J_1^u = [I_{M-1}, 0_{(M-1) \times 1}], J_2^u = [0_{(M-1) \times 1}, I_{M-1}], J_1^v = [I_{N-1}, 0_{(N-1) \times 1}],$ and $J_2^v = [0_{(N-1) \times 1}, I_{N-1}]$ are selection matrices [40]. Then, a joint Schur decomposition or a simultaneous diagonalization algorithm is performed on matrices $\Psi_u$ and $\Psi_v$ to acquire $\{\phi_l\}_{l=1}^L, \{\theta_l\}_{l=1}^L$ and their correct paring. On the other hand, for 2D ESPRIT, we first compute $\tilde{\Psi}_u = [J_1^u(M^\Xi \Diamond M^\Upsilon)]\dagger J_2^u(M^\Xi \Diamond M^\Upsilon)$ and $\tilde{\Psi}_v = [J_1^v(M^\Xi \Diamond M^\Upsilon)]\dagger J_2^v(M^\Xi \Diamond M^\Upsilon)$ where $J_1^u = [I_{M-1}, 0_{(M-1) \times 1}] \otimes I_N, J_2^u = [0_{(M-1) \times 1}, I_{M-1}] \otimes I_N, J_1^v = I_M \otimes [I_{N-1}, 0_{(N-1) \times 1}]$ and $J_2^v = I_M \otimes [0_{(N-1) \times 1}, I_{N-1}]$ are selection matrices following 2D ESPRIT principle [19]. Then, a joint Schur decomposition or a simultaneous diagonalization is conducted for matrices $\tilde{\Psi}_u$ and $\tilde{\Psi}_v$ to acquire $\{\phi_l\}_{l=1}^L, \{\theta_l\}_{l=1}^L$ and their paring. In terms of complexity, the most computationally demanding parts in the 1D ESPRIT are to calculate two pseudo-inverses $(J_1^uM^\Xi)\dagger$ and $(J_1^vM^\Upsilon)\dagger$ with size $(M-1) \times L$ and $(N-1) \times L$, respectively. But this is much simpler than the 2D ESPRIT, which requires the calculations of pseudo-inverses with size $(M-1)N \times L$ and $(N-1)M \times L$.

For 1D MUSIC, $\{u_l\}_{l=1}^L$ can be estimated by searching peaks of the spectrum $P_{\text{MUSIC}}(u) = [a^H(u)(I - P_u)a(u)]^{-1}$ where $P_u$ is the projection matrix onto the range space of $M^\Xi$, and then $\{v_l\}_{l=1}^L$ can be estimated by searching peaks of the spectrum $P_{\text{MUSIC}}(v) = [a^H(v)(I - P_v)a(v)]^{-1}$ where $P_v$ is the projection matrix onto the range space of $M^\Upsilon$. Then, the correct paring is obtained by utilizing the fact that only the vector $a(u_l) \Diamond a(v_l)$ with correct paring belongs to the range space of $A[u, v]$. More specifically, we pick up any $\hat{u} \in \{\hat{u}_l\}_{l=1}^L$ and $\hat{v} \in \{\hat{v}_l\}_{l=1}^L$ to compute inner product $[a(\hat{u}) \Diamond a(\hat{v})]^H(I - P_{u,v})[a(\hat{u}) \Diamond a(\hat{v})]$ where $P_{u,v}$ is the projection matrix onto the range space of $M^\Xi \Diamond M^\Upsilon$. Only the correct paring will make the inner product zero. After the paring procedure, $\theta_l$ is estimated by $\hat{\theta}_l = \arccos [(\hat{u}_l\lambda)/(2\pi d_1)]$ and $\phi_l$ is estimated by $\hat{\phi}_l = \arccos [(\hat{v}_l\lambda)/(2\pi d_2 \sin \hat{\theta}_l)]$. Since this approach only needs two 1D MUSIC searching steps, it has a much lower complexity than the 2D MUSIC algorithm, which requires searching in a 2D space. On the other hand, although $\hat{u}_l$ and $\hat{v}_l$ from 1D MUSIC usually touch the 1D Cramer-Rao bounds (CRBs) in finite sample regime [41], $\hat{\theta}_l$ and $\hat{\phi}_l$ are related to them through a nonlinear transformation, which will destroy the efficiency [42], and consequently the performance of $\hat{\theta}_l$ and $\hat{\phi}_l$ cannot touch the corresponding 2D DOA CRB. To fill

\textsuperscript{1}Reference [19] presents a general R-D ESPRIT framework for both matrix represented and tensor represented data, which obviously includes the 2D ESPRIT presented in this paper as a special case.
in the performance gap, we can further execute a 2D MUSIC search at the local neighborhood of $\hat{\theta}_l$ and $\hat{\phi}_l$ to obtain even better estimates.

VI. Simulation Results and Discussions

In this section, numerical simulations are presented to assess the performance of the proposed method for subspace identification and DOA estimation. A URA with size $M = 40, N = 30$ is considered and observations are taken in $T = 20$ snapshots. The vertical inter-antenna spacing $d_1$ and horizontal inter-antenna spacing $d_2$ are set to be $\lambda/2$. The data symbols are quadrature phase-shift keying (QPSK) signal with unit power and the path gains $\alpha_l(k)$ are drawn from zero-mean circularly-symmetric complex Gaussian distribution with unit variance, and without any correlation across $l$ and $k$. There are 5 dominant paths from the mobile terminal to the base station, with elevation DOAs uniformly selected from $\{[7^\circ, 13^\circ], [22^\circ, 28^\circ], [37^\circ, 43^\circ], [52^\circ, 58^\circ], [67^\circ, 73^\circ]\}$ and azimuth DOAs uniformly selected from $\{[25^\circ, 35^\circ], [55^\circ, 65^\circ], [85^\circ, 95^\circ], [115^\circ, 125^\circ], [145^\circ, 155^\circ]\}$ respectively. More specifically, the elevation DOA selected from $[7^\circ, 13^\circ]$ gets paired with the azimuth DOA picked from $[25^\circ, 35^\circ]$, and the other DOA pairs are also formed in corresponding order in the list. Two scenarios are considered: (1) each antenna will produce bad data with probability $\pi = 0.05$; (2) all the antennas work perfectly, i.e., $\pi = 0$. Each bad datum is independently drawn from a circularly-symmetric complex Gaussian distribution $p(e_{i,j,k}) = CN(e_{i,j,k} | 0, 100)$. The signal-to-noise ratio (SNR) is defined as $10 \log_{10}(1/\sigma^2)$ where $\sigma^2$ is the noise power. The proposed algorithm is initialized by setting $\{e_l^0, e_l^0, a_{i,j,k}^0, b_{i,j,k}^0, e_l^0, f_l^0\}$ as $10^{-6}$ for all $l, i, j$ and $k$, the upper bound on the path number $R$ is chosen as $\max\{M, N, T\}$, the initial $M^{X,0}, M^{Y,0}$ and $M^{\Pi,0}$ are drawn from $CMN(X | 0_{M \times R}, I_M, I_R), CMN(X | 0_{N \times R}, I_N, I_R)$ and $CMN(X | 0_{T \times R}, I_T, I_R)$ respectively, and $\{\Sigma^{X,0}, \Sigma^{Y,0}, \Sigma^{\Pi,0}\}$ are all set as $I_R$. The proposed algorithm terminates at the $t^{th}$ iteration when $(\|M^{X,t}, M^{Y,t}, M^{\Pi,t}\| - \|M^{X,t-1}, M^{Y,t-1}, M^{\Pi,t-1}\|)(\|M^{X,t-1}, M^{Y,t-1}, M^{\Pi,t-1}\|)^{-1} < 10^{-10}$. Each point in the figures is an average of 500 Monte-Carlo runs with different realizations of the DOAs, uplink signals, channels and bad data.

First, we examine the performance of subspace recovery, where the performance criterion is the largest principal angle (LPA). To measure the “distance” between two subspaces spanned by the columns of matrices $A$ and $B$, LPA is defined as $\cos^{-1}\{\sigma_{\min}\{\text{orth}(A)^H\text{orth}(B)\}\}$. Fig. 2 presents the convergence performance of the proposed algorithm at SNR=10 dB and
SNR=20 dB. From Fig. 2, it can be seen that the LPAs decrease significantly in the first few iterations and converge to stable values within 10 iterations. This figure not only shows the fast convergence property of the proposed subspace recovery algorithm, its robustness against bad data is also clearly demonstrated. Fig. 3 further depicts the performance of the proposed variational inference approach versus SNR, with comparison to the tensor HOSVD method [19], the matrix SVD approach [40] and the matrix robust PCA approach [39]. In Fig 3, the tensor HOSVD method and the matrix SVD approach are operated with perfect knowledge of path number while the proposed variational Bayesian approach and the matrix robust PCA method assume no knowledge of path number. From Fig. 3, it is seen that both the HOSVD and the proposed tensor approach perform better than the matrix SVD method no matter whether bad data exists or not, and the proposed algorithm outperforms the matrix robust PCA method irrespective of the existence of bad data. Moreover, while HOSVD offers the same performance as the proposed algorithm when $\pi = 0$, their performances are largely different in the presence of bad data when $\pi = 0.05$. In particular, HOSVD fails to mitigate the effects brought by bad data and shows no LPA improvement as SNR increases, but the proposed algorithm performs hardly distinguishable from the case of no bad data.

Then, we look at the DOA estimation performance obtained from the identified subspaces, where the performance criterion is the mean-square error (MSE) defined as $L^{-1} \sum_{l=1}^{L} [ (\theta_l - \hat{\theta}_l)^2 + (\phi_l - \hat{\phi}_l)^2 ]$. We consider three different ESPRIT methods: 1) 2D ESPRIT applied to $\mathbf{M}^{\Xi} \circ \mathbf{M}^{\Upsilon}$ (labeled as VB 2D ESPRIT); 2) 2D ESPRIT applied to subspace recovered by HOSVD (labeled as HOSVD ESPRIT); and 3) the proposed low-complexity 1D ESPRIT applied to $\mathbf{M}^{\Xi}$ and $\mathbf{M}^{\Upsilon}$ (labeled as VB 1D ESPRIT). Fig. 4 compares their MSEs versus SNR. It is seen that when $\pi = 0$, all the algorithms behave the same. However, when $\pi = 0.05$, the performance of HOSVD ESPRIT degrades significantly whereas the VB 2D ESPRIT and VB 1D ESPRIT show excellent robustness against the bad data. Notice that the VB 2D ESPRIT and VB 1D ESPRIT perform the same since the simulation setting satisfies condition $P2)$ in Property 1.

Finally, Fig. 5 compares the performance of the ESPRIT method and the MUSIC approach with the CRB derived in [43] as the performance reference. Since the VB 1D ESPRIT shows the best performance in term of accuracy and complexity, it is picked as a representative for ESPRIT-type methods. For MUSIC algorithm, we use the 1D MUSIC algorithm applied to $\mathbf{M}^{\Xi}$ and $\mathbf{M}^{\Upsilon}$ as proposed in Section V (labeled as VB 1D MUSIC). To obtain better estimates, local search
using a 2D MUSIC around the estimates provided by 1D VB MUSIC is also conducted. Since all the DOA estimators in Fig. 5 are based on the subspace identified from the proposed variational inference method, only the simulation results when \( \pi = 0.05 \) is presented. The same conclusion can be drawn from the case \( \pi = 0 \). From Fig. 5, it is seen that VB 1D MUSIC outperforms the ESPRIT-type algorithm but its performance cannot achieve the CRB, as predicted in Section V. However, the performance gap can be filled by further invoking a 2D MUSIC local search around the VB 1D MUSIC estimates.

VII. CONCLUSIONS

In this paper, a subspace estimator for DOA estimation in massive MIMO systems was proposed, with novel features of bad data mitigation and automatic source enumeration. The proposed algorithm exploits the properties from tensor represented data and offers robust recovered subspaces, readily for use by any traditional 2D DOA estimator. Furthermore, when the number of signal paths is less than the number of horizontal antennas and vertical antennas, the subspaces in elevation and azimuth domains can be separately estimated, breaking the original 2D DOA estimation problem into two 1D counterparts, further lowering the complexity. Simulation results demonstrated the excellent performance of the proposed subspace recovery method and subsequent DOA estimation in term of accuracy and robustness.

APPENDIX A

DERIVATIONS FOR \( Q(\Xi) \)

By substituting (15) into (14) and only taking the terms relevant to \( \Xi \), we directly have

\[
Q(\Xi) \propto \exp \left\{ \mathbb{E}_{\Pi_{\theta},\mathbb{R} \in \Xi} Q(\Theta) \left[ -\beta \| \mathcal{Y} - [\Xi, \mathcal{Y}, \Pi] - \mathcal{E} \|_F^2 - \text{Tr} \{ \Gamma \Xi^H \Xi \} \right] \right\}. \tag{54}
\]

After expanding the square of the Frobenius norm, the term inside expectation in (54) can be expressed as

\[
-\beta \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{T} \left\{ \left( \sum_{l=1}^{R} \Xi_{i,l} \mathcal{Y}_{j,l} \Pi_{k,l} \right)^* \left( \sum_{l=1}^{R} \Xi_{i,l} \mathcal{Y}_{j,l} \Pi_{k,l} \right) 
- 2 \Re \left[ \left( \sum_{l=1}^{R} \Xi_{i,l} \mathcal{Y}_{j,l} \Pi_{k,l} \right)^* \left( \mathcal{Y}_{i,j,k} - \mathcal{E}_{i,j,k} \right) \right] \right\} 
- \sum_{i=1}^{M} \Xi_{i,:} \Gamma \Xi_{i,:}^H. \tag{55}
\]
Then using the properties of summation, (55) can be written as

\[
\sum_{i=1}^{M} \left\{ -\beta \sum_{j=1}^{N} \sum_{k=1}^{T} \sum_{p=1}^{R} \sum_{q=1}^{R} \Xi_{i,p} \Psi_{j,p}^{*} \Pi_{k,p}^{*} = \Xi_{i,q} \Psi_{j,q} \Pi_{k,q} - \Xi_{i,:} \Gamma \Xi_{i,:}^{H} \right\}, \triangleq d_{1}
\]

\[
+2 \beta \Re \left[ \left( \sum_{j=1}^{N} \sum_{k=1}^{T} \sum_{l=1}^{R} \Xi_{i,l} \Psi_{j,l} \Pi_{k,l} \right)^{*} \left( \Psi_{i,j,k} - \mathcal{E}_{i,j,k} \right) \right]. \triangleq d_{2}
\]

For \( d_{1} \), noticing that the summands are all scalars, we have

\[
d_{1} = \sum_{p=1}^{R} \sum_{q=1}^{R} \Xi_{i,p}^{*} \left( (\Psi_{j,p}^{H} (\Psi_{j,q} \Pi_{k,q}^{*} \Xi_{i,q}) \right) \Xi_{i,q}
\]

\[
= \Xi_{i,:}^{*} \left[ (\Psi_{j,p}^{H} (\Psi_{j,q} \Pi_{k,q}^{*} \Xi_{i,q}) \right]^{T} \Xi_{i,:}.
\]

For \( d_{2} \), as all the summands are also scalars, it is easy to obtain that

\[
d_{2} = \sum_{l=1}^{R} \Xi_{i,l}^{*} \left( \sum_{j=1}^{N} \Psi_{j,l}^{*} \Pi_{j,l}^{H} \right) \left( \Psi_{j,l}^{H} (\Psi_{i,l} \Pi_{k,l}^{*} \Xi_{j,l} - \mathcal{E}_{i,j,k}) \right) \Xi_{i,:}^{*} \left[ (\Psi_{j,l}^{H} (\Psi_{i,l} \Pi_{k,l}^{*} \Xi_{j,l}) \right]^{T}.
\]

where the last line of (58) utilizes the definition of Khatri Rao product. Putting (57) and (58) into (56) and then taking expectations, the terms inside the exponent in (54) is

\[
\sum_{i=1}^{M} \left\{ -\Xi_{i,:}^{*} \left[ \mathbb{E}_{Q(\beta)} \left[ \beta \right] \mathbb{E}_{Q(\Psi)} \left[ \Psi \Pi \right] \left( (\Psi_{j,p}^{H} (\Psi_{j,q} \Pi_{k,q}^{*} \Xi_{i,q}) \right) + \mathbb{E}_{Q_{\Pi}^{*}(\gamma)} \left[ \Gamma \right] \right] \Xi_{i,:}^{T} \right\}, \triangleq [\Sigma_{\Psi}]^{-1}
\]

\[
+ 2 \beta \Re \left[ \Xi_{i,:}^{*} \left[ \Sigma_{\Xi} \right]^{-1} \left( \left[ \mathbb{E}_{Q(\beta)} \left[ \beta \right] \mathbb{E}_{Q(\Psi)} \left[ \Psi \right] \mathbb{E}_{Q(\Pi)} \left[ \Pi \right] \right] \left( (\Psi_{j,p}^{H} (\Psi_{j,q} \Pi_{k,q}^{*} \Xi_{i,q}) \right) \right) \right]^{T} \right]. \triangleq \mathbb{M}_{\Sigma_{\Xi}}}
\]
Finally, using (59), equation (54) can be expressed as
\[
Q(\Xi) \propto \exp \left\{ \sum_{i=1}^{M} \left[ -\Xi_i^* \left[ \Sigma \Xi \right]^{-1} \Xi_i^T + 2 \Re \left( \Xi_i^* \left[ \Sigma \Xi \right]^{-1} [M_i \Xi]^T \right) \right] \right\} \\
\propto \exp \left\{ -\text{Tr} \left\{ \Xi^* \left[ \Sigma \Xi \right]^{-1} \Xi^T - \left[ \Xi^* \left[ \Sigma \Xi \right]^{-1} \Xi^T - \Xi^* \left[ \Sigma \Xi \right]^{-1} [M \Xi]^T \right] \right\} \right\}.
\]  

(60)

**APPENDIX B**

**PROOF OF PROPERTY 2**

From the definitions of the Khatri Rao product, it can be shown that
\[
(A \odot B)^H (A \odot B) = \sum_{i=1}^{\kappa_1} \sum_{j=1}^{\kappa_2} A_{i,p}^* A_{i,q} B_{j,p}^* B_{j,q}.
\]

(61)

After taking the expectation, it follows that
\[
\mathbb{E}_{p(A)p(B)} [(A \odot B)^H (A \odot B)]_{p,q} = \sum_{i=1}^{\kappa_1} \sum_{j=1}^{\kappa_2} \mathbb{E}_{p(A)} [A_{i,p}^* A_{i,q}] \mathbb{E}_{p(B)} [B_{j,p}^* B_{j,q}]
\]
\[
= \sum_{i=1}^{\kappa_1} \sum_{j=1}^{\kappa_2} \left( (M_{i,p}^A)^* M_{i,q}^A + \Sigma_{p,q}^A \right) \left( (M_{j,p}^B)^* M_{j,q}^B + \Sigma_{p,q}^B \right).
\]

(62)

Using the definition of Hadamard product, it can be shown that (19) holds.

**APPENDIX C**

**DERIVATIONS FOR \( \bar{f} \)**

After expanding the square of the Frobenius norm of (35) and taking expectations, it can be shown that
\[
\bar{f} = \| Y - \left[ M^\Xi, M^Y, M^\Pi \right] - \mathbb{E}_{Q(\varepsilon)} [\varepsilon] \|_F^2 + \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{T} \sum_{p=1}^{R} \sum_{q=1}^{R} \Xi_{i,p}^* \Xi_{i,q} \varepsilon_{j,p} \varepsilon_{j,q} \Pi_{k,p} \Pi_{k,q}.
\]

\( \triangleq \xi_1 \)

(63)

For \( \xi_1 \), distributing the expectation to various terms results in
\[
\xi_1 = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{T} \sum_{p=1}^{R} \sum_{q=1}^{R} (M_{i,p}^\Xi)^* M_{i,q}^\Xi (M_{j,p}^Y)^* M_{j,q}^Y (M_{k,p}^\Pi)^* M_{k,q}^\Pi + MNT \sum_{p=1}^{R} \sum_{q=1}^{R} \Sigma_{q,p}^\Xi \Sigma_{q,p}^Y \Sigma_{q,p}^\Pi.
\]

\( \triangleq \xi_2 \)
\[ + M \sum_{j=1}^{N} \sum_{k=1}^{T} \sum_{p=1}^{R} \sum_{q=1}^{R} (M^Y_{j,p})^* M^Y_{j,q} (M^\Pi_{k,p})^* M^\Pi_{k,q} \Sigma_{q,p}^\varpi + NT \sum_{i=1}^{M} \sum_{p=1}^{R} \sum_{q=1}^{R} (M^\Xi_{i,p})^* M^\Xi_{i,q} \Sigma_{q,p}^\varpi + N \sum_{i=1}^{M} \sum_{q=1}^{R} \sum_{j=1}^{N} \sum_{p=1}^{R} (M^\Xi_{i,p})^* M^\Xi_{i,q} \Sigma_{q,p}^\varpi + M T \sum_{j=1}^{T} \sum_{k=1}^{M} \sum_{p=1}^{R} \sum_{q=1}^{R} (M^Y_{j,p})^* M^Y_{j,q} \Sigma_{q,p}^\varpi + M \sum_{i=1}^{M} \sum_{k=1}^{T} \sum_{p=1}^{R} \sum_{q=1}^{R} (M^\Xi_{i,p})^* M^\Xi_{i,q} \Sigma_{q,p}^\varpi + M N \sum_{k=1}^{T} \sum_{p=1}^{R} \sum_{q=1}^{R} (M^\Pi_{k,p})^* M^\Pi_{k,q} \Sigma_{q,p}^\varpi. \]

From the definition of tensor, \( \varpi_1 = \| [M^\Xi, M^Y, M^\Pi] \|_F^2 \). On the other hand, as all the summands in \( \eta_1 \) are scalars, it follows that
\[
\eta_1 = \sum_{p=1}^{R} \sum_{q=1}^{R} \left[ \left( \sum_{j=1}^{N} (M^Y_{j,p})^* M^Y_{j,q} \sum_{k=1}^{T} (M^\Pi_{k,p})^* M^\Pi_{k,q} \right) \Sigma_{q,p}^\varpi \right]
= \sum_{p=1}^{R} \sum_{q=1}^{R} \left[ (M^Y)^H M^Y \right]_{p,q} \left[ (M^\Pi)^H M^\Pi \right]_{p,q} \Sigma_{q,p}^\varpi
= \sum_{p=1}^{R} \sum_{q=1}^{R} \left\{ \left[ (M^Y)^H M^Y \right] \odot \left[ (M^\Pi)^H M^\Pi \right] \right\}_{p,q} \Sigma_{q,p}^\varpi
\]
where the last line utilizes the definition of Hadamard product. Further using the property
\[ \sum_{i,j}[A \odot B]_{i,j} = \text{Tr}\{AB^T\}, \]
we have \( \eta_1 = \text{Tr}\{ \left[ (M^Y)^H M^Y \right] \odot \left[ (M^\Pi)^H M^\Pi \right] \Sigma^\varpi \} \). Similarly, it can be shown that \( \eta_2 = \text{Tr}\{ \left[ (M^\Xi)^H M^\Xi \right] \odot \left[ (M^\Pi)^H M^\Pi \right] \Sigma^\varpi \} \); \( \chi_1 = \text{Tr}\{ \left[ (M^\Xi)^H M^\Xi \right] \Sigma^\varpi \} \); \( \chi_1 = \text{Tr}\{ \left[ (M^\Xi)^H M^\Xi \right] \Sigma^\varpi \} \); \( \chi_3 = \text{Tr}\{ \left[ (M^\Xi)^H M^\Xi \right] \Sigma^\varpi \} \); and \( \varpi_2 = \text{Tr}\{ \left[ (M^\Xi)^H M^\Xi \right] \Sigma^\varpi \} \).

Putting all these results into (64) and then putting the resultant \( \xi_1 \) into (63), it is easy to show that (36) holds.

REFERENCES


Fig. 1: System Model

Fig. 2: Largest principle angle of subspace recovery versus iteration number
Fig. 3: Largest principle angle of subspace recovery versus SNR

Fig. 4: MSE of DOA Estimation using ESPRIT
Fig. 5: MSE of DOA Estimation using ESPRIT and MUSIC when $\pi = 0.05$

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