Energy Efficient Transmission in Multi-User MIMO Relay Channels with Perfect and Imperfect Channel State Information

Shiqi Gong, Student Member, IEEE, Chengwen Xing, Member, IEEE, Nan Yang, Member, IEEE, Yik-Chung Wu, Senior Member, IEEE and Zesong Fei, Member, IEEE

Abstract—We design novel transmission strategies to maximize the energy efficiency (EE) of the uplink multi-user MIMO relay channel. In this channel, $K$ multi-antenna users communicate with a multi-antenna base station (BS) through a multi-antenna relay. To achieve the goal of EE maximization, we propose new iterative algorithms to jointly optimize the multi-user precoder and the relay precoder under transmit power constraints for two cases. In the first case, the perfect global channel state information (CSI) is available, while in the second case, the CSI between the relay and the BS is imperfect. To surmount the non-convexity of our formulated EE optimization problems in both cases, we introduce the parameter subtractive function into the proposed algorithms. Then the EE parameter in the parameter subtractive function is updated by the Dinkelbach’s algorithm in the perfect CSI case, and by the bisection method in the imperfect CSI case. Moreover, in the perfect CSI case the relay precoder is optimized by the diagonalization operation and the multi-user precoder is optimized based on the weighted minimum mean square error method. Differently, in the imperfect CSI case we apply the sign-definiteness lemma to promote the semidefinite programming formulation of the EE optimization problem. Furthermore, we present numerical results to demonstrate that our proposed iterative algorithms have a good convergence rate in both cases. In addition, we show that our proposed iterative algorithms achieve a higher EE performance than the existing algorithms in both CSI cases.

Index Terms—Energy efficiency, multi-user MIMO, relay channels, iterative algorithms, convex optimization.

I. INTRODUCTION

THE multiple-input multiple-output (MIMO) technology has been widely applied to support the ever-growing user rate requirements and the increasing user access by enabling prominent multiplexing and diversity gain [1]–[3]. The wireless relay technology has emerged as a key advocate for providing reliable communication coverage and high throughput when the direct link between the source and the destination experiences deep fading [4]–[6]. Motivated by these benefits, numerous research efforts have been devoted to examine the error performance or the rate performance of relay-aided MIMO networks over the past few years, e.g., [7]–[10].

Recently, the concept of green communications [11] has been manifested in design and implementation of wireless networks. Traditionally, the research target in green communications is to minimize the transmit power while satisfying the prescribed quality of service (QoS) requirements. However, such a target is usually complicated to achieve in practice. Against this background, the concept of energy efficiency (EE) has been defined and adopted in designing green communications. In this concept, the EE is defined as the ratio between the achievable data rate and the energy consumption. Taking the EE as the major performance metric, a growing body of studies have investigated the energy-efficient transmission strategy for improving the quality of various wireless communication systems, such as the uplink MIMO system [12]–[14], the multi-cell multi-user system [15]–[18], and the interference-limited system [19].

In the aforementioned studies [12]–[19], a key assumption adopted is that the perfect channel state information (CSI) is available at each communication node. We note that this assumption does not always hold in practical scenarios where feedback and quantization errors may occur. This has inspired a great extent of studies contributing to the EE improvement of wireless communication systems without perfect CSI, e.g., [20]–[24]. When the statistical CSI is available, [20] determined the optimal EE precoding of the two-hop relay channel, and [21] demonstrated that the diagonalization operation can be used for robust EE optimization in a single-cell relay-aided network. Differing from [20], [21], [22]–[24] focused on the robust EE optimization from the worst-case CSI perspective. For example, the maximum worst-case EE of the multi-cell multi-user network was achieved by an alternating optimization algorithm [22].

In this paper, we maximize the EE performance of the uplink in the multi-user MIMO relay channel for both perfect and imperfect CSI cases, which are of theoretical and practical significance. To the best knowledge of the authors, such maximization has not been conducted in the literature. In the
considered channel, $K$ multi-antenna users transmit to a multi-antenna base station (BS) with the aid of a multi-antenna amplify-and-forward (AF) relay. In the perfect CSI case, we assume that the precise channel knowledge is available at each communication node. In the imperfect CSI case, we assume that the precise channel knowledge between the relay and the BS is unknown. To maximize the EE of the considered channel, we jointly optimize the multi-user precoder and the relay precoder. To this end, we propose new iterative algorithms to tackle the non-convexity of the formulated EE maximization problem in both cases. The novel contributions of our work are summarized as follows:

1) In the perfect CSI case, we first show that the diagonalization operation is optimal to design the relay precoder when the multi-user precoder is given. Then we apply the fractional programming method, i.e., the parameter subtractive function, to determine the optimal relay precoder by solving a standard convex optimization problem. Once the optimal relay precoder is determined, we jointly use the weighted minimum mean square error (WMMSE) method and the parameter subtractive function to iteratively determine the optimal multi-user precoder. We clarify that the WMMSE method is applicable to the case where the signal-to-interference-plus-noise ratio (SINR) and the MSE [25] are known. Finally, we use the Dinkelbach’s algorithm [26] to update the key parameters in the parameter subtractive function.

2) In the imperfect CSI case, we adopt the norm bounded error model to characterize the CSI between the relay and the BS. To maximize the robust EE based on the WMMSE method and the parameter subtractive function in this case, we first introduce the sign-definiteness lemma [27] to transform the robust EE optimization problem into a series of semidefinite programming (SDP) subproblems, which can be efficiently solved by the interior point method. Then we propose a two-layer iterative optimization algorithm. In the inner optimization, the transformed SDP subproblems are solved iteratively. In the outer optimization, the bisection method over the parameter subtractive function is applied to find the optimal EE.

We present numerical results to examine the effectiveness of our proposed iterative algorithms. First, we show that our proposed algorithms have a good convergence rate, indicating that the algorithms are stably converged in both perfect and imperfect CSI cases. Second, we show that our proposed iterative algorithms achieve a higher maximum EE than the benchmark algorithms, namely, the SINR-scalarized algorithm [14] in the perfect CSI case and the non-robust algorithm in the imperfect CSI case. Third, we examine the impact of system parameters, e.g., the transmit power, the circuit power consumption, and the CSI error threshold, on the achievable maximum EE.

**Notation:** Normal, bold lower-case letters, and bold upper-case letters denote scalars, vectors, and matrices, respectively. $\cdot^T$, $\cdot^H$, $\cdot^{-1}$, and $\cdot^\dagger$ denote the transpose, conjugate transpose, inverse, and pseudo-inverse of a matrix, respectively. $\cdot$, $\| \cdot \|$, and $\| \cdot \|_F$ denote the scalar absolute value, Euclidean norm, and Frobenius norm, respectively. $\otimes$ and $\odot$ denote the Kronecker product and the elementwise multiplication of two matrices. $E[\cdot]$ denotes expectation. $\text{Tr}(\cdot)$ and $\det(\cdot)$ denote the trace and determinant of a matrix, respectively. $\nabla$ denotes the differential operator. $I_N$ denotes an $N \times N$ identity matrix. $0$ denotes a zero matrix or vector with the appropriate dimension. $A \succeq 0$ indicates that $A$ is a positive semidefinite matrix. $\text{diag}(a_1, a_2, \cdots, a_K)$ denotes a diagonal matrix with the diagonal elements $a_1, a_2, \cdots, a_K$. $\text{BLKdiag}\{A_1, A_2, \cdots, A_K\}$ is the block diagonal matrix with the diagonal elements $A_1, A_2, \cdots, A_K$.

## II. System Model and Problem Formulation

As depicted in Fig. 1, we consider the uplink multi-user MIMO relay channel where $K$ users simultaneously transmit information to the same BS with the assistance of a relay. In this channel, the $k$th user, the BS, and the relay are equipped with $N_S$, $N_R$, and $N_B$ antennas, respectively, where $k \in \{1, \cdots, K\}$. We assume that the wireless channel between any two nodes is subject to quasi-static and frequency-flat fading. To be specific, we denote $H_{S_k R} \in \mathbb{C}^{N_R \times N_S}$ as the channel matrix from $k$th user to the relay and $H_{R B} \in \mathbb{C}^{N_B \times N_R}$ as the channel matrix from the relay to the BS. Moreover, we assume that all nodes in this MIMO relay channel operate in the half-duplex mode. As such, the communication between $K$ users and the BS is completed within two consecutive time slots, as follows:

In the first time slot, $K$ users simultaneously transmit multiple data streams to the relay. The received signal at the relay is given by

$$\mathbf{y}_R = \sum_{k=1}^{K} H_{S_k R} \mathbf{p}_k \mathbf{s}_k + \mathbf{n}_R,$$

where $\mathbf{p}_k \in \mathbb{C}^{N_B \times d_{sk}}$ denotes the precoding matrix of the $k$th user, $\mathbf{s}_k \in \mathbb{C}^{d_{sk}}$ denotes the data streams transmitted
by the $k$th user that satisfies $\mathbb{E}[\|s_k\|^2] = 1$, $d_{sk}$ denotes the number of the transmitted data streams from the $k$th user, and $n_R \sim \mathcal{CN}(0, \sigma^2_R \mathbf{I})$ denotes the additive Gaussian noise vector received at the relay. Here, the transmit power of the $k$th user, denoted by $P_{Sk}$, needs to satisfy $P_{Sk} = \text{Tr}(\mathbf{P}_k \mathbf{P}_k^H) \leq P_{S,k}^{\max}$, where $P_{S,k}^{\max}$ is the maximum transmit power of each user. We next define $N_S \triangleq \sum_{k=1}^{K} N_{Sk}$ and $d \triangleq \sum_{k=1}^{K} d_{sk}$. To support the users to transmit $d$ independent data streams, we assume $N_B \geq N_R$, $N_R \geq d$ and $N_S \geq d$ throughout this paper.

When the relay receives the signals from $K$ users, it processes the received signals using the amplify-and-forward (AF) strategy [28]. As such, the retransmitted signal vector at relay, $y'_R$, is given by

$$y'_R = \mathbf{W}_R y_R,$$

where $\mathbf{W}_R \in \mathbb{C}^{N_R \times N_B}$ denotes the precoding matrix at the relay. Accordingly, the transmit power at the relay is given by

$$P_R = \sum_{k=1}^{K} \text{Tr} \left( \mathbf{W}_R \left( \mathbf{H}_{Sk} \mathbf{P}_k \mathbf{P}_k^H \mathbf{H}_{Sk}^H + \sigma^2_R \mathbf{I}_{N_R} \right) \mathbf{W}_R^H \right).$$

In the second time slot, the relay forwards $y'_R$ to the BS. Thus, the received signal at the BS is expressed as

$$y_B = \sum_{k=1}^{K} \mathbf{H}_{RB} \mathbf{W}_R \mathbf{H}_{Sk} \mathbf{P}_k s_k + \mathbf{H}_{RB} \mathbf{W}_R n_R + n_B,$$

where $n_B \in \mathbb{C}^N \mathcal{N}(0, \sigma^2_B \mathbf{I})$ is the received additive Gaussian noise vector at the BS and $s = [s_1, \ldots, s_K]^T$. By defining two new matrices, $\mathbf{H}_{UP} \in \mathbb{C}^{N_R \times N_S}$ and $\mathbf{P} \in \mathbb{C}^{N_S \times d}$, as

$$\mathbf{H}_{UP} = [\mathbf{H}_{S1,R}, \ldots, \mathbf{H}_{Sk,R}] \quad \text{and} \quad \mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{P}_K \end{bmatrix},$$

we rewrite (4) as

$$y_B = \mathbf{H}_{RB} \mathbf{W}_R \mathbf{H}_{UP} \mathbf{P} s + \mathbf{H}_{RB} \mathbf{W}_R n_R + n_B.$$  

Then the achievable rate of the uplink multi-user MIMO relay channel is expressed as

$$R_{\text{sum}} = \frac{1}{2} \log \det (\mathbf{I}_{N_B} + T_1),$$

where

$$T_1 = \frac{\mathbf{H}_{RB} \mathbf{W}_R \mathbf{H}_{UP} \mathbf{P} \mathbf{P}^H \mathbf{H}_{UP}^H \mathbf{H}_{RB}^H}{\sigma^2_B \mathbf{I}_{N_B} + \sigma^2_R \mathbf{H}_{RB} \mathbf{W}_R \mathbf{W}_R^H \mathbf{H}_{RB}^H}.$$  

Here, the factor $\frac{1}{2}$ indicates that the communication process over the entire channel is performed within two consecutive time slots [28].

We find that the $K$ users, the relay, and the BS in the first time slot are in the mode of transmission, reception, and leisure, respectively. In the second time slot, differently, the $K$ users, the relay, and the BS are in the mode of leisure, transmission, and reception, respectively. Therefore, the amount of the consumed energy in the entire communication process is calculated as

$$E(\mathbf{P}, \mathbf{W}_R) = \frac{1}{2} \left( \sum_{k=1}^{K} \frac{P_{Sk}}{\xi_{Sk}} + \frac{P_R}{\xi_R} + P_C \right),$$

where $P_C$ denotes the circuit power consumption of the uplink multi-user MIMO relay channel [29], $\xi_{Sk}$ and $\xi_R$ denote the power amplifier efficiency at the $k$th user and the relay, respectively. Here, we assume $\xi_{Sk} = \cdots = \xi_{Sk} = \xi_R = 1$ for brevity. Based on (6) and (8), the achievable EE of the uplink multi-user MIMO relay channel is defined as

$$E(\mathbf{P}, \mathbf{W}_R) = \frac{R_{\text{sum}}}{E(\mathbf{P}, \mathbf{W}_R)} = \frac{\log \det (\mathbf{I}_{N_B} + T_1)}{\text{Tr}(\mathbf{P} \mathbf{P}^H)} + \text{Tr}(T_2) + P_C,$$

where $T_2 = \mathbf{W}_R \left( \mathbf{H}_{UP} \mathbf{P} \mathbf{P}^H \mathbf{H}_{UP}^H + \sigma^2_R \mathbf{I}_{N_R} \right) \mathbf{W}_R^H$. Based on (9), the EE maximization problem is formulated as

$$\max_{\mathbf{P} = \text{blkdiag}(\mathbf{P}_1, \ldots, \mathbf{P}_K)} \frac{\log \det (\mathbf{I}_{N_B} + T_1)}{\text{Tr}(\mathbf{P} \mathbf{P}^H)} \leq \frac{P_{S,k}^{\max}}{\xi_{Sk}}, \quad \forall k \in \{1, \ldots, K\}, \quad \text{Tr}(T_2) \leq \frac{P_{R}^{\max}}{\xi_R},$$

where $\mathbf{I}_k$ is a block diagonal matrix with the $k$th diagonal element being an identity matrix and other diagonal elements being zero matrices.

We find that the problem given by (10) is nonconvex, due to the non-linear fractional optimization objective function and the coupled optimization variables. Hence, it is difficult to derive the optimal solution to (10) from the global optimization perspective. To address this issue, we apply an alternative algorithm to optimize the multi-user precoder $\mathbf{P}$ and the relay precoder $\mathbf{W}_R$ iteratively, which ultimately maximizes the EE of the uplink multi-user MIMO relay channel. This method is presented in the following section.

### III. EE Maximization Under Perfect CSI

In this section, we aim to achieve the maximum EE of the uplink multi-user MIMO relay channel under the perfect global CSI case. To this end, we propose a new algorithm to optimize the relay precoder $\mathbf{W}_R$ and the multi-user precoder $\mathbf{P}$ iteratively. In this algorithm, the optimal structure of $\mathbf{W}_R$ is firstly determined for a given $\mathbf{P}$. Then the optimal solution of $\mathbf{W}_R$ is determined by solving a convex optimization problem. For a given $\mathbf{W}_R$, we utilize the equivalent relationship between the SINR and the MSE to reformulate (10), which promotes the iterative optimization of $\mathbf{P}$. Generally, the conventional sum-rate maximization problem in MIMO network is difficult to solve directly and globally, therefore, an alternative approach in [13] was proposed to transform it into an equivalent weighted sum MSE minimization (WMMSE) problem with a special weighting matrix depending on the optimal beamforming matrix. The WMMSE method is also considered to be a distributed linear transceiver design method. In addition, we can theoretically demonstrate that the WMMSE method converges to at least a
locally optimal solution of the optimization problem and has low computational complexity. The detailed iterative optimization of $W_R$ and $P$ is shown in the following subsections.

### A. Optimization of $W_R$

Considering the case where the perfect global CSI is available at the relay and the BS, we compute the singular value decomposition (SVD) of $H_{RB}$ and $H_{UP}$ for a given $P$ as

$$H_{RB} = U_H \Sigma_H^\frac{1}{2} V_H^H$$  \hspace{1cm} (11)

and

$$H_{UP} = U_D \Sigma_D^\frac{1}{2} V_D^H,$$  \hspace{1cm} (12)

respectively, where $U_H \in \mathbb{C}^{N_R \times N_R}$, $\Sigma_H^\frac{1}{2} \in \mathbb{C}^{N_R \times N_R}$, $V_H \in \mathbb{C}^{N_R \times N_R}$, $U_D \in \mathbb{C}^{N_R \times N_R}$, $\Sigma_D^\frac{1}{2} \in \mathbb{C}^{N_R \times N_R}$, and $V_H \in \mathbb{C}^{N_R \times N_R}$. Then the optimization problem in (10) is re-expressed as

$$\max_{W_R} \frac{\log \det (I_{N_R} + T_1)}{\text{Tr}(T_2)} + C_p,$$

s.t. $\text{Tr}(T_2) \leq P_R^\text{max},$  \hspace{1cm} (13)

where $C_p = \text{Tr}(PP^H) + P_C$ is a constant. Clearly, the problem given by (13) is a nonconvex fractional problem with respect to $W_R$. Thus, it is difficult to be solved from the global optimization perspective. This motivates us to resort to the diagonalization operation, which is the optimal method to solve the optimization problem similar to (13). Indeed, the diagonalization operation can maximize the numerator and the denominator of the objective function in (13) simultaneously. As such, we present the optimal structure of $W_R$ for a given $P$ in the following proposition.

**Proposition 1.** Assuming that the SVD of $W_R$ is $W_R = U_R \Sigma_R^\frac{1}{2} V_R^H$, where $U_R \in \mathbb{C}^{N_R \times N_R}$, $\Sigma_R^\frac{1}{2} \in \mathbb{C}^{N_R \times N_R}$, $V_R \in \mathbb{C}^{N_R \times N_R}$. Then the optimal relay precoder $W_R$ that solves (13) is the one which satisfies $U_R = V_H$ and $V_R = U_D$.

**Proof:** We first rewrite the objective function in (13) as

$$\hat{E}(W_R),$$

which is given by (14) at the top of the next page, where we have $H_{RB} = H_{RB}H_{RB}^H = U_D \Sigma_D U_D^H$, $Z = H_{RB}W_R \left(\sigma_R^2 I_{N_R} + D\right)^{-1} Z^H$, and $T_3 = Z \sigma_R^2 \Sigma_D^2 U_D^{-1} Z^H$.

Here, we define $U_Z \in \mathbb{C}^{N_R \times N_R}$, $\Sigma_Z^\frac{1}{2} \in \mathbb{C}^{N_R \times N_R}$, and $V_Z \in \mathbb{C}^{N_R \times N_R}$. Based on Lemma 1 in the Appendix, it is concluded that the first term of the denominator in the last line of (14) is minimized when $U_Z = U_H$. Moreover, based on Lemma 2 in the Appendix, the maximum value of the numerator in the last line of (14) is achieved when $V_Z = U_D$. We note that such derivations of $Z$ are feasible for the problem in (13) by optimizing the elements of the diagonal matrix $\Sigma_Z$ to satisfy the power constraint $Tr \left(H_{RB}^H ZZ^H (H_{RB}^H)^H\right) \leq P_R^\text{max}$. Furthermore, by substituting $U_Z = U_H$ and $V_Z = U_D$ into the expression for $Z$, we have

$$Z = U_H \Sigma_Z^\frac{1}{2} U_H^H.$$

It is clear that (15) holds only when $U_R = V_H$ and $V_R = U_D$. Therefore, the proof is completed.

Based on Proposition 1, we denote $\lambda_{l,R}$, $\lambda_{l,D}$, and $\lambda_{l}$ as the $l$/th diagonal element of matrices $\Sigma_H = \Sigma_H^\frac{1}{2} (\Sigma_H^\frac{1}{2})^H$, $\Sigma_D = \Sigma_D^\frac{1}{2} (\Sigma_D^\frac{1}{2})^H$, and $\Sigma_R = \Sigma_R^\frac{1}{2} (\Sigma_R^\frac{1}{2})^H$, respectively, where $l = \{1, \cdots, N_R\}$. We then define $\Lambda_R = \{\lambda_{l,R}, \cdots, \lambda_{N_R,R}\}$. Thus, the EE maximization problem given by (13) is reformulated as

$$\max_{\Lambda_R} \frac{\sum_{l=1}^{N_R} \log \left(1 + \frac{\lambda_{l,R}}{\sigma_R^2 + \sigma_R^2 \lambda_{l,R}}\right)}{\sum_{l=1}^{N_R} \lambda_{l,R} (\sigma_R^2 + \lambda_{l,R}) + C_p},$$

s.t. $\sum_{l=1}^{N_R} \lambda_{l,R} (\sigma_R^2 + \lambda_{l,R}) \leq P_R^\text{max},$

$$\lambda_{l,R} \geq 0, \forall l \in \{1, \cdots, N_R\}.$$

It is observed that (16) is quasi-concave over $\Lambda_R$. According to [30], [31], the Theorem 1 in Appendix II can be utilized to efficiently solve (16). Specifically, by applying Theorem 1 to the quasi-concave problem (16), we confirm that the parameter $\mu$ that causes the zero output of the corresponding parametric subfunction is also the optimal solution to the problem in (16). In the following, we write the parametric subfunction function of (16) as

$$F(\mu, P) = \max_{\Lambda_R} \frac{\sum_{l=1}^{N_R} \log \left(1 + \frac{\lambda_{l,R}}{\sigma_R^2 + \sigma_R^2 \lambda_{l,R}}\right)}{\sum_{l=1}^{N_R} \lambda_{l,R} (\sigma_R^2 + \lambda_{l,R}) + C_p},$$

s.t. $\sum_{l=1}^{N_R} \lambda_{l,R} (\sigma_R^2 + \lambda_{l,R}) \leq P_R^\text{max},$

$$\lambda_{l,R} \geq 0, \forall l \in \{1, \cdots, N_R\}.$$

For fixed $\mu$ and $P$, we find that the problem in (17) is a standard convex optimization problem over $\Lambda_R$. Thus, we resort to the Lagrangian multiplier method to derive a closed-form solution of $\Lambda_R$ from (17). Specifically, we include a Lagrangian multiplier $\tau_p \geq 0$ into the constraint of (17),
resulting in the Lagrange function given by

\[
L(\tau_p, \mu, \mathbf{A}_R) = \sum_{l=1}^{N_R} \log \left( 1 + \frac{\lambda_{l,R} \mu_{l}}{\sigma_B^2 + \sigma_{l,R}^2 \lambda_{l,R}} \right) - \mu \left( \sum_{l=1}^{N_R} \lambda_{l,R} (\sigma_B^2 + \lambda_{l,R} \mu_{l}) \right) - \tau_p \left( \sum_{l=1}^{N_R} \lambda_{l,R} (\sigma_B^2 + \lambda_{l,R} \mu_{l}) - P_{R_{\text{max}}} \right)
\]

By taking the derivative of \( L(\tau_p, \mu, \mathbf{A}_R) \) with respect to \( \lambda_{l,R} \), \( \forall l \), we obtain the optimal \( \lambda_{l,R}^* \) as

\[
\lambda_{l,R}^* = \max \left\{ \lambda_{l,R}^*(\tau_p), 0 \right\}, \quad \lambda_{l,R}^*(\tau_p) = \frac{-b + \sqrt{b^2 - 4c(\tau_p)}}{2}
\]

where

\[
b = \frac{\sigma_B^2 (2\sigma_B^2 + \lambda_{l,R})}{\sigma_B^2 \lambda_{l,R}^* (\sigma_B^2 + \lambda_{l,R})}, \quad c(\tau_p) = \frac{\sigma_B^2 \lambda_{l,R}^* (\sigma_B^2 + \lambda_{l,R})^2 - \sigma_B^2 \lambda_{l,R}^* (\sigma_B^2 + \lambda_{l,R})}{\lambda_{l,R}^* (\sigma_B^2 + \lambda_{l,R})}
\]

(19)

(21)

Here, the optimal value of \( \tau_p \) should also satisfy the complementarity slackness condition of the power constraint of (17), which is

\[
\tau_p \left( \sum_{l=1}^{N_R} \lambda_{l,R}^* (\sigma_B^2 + \lambda_{l,R}) - P_{R_{\text{max}}} \right) = 0
\]

(22)

Considering the monotonicity of \( \lambda_{l,R}^*(\tau_p) \) with respect to \( \tau_p \), the one-dimensional bisection method is utilized to find the optimal value of \( \tau_p \) [25]. Besides, due to the fact that \( \lambda_{l,R} \geq 0 \) is also required, the iterative water-filling algorithm can be applied to ultimately derive the optimal \( \lambda_{l,R}^* \), \( \forall l \in \{1, \cdots, N_R\} \) for the problem in (17). Further, the optimal value of \( \mathbf{W}_R \) is derived as

\[
\mathbf{W}_R^* = \mathbf{U}_R \Sigma_{R}^{-\frac{1}{2}} \mathbf{U}_R^H = \mathbf{V}_H \Sigma_{R}^{-\frac{1}{2}} \mathbf{U}_D^H
\]

(23)

where \( \Sigma_{R}^{-\frac{1}{2}} = \text{diag} \left( \sqrt{\lambda_{1,R}^*}, \cdots, \sqrt{\lambda_{N_R,R}^*} \right) \). In this subsection, we determine the optimal relay precoder, \( \mathbf{W}_R^* \), based on Proposition 1 and (23) for fixed \( \mathbf{P} \) and \( \mu \). Once \( \mathbf{W}_R^* \) is found, the multi-user precoder \( \mathbf{W} \) needs to be optimized to maximize the EE of the uplink multi-user MIMO relay channel. We next present the iterative optimization of \( \mathbf{P} \) in the following subsection.

\subsection{B. Optimization of \( \mathbf{P} \)}

To optimize the multi-user precoder \( \mathbf{P} \), we utilize the WMMSE method, which depends on the equivalent relationship between the SINR and the weighted MSE, to weaken the non-linearity of the expression of the achievable rate. Specifically, by applying this method, the achievable rate given by (6) can be transformed into a WMMSE problem by introducing some auxiliary variables. It is noted that the WMMSE method is generally considered to be a distributed linear transceiver design method, which converges to at least a locally optimal solution of the optimization problem and has low computational complexity [25].

With the WMMSE method, we firstly define a linear equalizer \( \mathbf{G}_s \) to recover the transmit data streams \( s = [s_1, \cdots, s_K]^T \) for \( K \) users, i.e., \( \tilde{s} = \mathbf{G}_s \mathbf{y}_B \). Correspondingly, the MSE covariance matrix is given by

\[
\mathbf{M}(\mathbf{G}_s) = \mathbb{E} \left[ (\tilde{s} - s)(\tilde{s} - s)^H \right] = (\mathbf{T}_4 \mathbf{H}_{\text{up}} \mathbf{P} - \mathbf{I}_d) (\mathbf{T}_4 \mathbf{H}_{\text{up}} \mathbf{P} - \mathbf{I}_d)^H + \sigma_n^2 \mathbf{T}_4 \mathbf{T}_4^H + \sigma_B^2 \mathbf{G}_s \mathbf{G}_s^H
\]

(24)

where \( \mathbf{T}_4 = \mathbf{G}_s \mathbf{H}_{\text{RB}} \mathbf{W}_R^* \). Then the \( \mathbf{P} \)-related WMMSE problem is formulated as

\[
\max_{\mathbf{W}_S, \mathbf{G}_s} \log \det \left( \mathbf{W}_S \right) - \mathbb{E} \left[ (\tilde{s} - s)(\tilde{s} - s)^H \right] \quad \text{s.t.} \quad \mathbf{P} = \text{BLKdiag} \left\{ \mathbf{P}_1, \cdots, \mathbf{P}_K \right\}, \quad \mathbf{T}_4 \mathbf{P} \mathbf{P}^H \mathbf{T}_4^H \leq P_{R_{\text{max}}}^\text{S}, \quad \forall k \in \{1, \cdots, K\}, \quad \mathbf{T}_2 \leq P_{R_{\text{max}}}^\text{R}
\]

(25)

(26)

Obviously, the problem given by (26) is nonconvex, due to the nonlinear fractional objective function, which is difficult to solve globally. To tackle this non-convexity, an iterative optimization among \( \mathbf{P}, \mathbf{W}_S, \) and \( \mathbf{G}_s \) is proposed. In fact, only one variable among \( \mathbf{P}, \mathbf{W}_S, \) and \( \mathbf{G}_s \) is optimized in each iteration, while other variables are fixed. We next detail the iterative optimization of \( \mathbf{P} \) for a given \( \mathbf{W}_R^* \) as follows:

\[
\mathbf{P} = \text{BLKdiag} \left\{ \mathbf{P}_1, \cdots, \mathbf{P}_K \right\}, \quad \mathbf{T}_4 \mathbf{P} \mathbf{P}^H \mathbf{T}_4^H \leq P_{R_{\text{max}}}^\text{S}, \quad \forall k \in \{1, \cdots, K\}, \quad \mathbf{T}_2 \leq P_{R_{\text{max}}}^\text{R}
\]

(26)
1) Optimization of $G_S$: To realize the equivalent rate substitution by the MSE covariance matrix, we can obtain the optimal linear equalizer by minimizing the MSE of the recovered signals $\hat{s}$ [25], i.e.,

$$G_S^* = \arg\min_{G_S} \text{Tr}(M(G_S)).$$

By calculating $\partial \text{Tr}(M(G_S))/\partial (G_S) = 0$, we obtain the optimal linear equalizer as

$$G_S^* = P^H H_{UP}^H W_R^H H_{RB}^H \left(\sigma_B^2 N_B + H_{RB} T_2 H_{RB}^H\right)^{-1}.$$  \hspace{1cm} (27)

Then the corresponding MSE matrix is given by

$$M(G_S^*) = I_d - G_S^* H_{RB} W_R H_{UP}.$$ \hspace{1cm} (28)

2) Optimization of $W_S$: Observing the problem given by (26), we find that the positive definite weighting matrix $W_S$ only exists in the numerator of the optimization objective function in (26). Therefore, given the multi-user precoder $P$ and the linear equalizer $G_S$, the $W_S$-related EE optimization problem is expressed as

$$\max_{W_S} \log \det(W_S) - \text{Tr}(W_S M(G_S)).$$ \hspace{1cm} (29)

Following a similar procedure to the derivation of $G_S$, we can derive the optimal $W_S$ as

$$W_S^* = M(G_S)^{-1}.~~~~(30)$$

3) Optimization of $P$: It is evident that the multi-user precoder $P$ exists in both the denominator and numerator of the objective function in (26). Moreover, when $W_S$ and $G_S$ are determined, the optimization objective function in (26) is proven to be a quasi-concave function with respect to $P$, which is due to the concave numerator and the convex denominator. Thus, we use Theorem 1 to transform the EE optimization problem in (26) into a parametric subtractive form, which is given by

$$\hat{F}(\mu, W_S, G_S)$$

$$= \max_{P} \log \det(W_S) - \text{Tr}(W_S M(G_S)) + \text{Tr}(I_d)$$

$$- \mu \left(\text{Tr}(P P^H) + \text{Tr}(T_2) + P_C\right),$$

s.t. $P = \text{BLKdiag}\{P_1, \cdots, P_K\}$,

$$\text{Tr}(\hat{I}_k P P^H) \leq P_{S}^\text{max}, \forall k \in \{1, \cdots, K\}$$

$$\text{Tr}(T_2) \leq P_{R}^\text{max}.~~~(31)$$

Performing some mathematical transformations, the problem in (31) is rewritten as

$$\min_{P} \|P^H A_p^{\frac{1}{2}} - B_p (A_p^{-\frac{1}{2}})^H\|_F$$

s.t. $P = \text{BLKdiag}\{P_1, \cdots, P_K\}$,

$$\|P^H \hat{I}_k P P^H\|_F \leq \sqrt{P_{S}^\text{max}}, \forall k \in \{1, \cdots, K\}$$

$$\|P^H H_{UP}^H W_R^H\|_F \leq \sqrt{P_{S}^\text{max}} - \sigma_R^2 \text{Tr}(W_R W_R^H),$$ \hspace{1cm} (32)

where $B_p = W_S T_2 H_{UP}$, $\hat{I}_k = \hat{I}_k^\frac{1}{2} (\hat{I}_k^\frac{1}{2})^H$, and

$$A_p = A_p^{\frac{1}{2}} (A_p^{-\frac{1}{2}})^H$$

$$\mu I_{N_S} + H_{UP}^H W_R^H (\mu I_{N_R} + H_{RB}^H G_S^H W_S G_S H_{RB}) W_R H_{UP}$$

\text{diagonalization operation in Section III-A and the WMMSE.}

\textbf{Algorithm 1:} Proposed iterative algorithm for EE maximization of the uplink multi-user relay channel.

\textbf{Initialize:} The multi-user beamforming $P^{(0)}$ satisfying\hspace{1cm} $\text{Tr}(I_k P P^H) \leq P_{S}^\text{max}, \forall k \in \{1, \cdots, K\}$;\hspace{1cm} $\text{The relay processing matrix } W_R$;\hspace{1cm} $\text{The parameter } \mu^{(0)}$;\hspace{1cm} $\text{The outer iteration index } \alpha = 0.$

1: \textbf{repeat} \hspace{1cm} 2: \text{Calculate } W_R^{(\alpha+1)} \text{ by (23), given } P^{(\alpha)} \text{ and } \mu^{(\alpha)}.$

3: \text{Obtain } P^{(\alpha+1)} \text{ by solving (26) using the WMMSE method, given } W_R^{(\alpha+1)} \text{ and } \mu^{(\alpha)}.$

4: \text{Calculate } \mu^{(\alpha+1)} \text{ by (34), given } W_R^{(\alpha+1)} \text{ and } P^{(\alpha+1)}.$

5: \text{Set } \alpha = \alpha + 1.$

6: \text{until } F (\mu^{(\alpha)}, P^{(\alpha)}) \approx 0.$

\textbf{Output:} The maximum EE $\mu$, the optimal multi-user precoder $P$, and the relay precoder $W_R.$

Clearly, the problem in (32) is a convex optimization problem with the Frobenius norm based objective function and constraints. Particularly, it can be transformed into the standard convex quadratically constrained quadratic programming (QCQP) problem utilizing the identity $\|\text{vec}(A)\|_F = \|A\|_F$, which is

$$\min_{P} \|\text{vec}(P^H A_p^{\frac{1}{2}} - B_p (A_p^{-\frac{1}{2}})^H)\|^2$$

s.t. $P = \text{BLKdiag}\{P_1, \cdots, P_K\}$,

$$\|\text{vec}(P^H \hat{I}_k P P^H)\|^2 \leq P_{S}^\text{max}, \forall k \in \{1, \cdots, K\}$$

$$\|\text{vec}(P^H H_{UP}^H W_R^H)\|^2 \leq P_{R}^\text{max} - \sigma_R^2 \text{Tr}(W_R W_R^H).$$ \hspace{1cm} (33)

Notably, we find that the problem given by (33) is convex with respect to $P$ for fixed $W_S$, $G_S$, $W_R$, and $\mu$. Therefore, the problem given by (33) can be efficiently solved by the interior point method.

In conclusion, when the relay precoder $W_R$ is fixed, the optimal multi-user precoder $P$ aiming at maximizing the EE of the uplink multi-user relay channel can be obtained by the WMMSE method. As shown in [25], the WMMSE method converges to a stationary solution to the EE maximization problem in (26) with a low complexity.

\textbf{C. Optimization of } $\mu$

Once both the relay precoder $W_R$ and the multi-user precoder $P$ are optimized for a given parameter $\mu$, we utilize the Dinkelbach’s algorithm [15], [26] to perform the iterative update of $\mu$ until $F (\mu, P) \approx \hat{F} (\mu, W_S, G_S) \approx 0$ is reached. It is noted that with the Dinkelbach’s algorithm, the positive $\mu$ satisfying $F (\mu, P) \approx 0$ can be derived with a superlinear convergence. Specifically, the iterative update of $\mu$ is performed using the Newton descent method, which is shown as (34) at the top of next page, where $\alpha$ is the iteration number. Overall, by substituting the updated $\mu$ into (17) and (26), an iterative optimization between $W_R$ and $P$ is performed based on the
method in Section III-B, respectively. The detailed iterative optimization process is presented in Algorithm 1. Besides, the convergence of the proposed Algorithm 1 is characterized in the following theorem.

**Theorem 2.** The proposed Algorithm 1 converges to a locally optimal solution of the problem (10).

**Proof.** Please see the Appendix III.

So far we have solved the EE maximization problem under the assumption that the perfect CSI is available. We note that in practical system, especially at the transmitter, the perfect CSI is very difficult to obtain, due to several factors such as estimation, quantization, and feedback errors. Therefore, we focus on the imperfect CSI case in the next section.

**D. The computational complexity analysis**

It is noted that the computational complexity of the proposed Algorithm 1 mainly comes from the multi-user precoder optimization utilizing the WMMSE method. With the WMMSE method, all users’ precoder $P$, weighting matrices $W_S$ or linear equalizers $G_S$ can be updated simultaneously due to the decoupled updating steps among users when any other two types variables are fixed. Generally, the complexity of matrix inversion operation is $O(N_{dim}^3)$, where $N_{dim}$ denotes the matrix dimension. Besides, due to the fact that the QCQP problem is a special case of the standard SOCP problem, according to the literature [34], the corresponding computational complexity is $O(N_{socp}M_{socp}^3 + N_{socp}^3 M_{socp}^2)$ log(1/ε), where $M_{socp}$ and $N_{socp}$ denote the number of second order cone constraints and the dimension of the second order cone, respectively. Based on above analysis, we can approximately derive the per-iteration complexity of the WMMSE method for multi-user precoder optimization as

$$O_W = O(N_{dim}^3) + O_s(d_s N_{s} K^{3.5} + (d_s N_{s} K)^{2.5} K + d_s N_{R} + (d_s N_{R})^3) \log(1/\epsilon),$$

(35)

Thus the computational complexity of the proposed Algorithm 1 is derived as $(I_{inc} + I_{out}) \cdot O_W$, where $I_{inc}$ and $I_{out}$ denote the maximum iteration number of the WMMSE method and Algorithm 1, respectively.

**IV. EE Maximization Under Imperfect CSI**

In this section, we aim to achieve the maximum EE of the uplink multi-user MIMO relay channel under the imperfect CSI case. Specifically, we assume that the relay has the imperfect knowledge of the instantaneous CSI from the relay to the BS. We also assume that the relay has the perfect knowledge of the instantaneous CSI from the users to the relay, since it is easy for the receiver to obtain perfect CSI in practice. The assumption of CSI availability made in this section is widely adopted in literature, e.g., [32].

Based on the adopted assumption, we model the imperfect channel matrix from the relay to the BS as

$$H_{RB} = \tilde{H}_{RB} + \Delta_{RB},$$

(36)

where $\tilde{H}_{RB}$ represents the nominal channel and $\Delta_{RB}$ represents the channel uncertainty. In general, two common imperfect channel models are adopted to characterize the channel uncertainty. One formulates $\Delta_{RB}$ as a deterministic matrix with the bounded norm, while the other one considers $\Delta_{RB}$ as a statistical model of the unknown parameters. In our work, we only consider the norm bounded uncertainty model for the relay-to-BS channel, which is

$$\mathcal{H}_{RB} = \{\Delta_{RB}||\Delta_{RB}||_{F} \leq \epsilon_h\},$$

(37)

where $\epsilon_h$ is the channel error threshold. Based on (36) and (37), the robust multi-user and relay precoders for maximizing the EE of the uplink multi-user MIMO relay channel is designed. Mathematically, the precoder design is expressed as

$$\max_{P, W_B} \log \det (I_{N_B} + T_5)$$

s.t. $P = \text{BLKdiag}\{P_1, \cdots, P_K\}$

$$\text{Tr}(\tilde{I}_k PP^H) \leq P_{S, \max}^{\epsilon}, \forall k \in \{1, \cdots, K\},$$

$$\text{Tr}(T_2) \leq P_{R, \max}^{\epsilon},$$

(38)

where $T_5$ is obtained by replacing $H_{RB}$ with $H_{RB} + \Delta_{RB}$ in (7). Naturally, the optimal solutions to the problem given by (38) is robust in the sense that the EE is guaranteed in the presence of CSI perturbation. However, the problem in (38) is non-convex and difficult to solve from the global perspective. Therefore, we resort to the WMMSE method [25] to solve this problem effectively.

By applying the WMMSE method, the achievable worst-case rate in the imperfect CSI case is expressed as

$$\min_{\Delta_{RB}} R_{sum}$$

$$= \min_{\Delta_{RB}} \max_{w_S, g_S} \log \det (W_S) - \text{Tr}(W_S M(G_S)) + \text{Tr}(I_\delta).$$

(39)

Then the robust EE maximization problem in (38) is reformulated as

$$\max_{P} \frac{\min_{\Delta_{RB}} R_{sum}}{\text{Tr}(PP^H) + \text{Tr}(T_2) + P_C}$$

s.t. $P = \text{BLKdiag}\{P_1, \cdots, P_K\}$

$$\text{Tr}(\tilde{I}_k PP^H) \leq P_{S, \max}^{\epsilon}, \forall k \in \{1, \cdots, K\},$$

$$\text{Tr}(T_2) \leq P_{R, \max}^{\epsilon}.$$}

(40)

Similar to the optimization of $P$ in Section III-B, we first apply Theorem 1 to transform the problem in (40) into an equivalent parametric subtractive problem, which gives

$$\max_{w_S, g_S, p} \min_{\Delta_{RB}} \tilde{F}(\mu, \Delta_{RB})$$

s.t. $P = \text{BLKdiag}\{P_1, \cdots, P_K\}$

$$\text{Tr}(\tilde{I}_k PP^H) \leq P_{S, \max}^{\epsilon}, \forall k \in \{1, \cdots, K\},$$

$$\text{Tr}(T_2) \leq P_{R, \max}^{\epsilon},$$

(41)
where
\[
\tilde{F}(\mu, \Delta_{RB}) = \log \det (W_S) - \log \det (W_{SM}(G_S)) + \log \det (I_d)
\]
\[- \mu (\log \det (P^{PP^H}) + \log \det (T_2)) + P_C. \tag{42}
\]

Then by introducing an arbitrary worst-case optimization variable \(\varsigma\) into (41), the epigraph form of the problem in (41) is given by
\[
\begin{align*}
\max_{W_S, G_S, \varsigma} &
\varsigma
\quad \text{s.t. } \varsigma \leq \tilde{F}(\mu, \Delta_{RB}), \forall \|\Delta_{RB}\| \leq \epsilon_h, \\
\text{P} & = \text{BLKdiag}\{P_1, \ldots, P_K\}, \\
\text{Tr} \left( \tilde{I}_k PP^H \right) & \leq P_{\max}, \forall k \in \{1, \ldots, K\}, \\
\text{Tr} (T_2) & \leq P_{\max}. \tag{43}
\end{align*}
\]

To address the problem in (43), we first perform the following mathematical transformations:
\[
\begin{align*}
\text{Tr} \left( \tilde{I}_k PP^H \right) &= \left\| \left( \tilde{I}_k \right)^H P \right\|_F^2, \forall k \in \{1, \ldots, K\}, \\
\text{Tr} (T_2) &= \left\| W_R \text{H}_{UP} P \right\|_F^2 + \sigma_R^2 \left\| W_R \right\|_F^2, \tag{44}
\end{align*}
\]
and
\[
\begin{align*}
\text{Tr} (W_{SM}(G_S)) &= \left\| P^H \text{H}_{UP} T_6^H W_S \frac{1}{2} - W_S \frac{1}{2} \right\|_F^2 + \sigma_R^2 \left\| \text{H}_G W_S \frac{1}{2} \right\|_F^2, \tag{45}
\end{align*}
\]
where \(T_6 = G_S \left( \text{H}_{RB} + \Delta_{RB} \right) W_R\). We take (45) as an example. From (45) we see that the optimization variables \(W_S, G_S, P\), and the CSI bounded uncertainty \(\Delta_{RB}\) are coupled, which undoubtedly makes the problem in (43) difficult to solve. In order to separate \(\Delta_{RB}\) from (45), we rewrite (45), (44), and (45) as the norm-squared vectors. Specifically, we rewrite (45) as
\[
\text{Tr} (T_2) = \left\| \frac{\text{vec} (W_R \text{H}_{UP} P)}{\sigma_R \text{vec} (W_R)} \right\|_2^2, \tag{47}
\]
and rewrite (45) as (48) at the top of next page. Furthermore, based on the identity \(\text{vec} (ABC) = (C^T \otimes A) \text{vec} (B)\), we re-express \(m\) in (48) as a combination of a deterministic term \(\bar{m}\) and a vectorial CSI uncertainty term \(\text{vec} (\Delta_{RB})\), which is
\[
\begin{align*}
\bar{m} &= \bar{m} + \text{vec} (\Delta_{RB}) = \bar{m} + M \text{vec} (\Delta_{RB}) \\
&= \left[ \begin{array}{c} \text{vec} \left( \text{H}_{UP} \text{H}_{RB} G_{S}^H W_S^\frac{1}{2} \right) - \text{vec} (W_S^\frac{1}{2}) \\
\sigma_R \text{vec} \left( \text{H}_{RB}^H G_S^H W_S^\frac{1}{2} \right) \\
\sigma_S \text{vec} (G_S^H W_S^\frac{1}{2}) \\
\left( \text{G}_S^H W_S^\frac{1}{2} \right)^T \otimes \text{H}_{RB}^H \text{H}_{UP}^H W_R \\
\text{vec} (\Delta_{RB}). \end{array} \right. \tag{49}
\end{align*}
\]
Combining (47) and (48) with (49), the EE-related constraint \(\varsigma \leq \tilde{F}(\mu, \Delta_{RB})\) in (43) is rewritten as
\[
\begin{align*}
\| \bar{m} \|_2^2 + \mu \| \text{vec} (P) \|_2^2 + \mu \left[ \begin{array}{c} \| \text{vec} (W_R \text{H}_{UP} P) \|_2^2 \\
\sigma_R \text{vec} (W_R) \end{array} \right. \\
\leq \log \det (W_S) + \text{Tr} (I_d) - \mu P_C - \varsigma. \tag{50}
\end{align*}
\]
By realigning the three terms on the left hand side of (50), we obtain
\[
\| \bar{m} \|_2^2 \leq \log \det (W_S) + \text{Tr} (I_d) - \mu P_C - \varsigma, \tag{51}
\]
where
\[
\bar{m} = \left[ \begin{array}{c} m \sqrt{\mu \text{vec} (P)} \\
\sqrt{\mu \text{vec} (W_R \text{H}_{UP} P)} \\
\sqrt{\mu \text{vec} (W_R \text{H}_{RB} G_S^H W_S^\frac{1}{2}) - \text{vec} (W_S^\frac{1}{2})} \\
\sigma_R \text{vec} (W_R) \\
\| \text{vec} (\Delta_{RB}) \|_2 \end{array} \right]. \tag{52}
\]
and \(L_1 = \tilde{N}_d + N_d \bar{d} + N_R^2\). Then we promote the SDP formulation for the problem in (43). To this end, we first apply the schur complement lemma \([33]\) to recast the formulation (51) as a linear matrix inequality (LMI), which is
\[
\begin{align*}
\log \det (W_S) + \text{Tr} (I_d) - \mu P_C - \varsigma \leq 0, \tag{53}
\end{align*}
\]
\[ \text{Tr}(W_S M(G_S)) = \|m\|^2_2 \leq \left\| \begin{bmatrix} \text{vec} \left( P^H H_{WP}^H W_R^H \left( H_{RB} + \Delta_{RB} \right) G_S^H W_S^2 \right) - \text{vec} \left( W_S^2 \right) \\ \sigma_R \text{vec} \left( W_R^H \left( H_{RB} + \Delta_{RB} \right) G_S^H W_S^2 \right) \\ \sigma_S \text{vec} \left( G_S^H W_S^2 \right) \end{bmatrix} \right\|^2_2 . \] (48)

where \( L_2 = \left( N_B + \tilde{N}_S + 2N_R \right) \hat{d} + N_R^* \hat{d}^2 \). Then we substitute (52) into (53) to obtain

\[ \log \det(W_S) + \text{Tr}(I_d) - \mu P_C - \varsigma \begin{bmatrix} m^H m \\ I_{L_2} \end{bmatrix} \geq \begin{bmatrix} 0 \\ -M \text{vec} \left( \Delta_{RB} \right) \end{bmatrix} \]. (54)

Unfortunately, the revised LMI in (54) is semi-infinite and intractable, due to the existence of the channel bounded uncertainty \( \Delta_{RB} \). Therefore, we utilize the sign-definiteness lemma to deal with this semi-infinite constraint, which is presented in the following \textbf{Theorem 3} in Appendix II.

By properly determining the matrix parameters in \textbf{Theorem 3} as

\[ Z_1 = \begin{bmatrix} \log \det(W_S) + \text{Tr}(I_d) - \mu P_C - \varsigma \begin{bmatrix} m^H m \\ I_{L_2} \end{bmatrix} \\ 0 \begin{bmatrix} m^H m \\ I_{L_2} \end{bmatrix} \end{bmatrix}, \] (55)

\[ P_1 = \begin{bmatrix} 0_{N_B N_R \times 1}, -M^H \end{bmatrix}, \] (56)

\[ X_1 = \text{vec} \left( \Delta_{RB} \right), \] (57)

and

\[ Q_1 = [1, 0_{1 \times L_2}], \] (58)

the LMI in (54) is equivalent to

\[ Z_1 - \xi_1 Q_1^H Q_1 - \epsilon_1 P_1 \geq 0. \] (59)

Accordingly, the robust EE maximization problem in (40) is transformed into (60), which is shown at the top of the next page. Obviously, the problem in (60) is not semi-infinite. However, it is still non-convex due to the coupled optimization variables. As a result, we propose an alternative algorithm to optimize the variables \( \varsigma, P, W_R, W_S, \) and \( G_S \) iteratively. Specifically, in the outer optimization, the parameter \( \mu \) is optimized by utilizing the bisection method. In this method, the lower bound on \( \mu \) is 0, i.e., \( \mu^{\text{lb}} = 0 \), while the upper bound \( \mu^{\text{ub}} \) is calculated by the proposed algorithm utilizing the nominal channel \( \bar{H}_{RB} \) in Section III. Then in the inner optimization, we optimize the variables \( \varsigma, P, W_R, W_S, \) and \( G_S \) by firstly fixing a subset of the optimization variables, which reduces the problem in (60) to a convex SDP problem in the remaining variables. Then the standard convex optimization technique is applied to solve the problem in (60) effectively. We note that the same operation is performed for the optimization of other variables. In general, this iterative optimization process continues until the desired convergence is reached. The detailed iterative optimization process is presented in Algorithm 2.

\section*{V. Numerical Results}

In this section, we present numerical results to examine the EE performance achieved by our proposed iterative algorithms in both the perfect and imperfect CSI cases. Specifically, we consider the uplink multi-user MIMO relay channel with \( K = \{6, 8\}, N_S = \{2, 3\}, N_R = \{4, 6\}, N_B = 8, \) and \( d_s = 2 \). We assume that all channel coefficients are independently and identically complex Gaussian distributed with zero mean and unit variance. We also assume that the power of all received additive Gaussian noise is \( \sigma^2_{\text{INR}} = \sigma^2_{\text{INR}} = 1 \). Moreover, we clarify that the circuit power consumption \( P_C \) is mainly composed of the constant power consumption of all user transmit antennas, which is generally smaller than the user total transmit power and chosen proportionally to the number of transmit antennas according to [29]. Furthermore, we utilize the widely-adopted software toolbox CVX [35] to solve the standard convex optimization problems efficiently. In addition, we clarify that each point in the simulation results is obtained by averaging over 500 independent Monte Carlo trials [36].

\subsection*{A. Perfect CSI}

In this subsection, we consider the perfect global CSI of the uplink multi-user MIMO relay channel. Under this
Algorithm 2 Proposed algorithm for robust EE maximization of the uplink multi-user relay channel with imperfect CSI.

**Initialize:** The multi-user precoder \( \mathbf{P}(0) \) satisfying \( \text{Tr} \left( \mathbf{I}_k \mathbf{P} \mathbf{P}^H \right) \leq P_S^{\text{max}}, \forall k = 1, \cdots, K \);

- The weighting matrix \( \mathbf{W}_S^{(0)} \);
- The linear equalizer \( \mathbf{G}_S^{(0)} \);
- The arbitrary scalar \( \varsigma(0) \);
- The upper and lower bounds on \( \mu \), \( \mu^{\text{ub}} \) and \( \mu^{\text{lb}} \);
- The iteration index \( \kappa = 0 \).

1: repeat
2: \( \mu = \frac{1}{2} (\mu^{\text{ub}} + \mu^{\text{lb}}) \) and \( \kappa = 0 \).
3: repeat
4: Given fixed \( \varsigma^{(\kappa)}, \mathbf{W}_R^{(\kappa)}, \mathbf{W}_S^{(\kappa)}, \mathbf{G}_S^{(\kappa)} \), solve (60) to find \( \mathbf{P}^{(\kappa+1)} \).
5: Given fixed \( \varsigma^{(\kappa)}, \mathbf{P}^{(\kappa+1)}, \mathbf{W}_S^{(\kappa)}, \mathbf{G}_S^{(\kappa)} \), solve (60) to find \( \mathbf{W}_R^{(\kappa+1)} \).
6: Given fixed \( \mathbf{P}^{(\kappa+1)}, \mathbf{W}_R^{(\kappa+1)}, \mathbf{W}_S^{(\kappa+1)} \), solve (60) to find \( \mathbf{G}_S^{(\kappa+1)} \) and \( \varsigma^{(\kappa+1)} \).
7: if (the problem in (60) is feasible) then
8: \( \mu^{\text{lb}} = \mu \),
9: else
10: \( \mu^{\text{ub}} = \mu \).
11: end if
12: \( \kappa = \kappa + 1 \).
13: until \( |\varsigma^{(\kappa+1)} - \varsigma^{(\kappa)}| \leq \varepsilon \), where \( \varepsilon \) is a sufficiently small positive number.
14: Output: The maximum EE \( \mu \), the optimal multi-user precoder \( \mathbf{P} \), and the relay precoder \( \mathbf{W}_R \).

consideration, we compare the maximum EE achieved by our proposed iterative algorithm, i.e., Algorithm 1, with that achieved by the benchmark algorithm, i.e., the SINR-scalarized algorithm proposed in [14]. In the SINR-scalarized algorithm, the maximum EE beamforming is determined based on the scalarized SINR formulation, without exploring the multi-antenna diversity at each node.

Fig. 2 plots the achievable maximum EE versus the number of iteration for different \( N_S \) and \( N_R \). First, we observe that our proposed algorithm converges within almost 7 iterations, demonstrating that our algorithm has a good convergence rate. Second, we observe that our proposed algorithm achieves a much higher EE than the benchmark algorithm regardless of \( N_S \) and \( N_R \), since the multi-antenna diversity or multiplexing gain is exploited. This observation also demonstrates the performance advantage of our algorithm over the existing algorithm. Third, we observe that the increasing \( N_S \) and \( N_R \) naturally improves the achievable maximum EE, which is attributed to the increasing multi-antenna multiplexing gain.

Fig. 3 plots the achievable maximum EE versus the maximum user transmit power, \( P_S^{\text{max}} \), for different \( N_S \) and \( N_R \). From this figure, we first observe that for both two algorithms, the achievable maximum EE increases when \( P_S^{\text{max}} \) increases and finally becomes saturate, which is due to the adopted amplify-and forward (AF) strategy at relay. Specifically, with the AF strategy, both the received noise and desired signals at the relay are amplified and forwarded to the BS, leading to the fact that the received SINR at the BS is not arbitrarily high even with the sufficiently large user transmit power \( P_S^{\text{max}} \).

Again, we observe that our proposed algorithm achieves a higher EE than the benchmark algorithm, regardless of \( N_S \)
and $N_R$. Furthermore, we confirm that the similar observations can be made if we plot the achievable maximum EE versus the maximum relay transmit power, $P_R^{\max}$, the figure of which is omitted due to space consideration.

Fig. 4 plots the achievable maximum EE versus the circuit power consumption, $P_C$, for different $N_S$ and $N_R$. We observe that the maximum EE achieved by our proposed algorithm decreases when $P_C$ increases. This is due to the fact that the increasing $P_C$ causes higher energy consumption, but it does not improve the achievable rate of the uplink multi-user MIMO relay channel, thus impairing the EE optimization of the uplink multi-user MIMO relay channel. We again observe the EE advantage achieved by our proposed algorithm over the benchmark algorithm, which does not depend on $N_S$ or $N_R$.

In this section, we consider the imperfect CSI between the relay and the BS. Under this consideration, we choose the non-robust algorithm as the benchmark algorithm and demonstrate the performance advantage of our proposed robust algorithm, i.e., Algorithm 2, over it.

Fig. 5 plots the achievable maximum EE versus the number of iteration for different values of the channel error threshold, $\epsilon_h$. We observe that our proposed robust algorithm converges within almost 7 iterations, showing a good convergence rate. Also, we observe that the maximum worst-case EE achieved by our robust algorithm decreases when $\epsilon_h$ increases, since the larger channel error is optimized by the Algorithm 2. Furthermore, we observe that the maximum worst-case EE achieved by Algorithm 2 is naturally lower than the maximum EE achieved by Algorithm 1 in the perfect CSI case, which meets our expectation.
Fig. 6 plots the achievable maximum EE versus the maximum user transmit power, $P_{S}^{\text{max}}$, for different algorithms. It is clear that the achievable maximum EE of all three algorithms first increases and then stays nearly constant when $P_{S}^{\text{max}}$ increases, which is also attributed to the adopted AF strategy at relay. Second, we observe that the maximum worst-case EE achieved by our proposed robust algorithm is higher than that of the non-robust algorithm, demonstrating the effectiveness of Algorithm 2. Third, as expected we observe that the maximum worst-case EE is lower than the maximum perfect EE.

VI. CONCLUSION

In this paper, we proposed novel algorithms to maximize the EE of the uplink multi-user MIMO relay channel by jointly optimizing the multi-user precoder and the relay precoder. Notably, our algorithms were proposed for both the perfect global CSI case and the imperfect relay-to-BS CSI case. To address the non-convexity of the formulated EE optimization problems in both cases, we proposed iterative algorithms and the corresponding parametric subtractive function to optimize the multi-user precoder and the relay precoder. With the aid of numerical results, we demonstrated the good convergence rate of our proposed algorithms. We also demonstrated that our proposed algorithms achieve a higher EE gain than the existing algorithms in both cases. We further evaluated the impact of the transmit power, the circuit power consumption, and the CSI error threshold on this EE gain.

APPENDIX I

Lemma 1. We consider $\text{Tr}(AB)$ in which both $A$ and $B$ are Hermitian matrices with a proper dimension. It is demonstrated in [37] that $\text{Tr}(AB)$ is minimized when the eigenvalues of $A$ and $B$ are arranged in the opposite order and they are commute, i.e., $AB = BA$.

Lemma 2. We consider $\log |I_{N} + A^{-1}B|$ in which we have the Hermitian matrices $A \in \mathbb{C}^{N \times N}$ and $B \in \mathbb{C}^{N \times N}$ is a positive constant. It is demonstrated in [21] that $\log |I_{N} + A^{-1}B|$ can be minimized when the eigenvalues of $A$ and $B$ are arranged in the same order and they are commute, i.e., $AB = BA$.

APPENDIX II

Theorem 1. A fractional function $f(R) = \frac{M(R)}{N(R)}$ is a quasi-concave function if $M(R)$ is a concave function and $N(R)$ is a linear function with respect to the matrix variable $R$, respectively. Moreover, by defining the parametric subtractive function $F(\mu)$ as

$$F(\mu) = \max_{X} \{M(X) - \mu N(X)\},$$

where $\mu$ is an arbitrary scalar, we confirm that $F(\mu)$ is a convex and monotonically decreasing function. For the determined $\mu$, we can calculate $F(\mu)$ via the standard convex optimization method. In fact, the problem of maximizing the quasi-concave fractional function $f(R)$ is equivalent to finding the zero point of $F(\mu)$ with positive $\mu$.

**Proof:** Please refer to [30], [31].

**Theorem 3.** Considering the Hermitian matrix $Z$ and the arbitrary matrices $\{P_j, Q_j\}$, $j = 1, \ldots, K$, the semi-infinite LMI having the following form

$$Z \geq \sum_{j=1}^{K} (P_j^H X_j Q_j + Q_j^H X_j^H P_j), \quad \|X_j\| \leq \epsilon_j, \forall j$$

(62)

holds if and only if there exists nonnegative numbers $\xi_1, \ldots, \xi_K \in \mathbb{R}$, such that

$$\begin{pmatrix}
Z - \sum_{j=1}^{K} \xi_j Q_j^H Q_j & -\psi_1 P_1^H & \cdots & -\psi_K P_K^H \\
-\psi_1 P_1 & \xi_1 I & & \\
\vdots & & \ddots & \\
-\psi_K P_K & & \cdots & \xi_K I
\end{pmatrix} \succeq 0$$

(63)

is established.

**Proof:** Please refer to [27].

APPENDIX III

Firstly, according to **Theorem 1**, we define the equivalent objective function of problem (10) as

$$\mathcal{J}(P, W_R) = \log \det (I_{N_R} + T_1) - \mu(\text{Tr}(PP^H)) + \text{Tr}(T_2 + P_C),$$

(64)

where $\mu$ denotes a positive constant. It is noted that when the value of $\mathcal{J}(P, W_R)$ approaches zero, the maximum EE $\eta$ of the uplink multi-user MIMO relay channel is achieved. Furthermore, we assume the obtained optimal solutions in the $t$-th iteration of the proposed iterative algorithm as $\{P^{(t)}, W_R^{(t)}\}$. Given $P^{(t)}$, the optimal $W_R^{(t+1)}$ in the $(t+1)$-iteration is derived by solving the problem (17), whose objective function indicates the optimal structure of $\mathcal{J}(P, W_R)$. Since the problem (17) is a standard convex optimization problem with the globally optimal solution, we definitely have

$$\mathcal{J}(P^{(t)}, W_R^{(t+1)}) \geq \mathcal{J}(P^{(t)}, W_R^{(t)}),$$

(65)

Besides, when $W_R^{(t+1)}$ is fixed, the $P$ related optimization problem (26) is nonconvex and difficult to solve globally. Thus the WMMSE method is applied to transform the problem (26) into a series of subproblems. According to [25], the convergence of the WMMSE method has been demonstrated. Specifically, the WMMSE method is guaranteed to converge to a stationary point of (26). In fact, in our work, the actual optimization objective for multi-user precoder $P$ is

$$\mathcal{J}(P, W_R^{(t+1)}) = \log \det (W_S) - \text{Tr}(W_S M(G_S)) + \text{Tr}(I_d) - \mu(\text{Tr}(PP^H)) + \text{Tr}(T_2 + P_C),$$

(66)

which is equivalent to $\mathcal{J}(P, W_R)$ due to the equivalent relationship between the SINR and the weighted MSE. Considering that the current solutions $\{P^{(t)}, W_R^{(t+1)}\}$ is also feasible for (26), the obtained $P^{(t+1)}$ by solving (26) based on $W_R^{(t+1)}$ naturally satisfies

$$\mathcal{J}(P^{(t+1)}, W_R^{(t+1)}) \geq \mathcal{J}(P^{(t)}, W_R^{(t+1)}),$$

(67)
Furthermore, we combine (65) and (67) to obtain
\[ H(P^{(t+1)}, W_R^{(t+1)}) \geq H(P^{(t)}, W_R^{(t)}) \]
\[ \vdots \]
\[ \geq H(P^{(1)}, W_R^{(1)}) \]
\[ \geq H(P^{(0)}, W_R^{(0)}) , \]
(68)
which indicates that the objective function \( H(P, W_R) \) is non-decreasing in the iterative process of the proposed Algorithm 1.

To demonstrate the convergence of the proposed Algorithm 1, we also need to prove that \( H(P, W_R) \) tends to keep invariant with the increasing number of iterations. Firstly, we focus on the upper bound of \( H(P, W_R) \) as
\[ H(P, W_R) \leq \log \det(I_{N_B} + T_1) \]
\[ \leq \frac{1}{2} \max_{P, W_R} \log \det(I_{N_B} + \alpha_B^2 R_{BR} W_R H_{R,P} P P^H H_{R,P}^H W_R H_{R,B}^H) \]
(69)
where the last inequality actually indicates the achievable maximum rate of the uplink multi-user MIMO relay channel by neglecting the interference term. In fact, the achievable maximum rate can be derived utilizing the diagonal structure optimization in [21]. Overall, by combining the non-decreasing and upper bounded characteristics of the proposed Algorithm 1, it is concluded that the proposed Algorithm 1 converges. We can also prove that the proposed Algorithm 1 converges to a locally optimal solution, which is assumed as \( \{P^*, W_R^*\} \). Specifically, with the proposed Algorithm 1, the \( W_R \) related subproblem is a standard convex optimization problem. According to the gradient descent theory, the optimal \( W_R^* \) satisfies
\[ \text{Tr} (\nabla_{W_R} H(P^*, W_R)(P - W_R)) \leq 0 , \]
(70)
where \( \nabla_{W_R} H(P, W_R) \) denotes the gradient of function \( H \) with respect to \( W_R \). Besides, for the optimization of \( P \) utilizing the WMMSE method, the obtained \( P^* \) is also a stationary point of (26) as aforementioned, which satisfies
\[ \text{Tr} (\nabla_{P} H(P^*, W_R^*)(P - P^*)) \leq 0 , \]
(71)
in a certain feasible region of (10).

By adding (70) and (71), we find that for the auxiliary variable \( V = \{P, W_R\} \), \( \text{Tr} (\nabla_{V} H(V))(V - V^*) \leq 0 \) holds. Besides, due to the fact that the function \( H \) is continuous, we conclude that for a sufficiently small number \( \zeta > 0 \), the inequality \( \zeta H(V^*) \geq H(V) \) holds for any \( V \in \{V \mid \nabla V \cap \{V - V^*\} \leq \zeta \} \), where \( \{V \} \) denotes the feasible region of problem (10). As a result, the converged point \( V^* = \{P^*, W_R^*\} \) can be considered to be a locally optimal solution to the problem (10). So far, the proof for Theorem 2 is completed.

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Shiqi Gong (S’14) received the B.S. degree in Electronic Engineering in 2014 from Beijing Institute of Technology (BIT). Currently, she is pursuing the Ph.D. degree with the School of Electronic and Information, Beijing Institute of Technology. Her research interests are in the area of signal processing, physical-layer security, resource allocation, convex optimization.

Chengwen Xing (S’08–M’10) received the B.Eng. degree from XiDian University, Xian, China, in 2005 and the Ph.D. degree from the University of Hong Kong, Hong Kong, in 2010. Since September 2010, he has been with the School of Information and Electronics, Beijing Institute of Technology, Beijing, China, where he is currently an Associate Professor. From September 2012 to December 2012, he was a visiting scholar at the University of Macau. His current research interests include statistical signal processing, convex optimization, multivariate statistics, combinatorial optimization, massive MIMO systems, and high frequency band communication systems. Dr. Xing is an Associate Editor for the IEEE Transactions On Vehicular Technology, KSII Transactions on Internet and Information Systems, Transactions on Emerging Telecommunications Technologies, and China Communications.

Nan Yang (S’09–M’11) received the B.S. degree in electronics from China Agricultural University in 2005, and the M.S. and Ph.D. degrees in electronic engineering from the Beijing Institute of Technology in 2007 and 2011, respectively. He has been with the Research School of Engineering at the Australian National University since July 2014, where he currently works as a Future Engineering Research Leadership Fellow and a Senior Lecturer. Prior to this, he was a Postdoctoral Research Fellow at the University of New South Wales from 2012 to 2014 and a Postdoctoral Research Fellow at the Commonwealth Scientific and Industrial Research Organization from 2010 to 2012. He received the Exemplary Reviewer Award for the IEEE TRANSACTIONS ON COMMUNICATIONS in 2015 and 2016, the Top Reviewer Award from the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY in 2015, the IEEE ComSoc Asia-Pacific Outstanding Young Researcher Award and the Exemplary Reviewer Award of the IEEE WIRELESS COMMUNICATIONS LETTERS in 2014, and the Exemplary Reviewer Award of the IEEE COMMUNICATIONS LETTERS in 2013 and 2012. He is also a co-recipient of the Best Paper Awards from the IEEE GlobeCOM 2016 and the IEEE VTC 2013-Spring. He is currently serving in the Editorial Board of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, and TRANSACTIONS ON EMERGING TELECOMMUNICATIONS TECHNOLOGIES. His general research interests lie in the areas of communications theory and signal processing, with specific interests in massive multi-antenna systems, millimeter wave communications, cyber-physical security, and molecular communications.

Yik-Chung Wu received the B.Eng. (EEE) degree in 1998 and the M.Phil. degree in 2001 from the University of Hong Kong (HKU). He received the Croucher Foundation scholarship in 2002 to study Ph.D. degree at Texas A & M University, College Station, and graduated in 2005. From August 2005 to August 2006, he was with the Thomson Corporate Research, Princeton, NJ, as a Member of Technical Staff. Since September 2006, he has been with HKU, currently as an Associate Professor. He was a visiting scholar at Princeton University, in summers of 2011 and 2015. His research interests are in general areas of signal processing, machine learning and communication systems, and in particular distributed signal processing, Bayesian inference, and robust optimization. Dr. Wu served as an Editor for IEEE Communications Letters, is currently an Editor for IEEE Transactions on Communications and Journal of Communications and Networks.

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Zesong Fei (S’03-M’07) received the Ph.D. degree in Electronic Engineering in 2004 from Beijing Institute of Technology (BIT). He is now an Associate Professor in BIT and currently with the Research Institute of Communication Technology (RICT) of BIT, where he is involved in the design of the next generation high-speed wireless communication. His research interests include mobile communication, channel coding and modulation, cognitive radio and cooperative networking. He was chief investigator of China national Natural Science Fund project. He is the senior member of Chinese Institute of Electronics and China Institute of Communications.